MyLab HW 5.5 → Q 11 (Part d)

The population of a culture of bacteria has a growth rate given by $p'(t) = \frac{200}{(t+1)^r}$ bacteria per hour, for $t \geq 0$, where $r > 1$ is a real number. The increase in the population over the time interval $[0,t]$ is given by $\int_0^t p'(s)ds$. (Note that the growth rate decreases in time, reflecting the competition for food and space.) Complete steps (a) through (e) below.

a. Using the population model with $r = 2$, what is the increase in the population over the time interval $0 \leq t \leq 4$?

The increase in the population over the given interval is approximately 160 bacteria.
(Round down to the nearest integer as needed.)

b. Using the population model with $r = 3$, what is the increase in the population over the time interval $0 \leq t \leq 6$?

The increase in the population over the given interval is approximately 97 bacteria.
(Round down to the nearest integer as needed.)

c. Let ΔP be the increase in the population over a fixed time interval $[0,T]$. For fixed T, does ΔP increase or decrease with the parameter r? Explain.

A. ΔP is independent of the parameter r.

B. $\Delta P = \int_0^T \frac{200}{(t+1)^r} dt$; decreases as r increases

C. $\Delta P = \int_0^T \frac{200}{(t+1)^r} dt$; increases as r increases

d. A lab technician measures an increase in the population of 400 bacteria over the first 12-hr period $[0,12]$. Estimate the value of r that best fits this data point.

$r \approx$ ☐ (Round to the nearest thousandth as needed.)

You may use a graphing calculator on the HW.

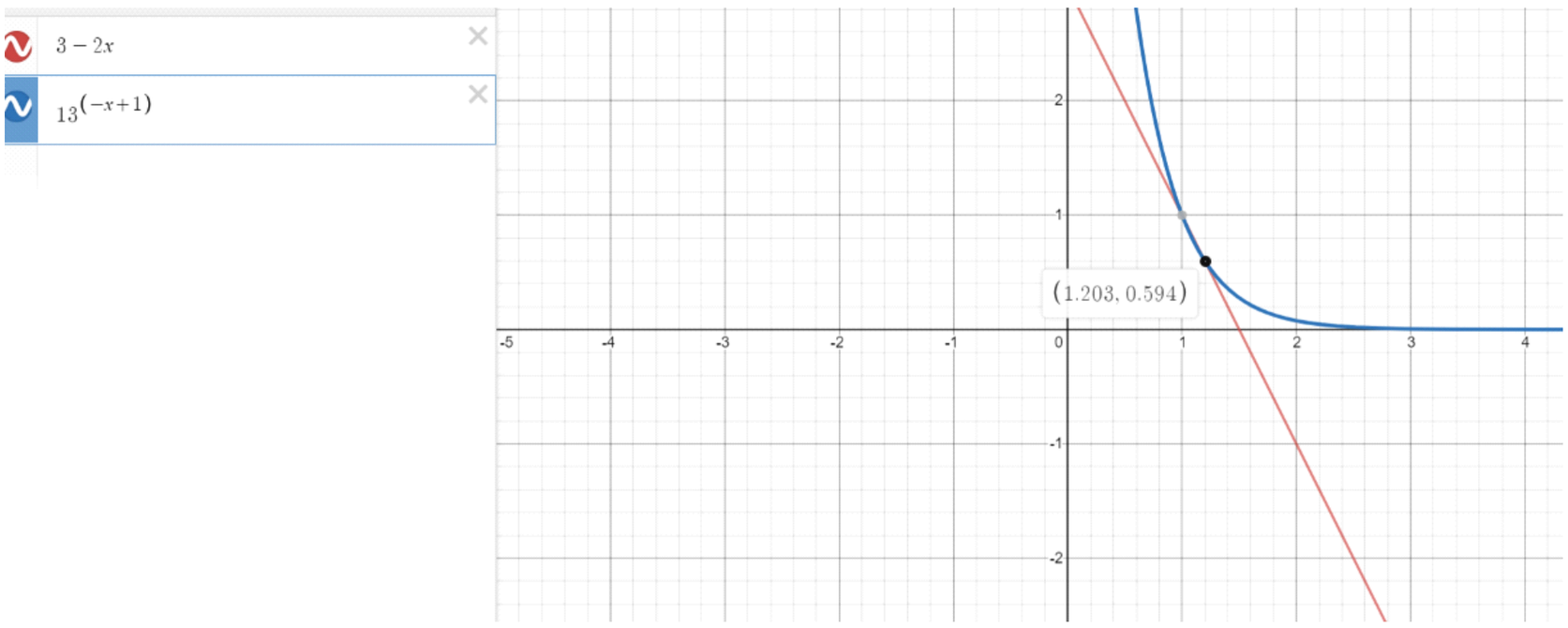
d) $400 = \int_0^{12} \frac{200}{(t+1)^r} \cdot dt$

$t+1 = u \Rightarrow dt = du$

$$2 = \int_0^{12} \frac{1}{u^r} \cdot du = \int_0^{12} u^{-r} \cdot du = \frac{u^{-r+1}}{(-r+1)} \Big|_0^{12}$$

$$2 = \frac{(t+1)^{(-r+1)}}{(-r+1)} \Big|_0^{12} = \frac{(12+1)^{-r+1}}{(-r+1)} - \frac{1^{(-r+1)}}{(-r+1)}$$

$$2 = \frac{13^{-r+1} - 1}{(-r+1)} \Rightarrow -2r+2 = 13^{(-r+1)} - 1 \Rightarrow -2r+3 = 13^{(-r+1)}$$



$-2r+3=13^{(-r+1)}$

Find the intersection points of

$y=-2x+3$, $y=13^{(-x+1)}$

(change r to x for graphing)

$x(r) \simeq 1.203$ is the final answer.