Midterm#5 Review - Part2

Friday, November 13, 2020 9:32 AM

Find abs. min/max. values for

$f(x) = \frac{9}{x} + x - 3$ on $[1, 9]$

Poll choices:

A) Abs. min at $x = 3$

B) Abs. min. at $x = 1$

C) Abs. min at $x = 9$

D) Abs. min at $x = 0$

E) Abs. min at $x = -3$

Step 1 $f(x)$ is NOT continuous at $x=0$
however, 0 is not in $[1,9]$

$f(x) = 9 \cdot x^{-1} + x - 3$

$f'(x) = -9x^{-2} + 1$

critical p. → $f'(x) = 0$ or DNE

$f'(x)=0 \Rightarrow \frac{-9}{x^2} + 1 = 0 \Rightarrow \frac{-9}{x^2} = -1 \Rightarrow 9 = x^2$

$x = 3$ $\quad \cancel{x=-3}$ not in $[1,9]$

only $x=3$ is a crit. P.

$f'(x)$ DNE : $x=0$, however, as stated before
$x=0$ is NOT in $[1,9]$

endpoints: $x = 1, 9$

Step 2

	x	$f(x) = \frac{9}{x} + x - 3$	
critical P	3	$f(3) = \frac{9}{3} + 3 - 3 = 3$	MIN
endpoints	1	$f(1) = 9 + 1 - 3 = 7$	MAX
	9	$f(9) = \frac{9}{9} + 9 - 3 = 7$	MAX

The absolute max. is 7, the abs. max points are: (1, 7) and (9, 7).

(It's OK that the abs. max. values occur at multiple x-values)

The abs. min. is 3, the abs. min point is: (3, 3)

Exp) Evaluate $\lim_{x \to \left(\frac{\pi}{2}\right)^-} \left[\left(x - \frac{\pi}{2}\right) \cdot \tan x\right]$

"OSP" $\lim_{x \to \left(\frac{\pi}{2}\right)^-} \left[\left(x - \frac{\pi}{2}\right) \cdot \tan x\right] \overset{\text{"OSP"}}{=} \boxed{\text{"}0 \cdot \infty\text{"}}$

Recall: "$\tan\left(\frac{\pi}{2}\right)^- = \frac{\sin\left(\frac{\pi}{2}\right)^-}{\cos\left(\frac{\pi}{2}\right)^-} = \frac{1}{0} = \infty$"

"Re-write to get $\frac{0}{0}$ or $\frac{\infty}{\infty}$"

$$\lim_{x \to \left(\frac{\pi}{2}\right)^-} \left[\left(x - \frac{\pi}{2}\right) \cdot \frac{1}{\cot x}\right]$$

$$= \lim_{x \to \left(\frac{\pi}{2}\right)^-} \left(\frac{\left(x - \frac{\pi}{2}\right)}{\cot x}\right) \overset{\text{"OSP"}}{=} \text{"}\frac{0}{0}\text{"}$$

"Use L.R"

$$\overset{H}{=} \lim_{x \to \left(\frac{\pi}{2}\right)^-} \left(\frac{1}{-\csc^2 x}\right) = \lim_{x \to \left(\frac{\pi}{2}\right)^-} \left(-\sin^2 x\right)$$

Recall: $\csc x = \frac{1}{\sin x}$

$$\overset{\text{"OSP"}}{=} -\sin^2\left(\frac{\pi}{2}\right) = -1$$

Find the indefinite integral:

$$\int \frac{x^2+\sqrt{x}+1}{x^2} \cdot dx$$

$$\int \frac{x^2}{x^2} \cdot dx + \int \frac{\sqrt{x}}{x^2} \cdot dx + \int \frac{1}{x^2} \cdot dx$$

$$= \int 1 dx + \int \frac{x^{1/2}}{x^2} \cdot dx + \int (x^{-2}) \cdot dx$$

$$= \int 1 \cdot dx + \int (x^{\frac{1}{2}-2}) \cdot dx + \int (x^{-2}) dx$$

$$= x + \int x^{-\frac{3}{2}} \cdot dx + \int (x^{-2}) \cdot dx$$

$$= x + \frac{x^{-1/2}}{-1/2} + \frac{x^{-1}}{-1} + C$$

$$= x - 2 \cdot x^{-1/2} - x^{-1} + C$$

$$= x - 2 \cdot \frac{1}{\sqrt{x}} - \frac{1}{x} + C$$