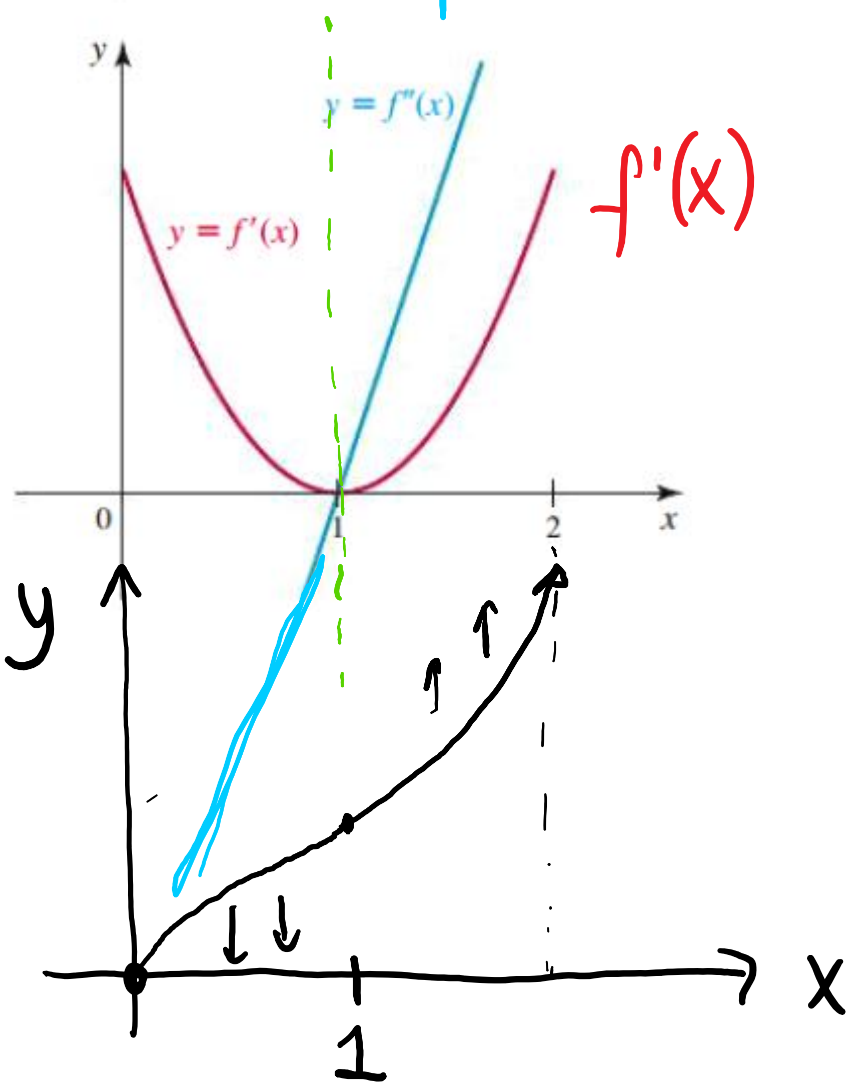
Graph of f, f', f''

Sunday, November 22, 2020 9:02 PM

47–48. *Use the graphs of f' and f'' to find the critical points and inflection points of f, the intervals on which f is increasing and decreasing, and the intervals of concavity. Then graph f assuming $f(0) = 0$.*



$f'(x) > 0$ on $(0, 2)$

$f(x)$ is incr. on $(0,2)$

$f'(1)=0$ (critical P. at $x=1$)

