# Midterm#3 Review - Part2

Friday, October 16, 2020 2:22 PM

Exp2) Calculate $f'(x)$ when $f(x)=\left(\frac{1}{2\sqrt{x}}+\frac{x^2}{4}-\pi^{2/3}\right)$

$$f'(x)=\left(\frac{1}{2\sqrt{x}}\right)'+\left(\frac{x^2}{4}\right)'+\underbrace{\left(-\pi^{2/3}\right)'}_{\text{constant}}$$

Recall:

$(x^n)'=n\cdot x^{n-1}$

$$=\left(\frac{1}{2}\cdot x^{-1/2}\right)'+\left(\frac{1}{4}\cdot x^2\right)+0$$

$$=\frac{1}{2}\cdot\left(-\frac{1}{2}\right)\cdot x^{-1/2-1}+\frac{1}{4}\cdot 2x$$

$$=-\frac{1}{4}\cdot x^{-3/2}+\frac{x}{2}$$

Exp2) Calculate $f'(0)$ when $f(x)=|x|$

"Find the slope of the tangent line to $f(x)$ at $x=0$"

**Solution:**

$$\boxed{f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}}=\lim_{h\to 0}\frac{|h|-0}{h}$$

$f(x)=|x|$

$a=0$

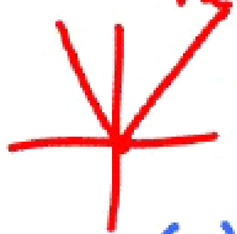
$f(a+h)=f(h)=|h|$

$f(a)=f(0)=|0|=0$

$$=\underbrace{\lim_{h\to 0}\frac{|h|}{h}}$$

$h\to 0$ LL 0 RL

Recall:



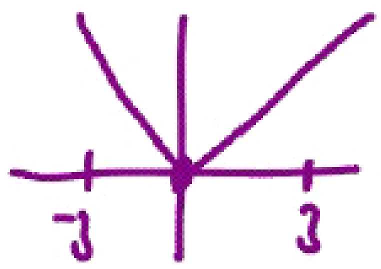
$f(x)=|x|$ $\begin{cases} f(x)=x, & x\geq 0 \\ f(x)=-x, & x<0 \end{cases}$

$$\lim_{h\to 0^-}\frac{|h|}{h}=\lim_{h\to 0^-}\frac{-h}{h}=\lim_{h\to 0}(-1)=-1$$

$$\lim_{h\to 0^+}\frac{|h|}{h}=\lim_{h\to 0^+}\frac{h}{h}=\lim_{h\to 0^+}(1)=1$$

$-1\neq 1$

$\underbrace{f'(0) \text{ DNE}}$





$f'(3)=1$

$f'(-3)=-1$

$f(x)=|x|$ is differentiable at every point EXCEPT $x=0$.

Exp1) Find the equation of the tangent line to $f(x) = x^2 - 1$ at $x = 2$.

Solution:

$f(x) = x^2 - 1$

$f(a) = f(2) = 2^2 - 1$

$f(2) = 3$ $P(2, 3)$

$f(2+h) = (2+h)^2 - 1$

$$m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$m_{tan} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$m_{tan} = \lim_{h \to 0} \frac{(4+4h+h^2-1) - 3}{h}$$

$$= \lim_{h \to 0} \left( \frac{\cancel{3}+4h+h^2-\cancel{3}}{h} \right)$$

$$= \lim_{h \to 0} \left( \frac{\cancel{h}(4+h)}{\cancel{h}} \right)$$

$$= \lim_{h \to 0} (4+h) \overset{\text{"DSP"}}{=} 4$$

Eq. of tangent line is:

$y - y_1 = m_{tan}(x - x_1)$

$$\boxed{y - 3 = 4(x - 2)}$$

Exp5) Calculate $f'(x)$ when $f(x) = e^x \cdot \cos x + \frac{\sqrt{x} \cdot \ln x}{x^3}$

$$f(x) = e^x \cdot \cos x + \frac{x^{1/2} \cdot \ln x}{x^3} = \underbrace{e^x \cdot \cos x}_{\text{product rule}} + x^{1/2 - 3} \cdot \ln x \quad \left(\frac{\ln x}{x^3} \neq \ln\left(\frac{x}{x^3}\right)\right)$$

$$f'(x) = \underbrace{e^x \cdot \cos x + e^x \cdot (-\sin x)} + \underbrace{\left(x^{-5/2} \cdot \ln x\right)'}_{\text{product rule}}$$

$$= e^x \cdot \cos x - e^x \cdot \sin x + \left(\frac{-5}{2} \cdot x^{-7/2} \cdot \ln x + x^{-5/2} \cdot \frac{1}{x}\right) \quad \left((\ln x)' = \frac{1}{x}\right)$$

$$= e^x(\cos x - \sin x) - \frac{5}{2} \cdot x^{-7/2} \cdot \ln x + x^{-7/2}$$

Recall:

$$\frac{x^{-5/2}}{x^1} = x^{-5/2 - 1} = x^{-\frac{5}{2} - \frac{2}{2}}$$
$$= x^{-7/2}$$

Exp6) Find the equation of the tangent line to $f(x)=\dfrac{x^2+5}{x+5}$ at $x=1$. Any form of equation is acceptable.

Solution: First find $m_{tan}\Big|_{x=1} = f'(1) = ?$

Then use the point $(x_1, y_1) = (1, f(1))$ to write the equation of the tangent line to $f(x)$ at $(1, f(1)) \Rightarrow$ $y - y_1 = m_{tan}(x - x_1)$

$$f'(x) = \left(\frac{x^2+5}{x+5}\right)' = \frac{(x^2+5)'(x+5) - (x^2+5)(x+5)'}{(x+5)^2}$$

$$f'(x) = \frac{2x(x+5) - (x^2+5)\cdot 1}{(x+5)^2}$$ (Do not simplify Eval. at $x=1$)

$$f'(1) = \frac{2\cdot 1(1+5) - (1^2+5)\cdot 1}{(1+5)^2}$$

$$f'(1) = \frac{2\cdot 6 - 6}{6^2} = \frac{6}{6^2} = \frac{1}{6} = m_{tan}$$

$$\left( f(x) = \frac{x^2+5}{x+5} \right) \Rightarrow f(1) = \frac{1^2+5}{1+5} = \frac{6}{6} = 1 \quad (1,1)$$

Use the point-slope form:

$m_{tan} = \frac{1}{6}$, $(1,1)$

$$y - y_1 = m_{tan}(x - x_1)$$

$$y - 1 = \frac{1}{6}(x-1)$$

Find the equation of the normal line to $f(x)$ at $x=1$.

$\underbrace{\text{normal line}}$ perpendicular to the tangent line at $x=1$.

$$m_{normal} = \frac{-1}{m_{tan}}$$

$$m_{normal} = \frac{-1}{\frac{1}{6}} = -6$$

eq. of the normal line:

$$y - 1 = -6(x-1)$$

# Midterm#3 Review Q

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**85. Finding derivatives from a table** Find the values of the following derivatives using the table.

| $x$ | 1 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|
| $f(x)$ | 3 | 1 | 9 | 7 | 5 |
| $f'(x)$ | 7 | 9 | 5 | 1 | 3 |
| $g(x)$ | 9 | 7 | 5 | 3 | 1 |
| $g'(x)$ | 5 | 9 | 3 | 1 | 7 |

