4.1: maxima and minima, extreme value theorem

4.7: l'Hospital's rule

4.9: antiderivatives, initial value problems

Midterm Exam #5: Sect 4.1, 4.7, 4.9

Exp) Find the value of a (if possible) given the limit of $x^{\frac{a}{2x}}$ as x approaches to positive infinity is calculated as e^0. (a is an unspecified non-zero constant)

Show all work.

A) a = 2 B) a = -2 C) a = 3 D) Not possible

$$\lim_{x\to\infty}\left(x^{\frac{a}{2x}}\right) = e^0 = 1$$

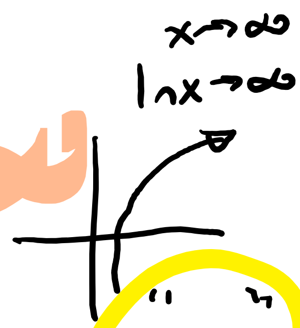
$$\lim_{x\to\infty}\left(x^{\frac{a}{2x}}\right) \overset{\text{"DSP"}}{=} \infty^{\cancel{\frac{a}{\infty}}\to 0} = \boxed{\infty^0}$$

"other indet. form"

use ln prop.

$\frac{\infty}{\infty}$, $\frac{0}{0}$ } L.R.

$$\ln y = \ln \lim_{x\to\infty}\left(x^{\frac{a}{2x}}\right) = \lim_{x\to\infty} \ln\left(x^{\frac{a}{2x}}\right)$$



$$\ln y = \lim_{x\to\infty}\left(\frac{a\cdot\ln(x)}{2x}\right) \overset{\text{"DSP"}}{=} \frac{a\cdot\ln(\infty)}{2\cdot\infty} = \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{H}{=} \lim_{x\to\infty}\left(\frac{a\cdot\frac{1}{x}}{2}\right) = \lim_{x\to\infty}\left(\frac{a}{2x}\right) \overset{\text{"DSP"}}{=} 0$$

$$\ln y = 0 \Rightarrow y = e^0 = 1 \text{ (Given)}$$

Consider the function $f(x) = ax^6 e^{-bx}$, where a and b are unspecified constants. Suppose f has a point of local maximum at $(2, 64e^{-2})$. Find the values of a and b. Show all work.

IF local extrema THEN $\underbrace{f'(c)=0 \text{ or DNE}}$

f has a point of local MAX at $\underbrace{(2, 64\cdot e^{-2})}$

$f(2) = 64\cdot e^{-2}$

$f'(2)=0$

$f(x) = ax^6 \cdot e^{-bx}$

$(e^{-2x})' = e^{-2x}\cdot(-2)$

$f'(x) = a\cdot 6\cdot x^5\cdot e^{-bx} + ax^6\cdot e^{-bx}\cdot(-b)$

$= a\cdot x^5\cdot e^{-bx}(6-b\cdot x)$

$f'(2)=0 \Rightarrow f'(2) = \underbrace{a\cdot 2^5}_{32a}\cdot e^{-2b}\cdot(6-2b) = \frac{0}{32a}$

$f'(2) = \cancel{32a}\cdot\frac{(6-2b)}{e^{2b}} = 0$ or DNE

$6-2b=0 \Rightarrow \boxed{b=3}$

$f(x) = a\cdot x^6\cdot e^{-3x}$

$f(2) = 64\cdot e^{-2}$

$f(2) = a\cdot \cancel{2^6}\cdot e^{-6} = \cancel{64}\, e^{-2}$

$a\cdot e^{-6} = e^{-2} \Rightarrow \boxed{a=e^4}$

Exp) An automobile starts from rest (that is, $v(0)=0$) and travels with constant acceleration $a(t)=k$ in such a way that 6 sec. after it begins to move, it has traveled 360 ft. from its starting point. What is k? (include unit) Show all work.