$f''(x) < 0$ on $(0,1)$ f $\cap$

$f''(x) > 0$ on $(1,2)$ $\cup$

PoI when concavity changes

$x=1$ is the PoI

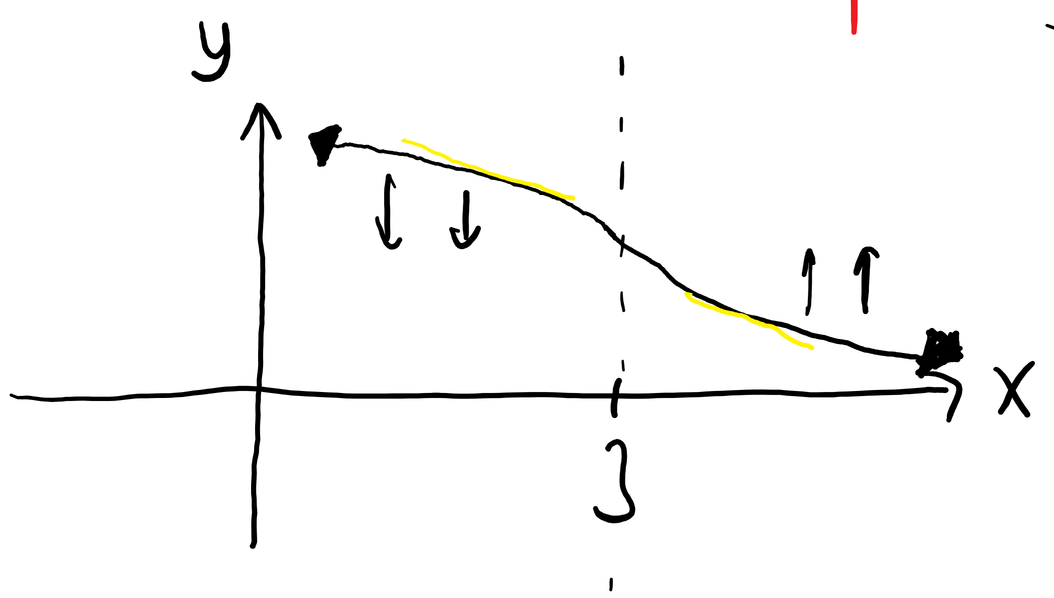
7–8. *Sketch a graph of a function f with the following properties.*

7. $f' < 0$ and $f'' < 0$, for $x < 3$ → f is decreasing, f is concave down $x<3$

$f' < 0$ and $f'' > 0$, for $x > 3$ → f is decreasing, f is concave up $x>3$

f' f''

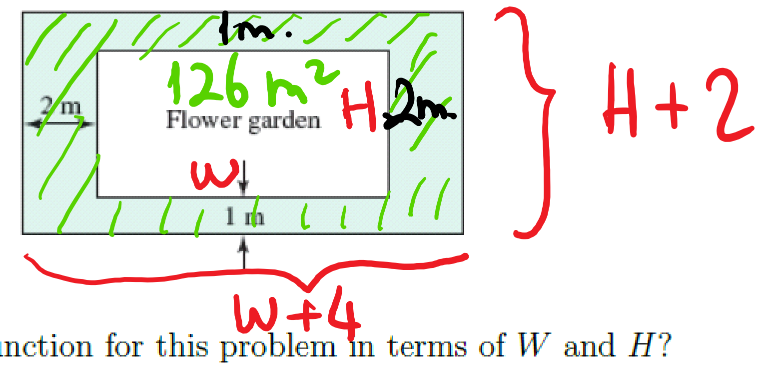
$\underbrace{\hspace{3cm}}_{\text{PoI at } x=3}$



Spring 2020 Final Q

Sunday, November 22, 2020 9:08 PM

A local park has hired you to construct a rectangular flower garden surrounded by a grass border that is 1 m wide on two sides and 2 m wide on the other two sides. (See the figure blow.) The area of the garden only (the small rectangle) must be 126 m^2. Your primary task is to find the dimensions of the garden that give the smallest possible combined area of the garden and the grass border. For this problem, let W be the horizontal width of the garden and let H be the vertical height of the garden.



$W \cdot H = 126$

$W = 252$ m.

$H = \frac{1}{2}$ m.

(a) What is the objective function for this problem in terms of W and H?

(b) What is the constraint equation for this problem in terms of W and H?

(c) Find the objective function in terms of W only.

(d) What is the interval of interest for the objective function? $W \to (0, \infty)$

(e) Find the values of W and H that minimize the total combined area.

