Midterm#4 Review: Sect 3.8, 3.11, 4.6

Sunday, October 25, 2020 9:54 PM

3.8: implicit differentiation, logarithmic differentiation
3.11: related rates and applications
4.6: linear approximation, marginal analysis

You try it!

Find the eq. of the tangent line to graph of

$x^2+(y-x)^3=9$ at $x=1$.

A) $y+3=\frac{-6}{5}(x-1)$

B) $y-3=\frac{5}{6}(x-1)$

C) $y-3=\frac{2}{3}(x-1)$

D) None of the above

Solution: Differentiate both sides wrt x:

$$2x + 3(y-x)^2 \cdot \left(\frac{dy}{dx} - 1\right) = 0$$

Subs. $(x,y) \to (1,3)$

$x=1$ subs. in
$x^2 + (y-x)^3 = 9$
yields $y=3$

$$2\cdot 1 + 3(3-1)^2\left(\frac{dy}{dx} - 1\right) = 0$$

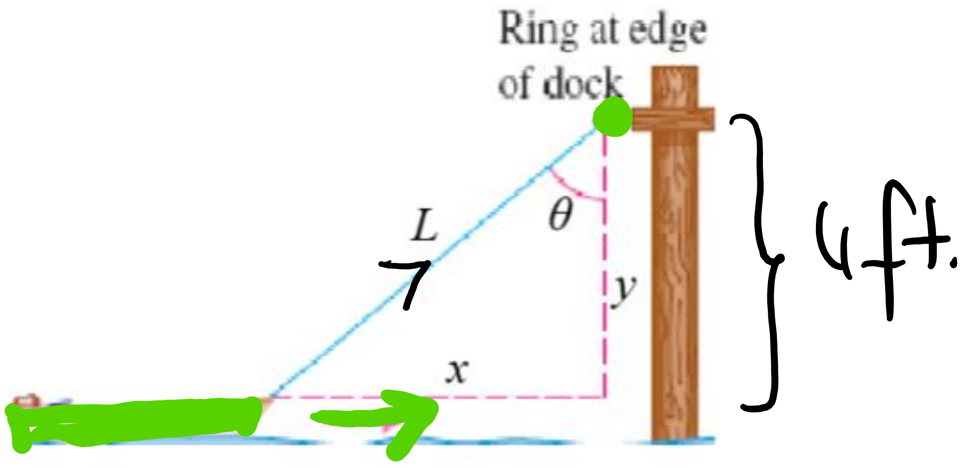
$$2 + 3\cdot 4\left(\frac{dy}{dx} - 1\right) = 0$$

$$12\left(\frac{dy}{dx} - 1\right) = -2$$

$$\frac{dy}{dx} - 1 = -\frac{1}{6} \Rightarrow \frac{dy}{dx} = \frac{5}{6} = m_{tan}$$

Eq. of the tangent line: $y - 3 = \frac{5}{6}(x-1)$

A boat is pulled toward a dock by a rope through a ring on the dock 4 ft above the front of the boat. The rope is hauled in at the rate of 12 ft/sec.



(a) Which of the marked variables (x, y, L, and θ) are changing over time?

(b) Write a mathematical equation that expresses the English sentence *"The rope is hauled in at the rate of 12 ft/sec"*.

(c) Is $\cos(\theta)$ increasing, decreasing, or constant?

(d) Write a mathematical expression for *"the rate at which the boat approaches the dock"*. $\rightarrow \frac{dx}{dt}$

(e) How fast in ft/sec is the boat approaching the dock when the rope is 5 ft long?

a) $x \rightarrow$ horizontal distance between the front of the boat and the dock

$x \downarrow$ (changing / decreasing)

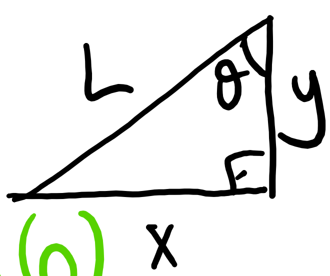
y (not changing, constant)

L (length of the rope) (decreasing)

θ (decreasing)

b) $\frac{dL}{dt} = -12$ ft/sec.

c) $\cos\theta$?