**d.** $\dfrac{d}{dx}(f(x)^3)\Big|_{x=5}$

$$= \left( \underbrace{3\cdot[f(x)]^2}_{\text{power rule}} \cdot \overbrace{\underbrace{f'(x)}_{\frac{df}{dx}}}^{\text{chain rule}} \right)\Bigg|_{x=5}$$

$$= 3\,[f(5)]^2 \cdot f'(5)$$

$$= 3\cdot 9^2 \cdot 5 = 1215$$

**72–76. Tangent lines** *Find an equation of the line tangent to each of the following curves at the given point.*

**73.** $y = 3x^3 + \sin x$, $(0, 0)$

$(x_1, y_1)$

$$y - y_1 = m(x - x_1)$$

$\hookrightarrow m_{tan}\big|_{x=0} = f'(0)$

$$\frac{dy}{dx} = 3\cdot 3\cdot x^2 + \cos x = 9x^2 + \cos x$$

$$m_{tan}\big|_{x=0} = \frac{dy}{dx}\Big|_{x=0} = \cancel{9\cdot 0^2}^{\,0} + \cos 0 = 1$$

$$y - 0 = 1(x - 0) \Rightarrow y = x$$

d. $\dfrac{d}{dx}(f(x))^3\Big|_{x=5} = 3f(5)^2 f'(5) = 3(9)^2\cdot 5 = 1215.$