(f) What horizontal width W of the garden will *maximize* the total area? None

$H \downarrow \quad W \uparrow$

a) Obj: MIN. combined area

Obj. F: $A(W,H) = (W+4)\cdot(H+2) \to A(W)$, $A(H)$

$W \cdot H = 126$

$H = \frac{126}{W}$

b) Constraint eq: $W \cdot H = 126$

c) $A(W,H) \to A(W)$

$$A(W) = (W+4)\cdot\left(\frac{126}{W} + 2\right) = 126 + 2W + \frac{504}{W} + 8$$

$$= 134 + 2W + 504 \cdot W^{-1}$$

e) MIN. $A(W)$

$A'(W) = 0$ or DNE ; $A'(W) = (134 + 2W + 504\cdot W^{-1})'$

$= 0 + 2 - 1 \cdot W^{-2} \cdot 504$

$= 2 - 504 \cdot W^{-2} = 0$

$= 2 - \frac{504}{W^2} = \frac{2W^2 - 504}{W^2}$

crit. P: $2W^2 - 504 = 0$, $W = 0$

$W^2 = 252$

$W = 6\sqrt{7}$

$(0, \infty)$

e) min. $A(w)$

$A'(w)=0$ or DNE ; $A'(w) = (134+2w+504\cdot w^{-1})'$

crit. P: $2w^2-504=0$, $w=0$

$w^2=252$

$w=6\sqrt{7}$

$(0,\infty)$

$= 0+2-1\cdot w^{-2}\cdot 504$

$= 2-504\cdot w^{-2} = 0$

$= \underbrace{2-\frac{504}{w^2}} = \frac{2w^2-504}{w^2}$

Sign chart

	0 ... 2	$6\sqrt{7}$	504 ... ∞
$A'(w)$	$-$		$+$
$A(w)$	↘	$\boxed{\text{local min}}$	↗

local min at $w=6\sqrt{7}$

$A''(w) = (2-504\cdot w^{-2})' = 0 \underbrace{+504\cdot(+2)\cdot w^{-3}}_{\text{pos.}}$

$A''(w) > 0$

$A(w)$ ↳ $w=6\sqrt{7}$

global min ~~local~~

When $w=6\sqrt{7}$ m. $A(w)$ is min.

$W\cdot H=126 \Rightarrow H=\frac{\cancel{126}^{21}}{\cancel{6}\sqrt{7}} = \frac{\cancel{21}^{3}\sqrt{7}}{\cancel{7}} = 3\sqrt{7}$ m.

Question with an unknown constant

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Identity the critical points and the inflection points of $f(x) = (x - a)(x + a)^3$, for $a > 0$.

$a \to (+)$ constant

Critical P.

$f'(x) = 0$ or DNE

$$f'(x) = 1 \cdot (x+a)^3 + (x-a) \cdot 3(x+a)^2 \cdot 1$$

$$= (x+a)^2 \cdot \left((x+a)^1 + 3(x-a) \right)$$

$$= (x+a)^2 (x+a+3x-3a) = (x+a)^2 (4x-2a)$$

$$f'(x) = \underbrace{(x+a)^2}_{0} \cdot \underbrace{(4x-2a)}_{0} = 0$$

$x+a=0$

$x=-a$

$4x-2a=0$

$\frac{4x}{4} = \frac{2a}{4}$; $x = \frac{a}{2}$

PoI

$$\boxed{f''(x)} = \left[(x+a)^2 \cdot (4x-2a) \right]'$$

$$= \cancel{2}(\cancel{x+a}) \cdot 1 \cdot \underbrace{(4x-2a)}_{\cancel{2}(2x-a)} + (x+a)^2 \cdot 4$$

$$= 4(x+a) \left[(2x-a) + (x+a)^1 \right]$$

$f''(-3a)$

$$= 4(x+a)(3x) = \boxed{12x(x+a)}$$

Second-order crit. P: $x=0$ $x=-a$ (a is pos.)

PoI

$x=-a, 0$

sign chart

	$-\infty$ ($-3a$)	$-a$	($-\frac{a}{2}$)	0	($3a$) $+\infty$
f''	$12x(x+a)$ $\ominus \cdot \ominus = \oplus$	0	$\ominus \cdot \oplus = \ominus$	0	$12 \cdot 3a(3a+a)$ $\oplus$
f		PoI		PoI