$\cos\left(\frac{\pi}{2}\right)$ $\cos(\pi/6)$ $\cos(0)$

$\theta \rightarrow \pi/6$ . . . 0

$\cos\theta = \frac{Adj}{H} = \frac{y}{L}$

Increasing

$y=5$

$\cos\theta$

$\frac{5}{3}$ vs $\frac{5}{2}$

1.6 2.5

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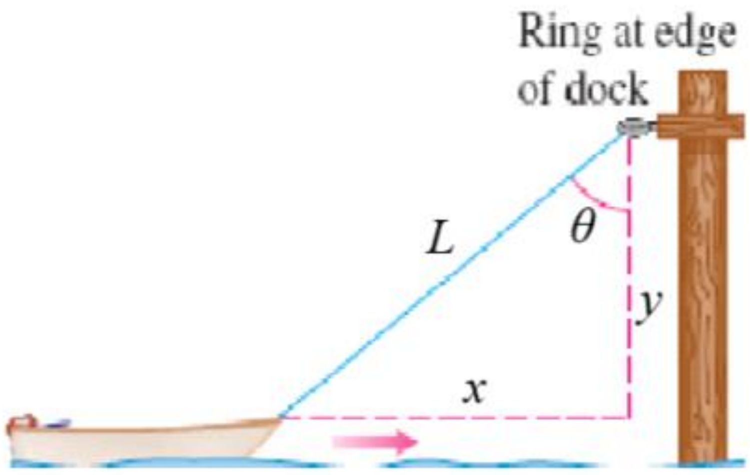

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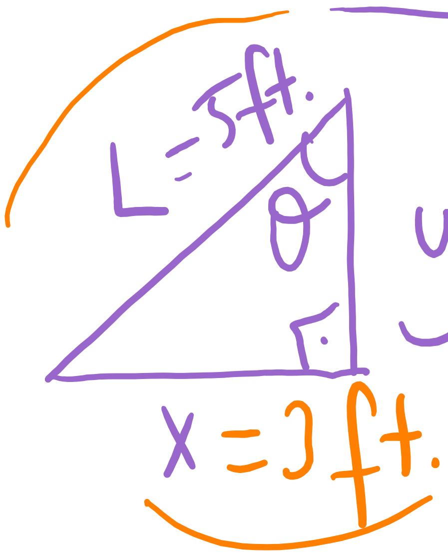
(c) Is $\cos(\theta)$ increasing, decreasing, or constant?

(d) Write a mathematical expression for *"the rate at which the boat approaches the dock"*.

(e) How fast in ft/sec is the boat approaching the dock when the rope is 5 ft long?

e) when $L = 5$ ft. $\quad \frac{dx}{dt} = ?$



$y = 4$ ft. (constant)

$$x^2 + y^2 = L^2$$

Diff. both sides w/t time

$$\cancel{2}x \cdot \frac{dx}{dt} + 0 = \cancel{2}L \cdot \frac{dL}{dt}$$

$$x \cdot \frac{dx}{dt} = L \cdot \frac{dL}{dt}$$

$$3 \cdot \frac{dx}{dt} = 5 \cdot -12$$

$$\frac{dx}{dt} = -\frac{60}{3} = -20 \text{ ft/sec}$$

The boat is approaching the dock at 20 $\frac{\text{ft}}{\text{sec}}$. when $L = 5$ft.

a. *Write the equation of the line that represents the linear approximation to the following functions at the given point a.*
b. *Use the linear approximation to estimate the given quantity.*
c. *Compute the percent error in your approximation,* $100|\text{approximation} - \text{exact}|/|\text{exact}|$, *where the exact value is given by a calculator.*

$f(x) = \ln(1 + x); a = 0; f(0.9)$

$$y = L(x) = f(a) + f'(a)(x-a) \qquad [y - y_1 = m(x - x_1)]$$

$$f(x) = \ln(1+x)$$

$$a = 0$$

$$f(a) = f(0) = \ln(1+0) = \ln 1 = 0$$

$$f'(x) = [\ln(1+x)]' = \frac{(1+x)'}{1+x} = \frac{1}{1+x}$$

$$f'(a) = f'(0) = \frac{1}{1+0} = 1$$

$$L(x) = 0 + 1 \cdot (x - 0) = x$$

$$f(0.9) \approx L(0.9) = 0.9$$

c) $\%\ \text{error} \Rightarrow \dfrac{100 \cdot |\text{approx.} - \text{exact}|}{|\text{exact}|}$

$\text{exact} \rightarrow \ln(1+0.9) \approx 0.641853$

$\text{approx} \rightarrow L(0.9) = 0.9$

$\%\ \text{error} \Rightarrow \dfrac{100 \cdot |0.9 - 0.641853|}{|0.641853|}$

$\approx 40.2\%$