A) $k=5\ ft/sec$ B) $k=5\ ft/sec^2$ C) $k=20\ ft/sec^2$ D) Not possible

Given: $v(0)=0$ $\underbrace{s(6)=360\ ft.}$

$a(t)=k$

$\int 2\cdot dt = 2t + C$

Asked: $k=?$

$$v(t)=\int a(t)\cdot dt=\int k\cdot dt = kt + C_1$$

$$v(0)=0 \Rightarrow v(0)=k\cdot 0 + C_1 = 0 \Rightarrow C_1 = 0$$

$$v(t)=k\cdot t$$

$$s(t)=\int v(t)\cdot dt = \int k\cdot t\cdot dt = k\cdot\frac{t^2}{2}+C_2$$

$$s(6)=360\ ft. \Rightarrow s(6)=k\cdot\frac{6^2}{2}+C_2=360$$

$s(0)=0$ (Initial Value) $s(t)=k\cdot\frac{t^2}{2}+C_2$

$C_2=0 \Rightarrow s(6)=k\cdot 18=360$ $s(0)=0+C_2=0$

$$\boxed{k=20\ ft/sec^2}$$

$$\boxed{C_2=0}$$

$s(0) = 0 + C_2 = 0$

$\boxed{C_2 = 0}$

$k = 20\ ft/sec^2$

Exp) Calculate the limit or determine that it DNE.

$f'(x)$ is continuous with $f(0) = -2$, $\underbrace{f(2) = 3}_{\cancel{f'(2)}\,?}$.

$$\lim_{x \to 0} \left(\frac{f(x^2+2) - 3\cdot\cos(4x)}{\sin x} \right)$$

A) Not enough info B) DNE C) ∞ D) $-\infty$ E) $3/2$ F) 0

$$\lim_{x \to 0} \left(\frac{f(x^2+2) - 3\cdot\cos(4x)}{\sin x} \right) \overset{\text{"DSP"}}{=} \frac{f(2) - 3\cdot\cos 0 \to 1}{\sin 0 \to 0}$$

$$= \frac{3-3}{0} = \text{"}\frac{0}{0}\text{"} \checkmark \text{ L.R.}$$

$$\overset{H}{=} \lim_{x \to 0} \left(\frac{f'(x^2+2)\cdot(2x) + 3\cdot(+\sin(4x))\cdot 4}{\cos x} \right)$$

$$\overset{\text{"DSP"}}{=} \frac{f'(2)\cdot 0 + 3\cdot 4 \sin 0 \to 0}{\cos 0} = \frac{0+0}{1} = 0$$

$\boxed{F}$

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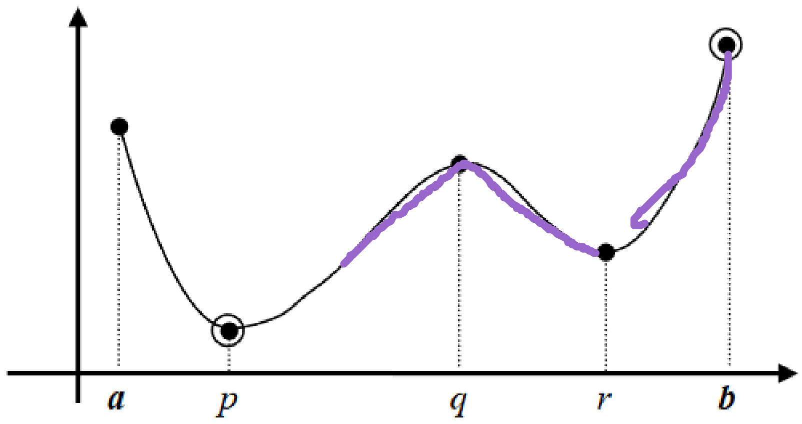

[a, b]

Use the graph to answer the questions below.

( )

Local Maximum at: q

Local Minimum at: b, r

[ ]

Global (Absolute) Maximum at: b

Global (Absolute) Minimum at: p