**73** $y' = 9x^2 + \cos x$. At $x = 0$, $y' = 1$. So the tangent line is given by $y - 0 = 1(x - 0)$, or $y = x$.

# Midterm#3 Review Q

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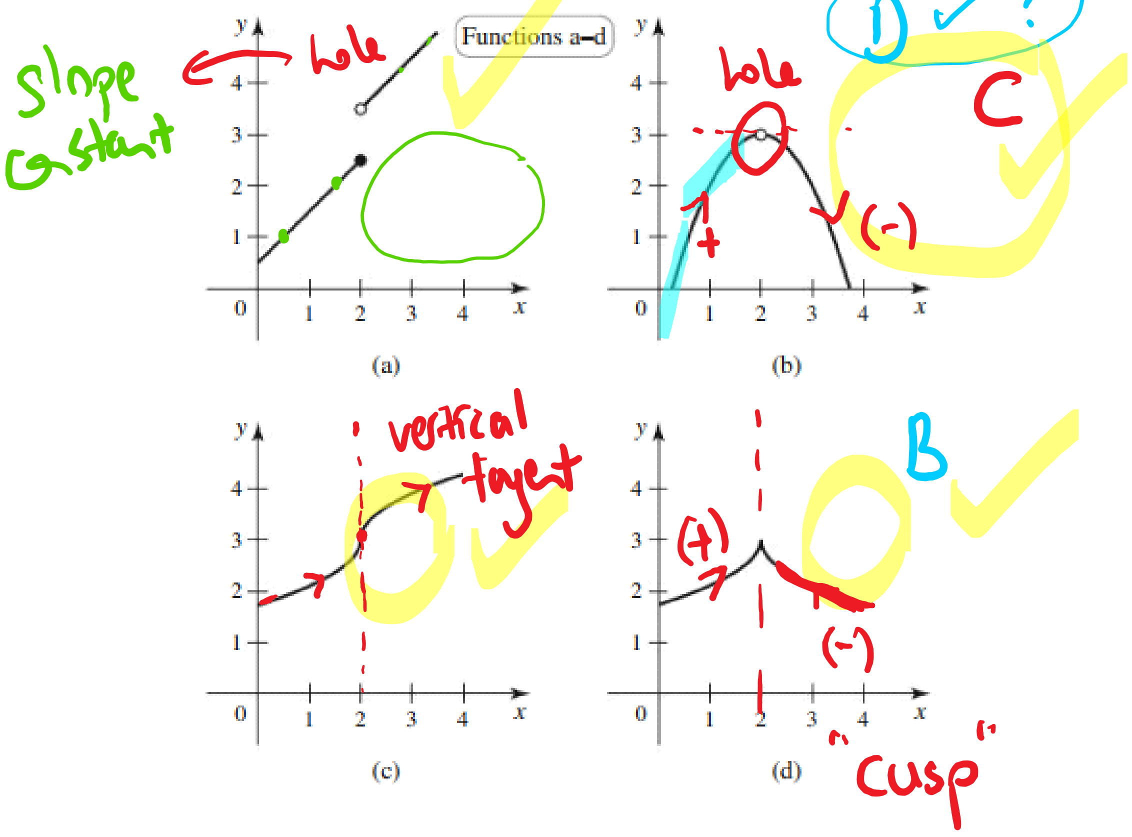
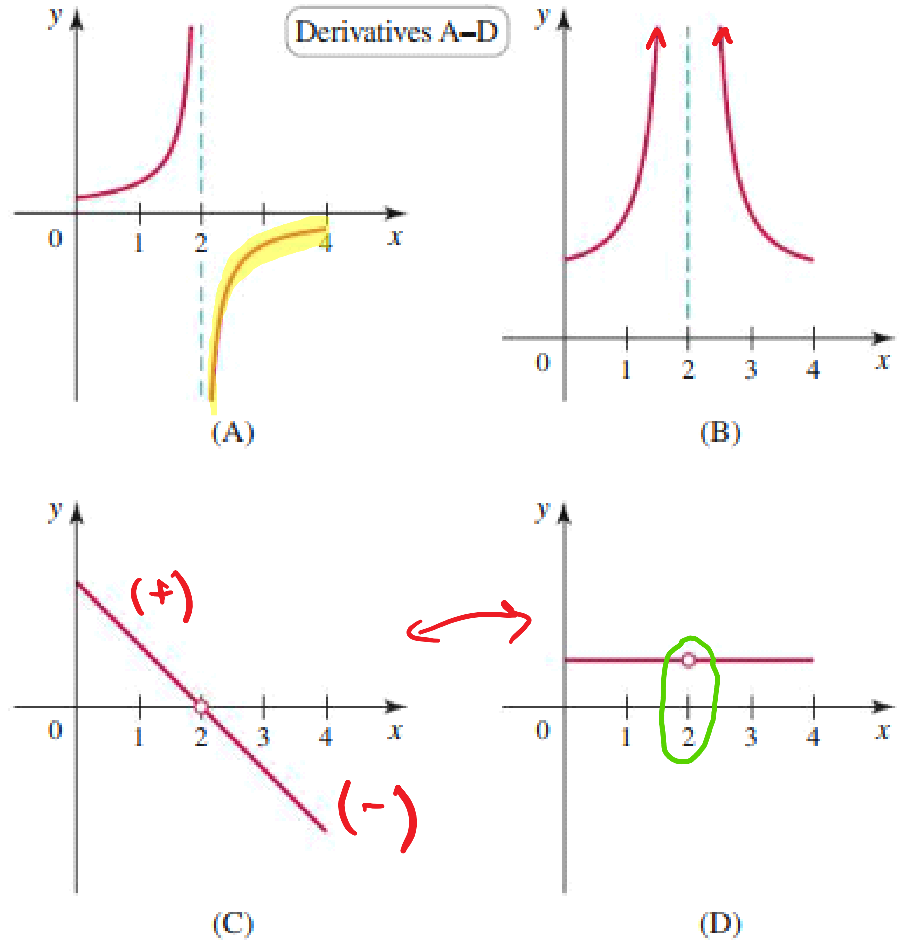
Original F. (+) (-) m=0 Derivative F.

**81. Matching functions and derivatives** Match the functions a–d with the derivatives A–D.




**1. Explain why or why not** Determine whether the following statements are true and give an explanation or counterexample.

a. The function $f(x) = |2x + 1|$ is continuous for all $x$; therefore, it is differentiable for all $x$. False

b. If $\frac{d}{dx}(f(x)) = \frac{d}{dx}(g(x))$, then $f = g$. False

b. $f(x) = x^2 + 6$ $\quad f'(x) = 2x$

$g(x) = x^2 + 11$ $\quad g'(x) = 2x$

a. Just because $f(x)$ is cont. for all $x$ doesn't necessarily mean it's diff.

for all $x$. There's a <u>corner</u> p. at $x = -1/2$
$(2x+1=0)$