Midterm#4 Review-Part2

Tuesday, October 27, 2020 12:43 PM

) Use linear approximation to estimate the value of $\cos\left(\frac{\pi}{2}+0.01\right)$

$$L(x)=f(a)+f'(a)(x-a)$$

$f(x)=\cos(x)$

$a \rightarrow$ known value $\left(\frac{\pi}{2}\right)$

$x \rightarrow \frac{\pi}{2}+0.01$

Note:
we expect the approx. to be (-)
$\cos\left(\frac{\pi}{2}+0.01\right)$ is in the 2nd Q.

$$f(a)=f\left(\frac{\pi}{2}\right)=\cos\left(\frac{\pi}{2}\right)=0$$

$$f'(x)=-\sin(x)$$

$$f'(a)=f'\left(\frac{\pi}{2}\right)=-\sin\left(\frac{\pi}{2}\right)=-1$$

$$L(x)=f(a)+f'(a)(x-a)$$

$$L\left(\frac{\pi}{2}+0.01\right)=f\left(\frac{\pi}{2}\right)+f'\left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}+0.01-\frac{\pi}{2}\right)$$

$$=0+(-1)(0.01)=-0.01$$

With calc: (in radians) $\cos\left(\frac{\pi}{2}+0.01\right)\approx -0.00999983$

≈ -0.010

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Be careful! Estimate the cost of producing $(x+1)^{th}$ units:

$C'(x)$

Estimate the revenue from producing $(x+1)^{th}$ unit:

$R'(x)$

Exp) $C(q) = 3q^2+q+500$ [$q \rightarrow$ # of units produced]

a) compute the actual cost of producing the 41^{th} unit.
{cost of producing 41 units minus 40 units}

$\Delta C = C(41) - C(40)$

$(3\cdot 41^2+41+500) - (3\cdot 40^2+40+500)$

$3\cdot 41^2+41+500 - 3\cdot 40^2 - 40 - 500$

$3(41^2-40^2)+1$

$3(41-40)(41+40)+1$

$3\cdot 1\cdot 81+1 = 243+1 = \underbrace{\$244}_{\text{actual cost}}$

Midterm#4 Review

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Exp) A person standing at the end of a pier 12 ft. above the water and is pulling in a rope attached to a rowboat at the rate of 6 ft./min. How fast is the boat moving when it's 16 ft. from pier?

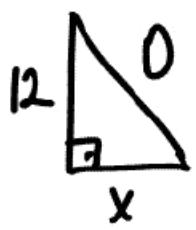
Solution:

when $\frac{dD}{dt} = -6\frac{ft}{min}$

(D is decreasing by time)

AND $x = 16$ ft.

what's $\frac{dx}{dt} = ?$

Use the right Δ: 

$\Rightarrow 12^2 + x^2 = D^2$

$$144 + x^2 = D^2$$

Differentiate both sides wrt time

use units as a hint!

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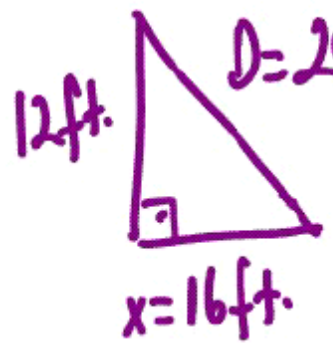
$$0 + 2x \cdot \frac{dx}{dt} = 2D \cdot \frac{dD}{dt}$$

Subs. given info @ specific time:

$x = 16\text{ft.}, \quad \frac{dD}{dt} = -6\frac{\text{ft}}{\text{min}}$



(3-4-5 special tri.)

$$x \cdot \frac{dx}{dt} = D \cdot \frac{dD}{dt}$$

$$16 \cdot \frac{dx}{dt} = 20 \cdot (-6)$$

$$\frac{dx}{dt} = \frac{-120}{16} = -7.5\frac{\text{ft}}{\text{min.}}$$

Distance between the boat & pier is decreasing.

Midterm#4 Review

Sunday, October 25, 2020 10:20 PM

Exp) A person standing at the end of a pier 12 ft. above the water and is pulling in a rope attached to a rowboat at the rate of 6 ft./min. How fast is the boat moving when it's 16ft. from pier?