Dr. Tabanli's Exam #3 Review for Sections 13-15, 21-23

Note: You will not be given any geometric formulas on the test. Please make sure to have a formula sheet of: perimeter/area/volume of circle, disc, rectangle, triangle, cylinder, cone, sphere, etc. Answers must be justified using techniques that have been taught in this course.

3.6: Derivative as rates of change

1. A coffee vendor has collected data on the price of coffee in her store over the last year. The price of coffee t weeks since when the data collection began was

$$p(t) = 0.02t^2 - 0.1t + 6$$

dollars per pound. **For each part, you must give correct units as part of your answer.**

   (a) How much did the price of one pound of coffee increase in the first ten weeks after the data collection began?

   (b) What was the average rate at which the price of one pound of coffee changed over the same ten-week period mentioned in part (a)?

   The vendor also found that, in a given week, the local consumers bought approximately

$$D(p) = \frac{2500}{p^2 + 1}$$

   pounds of coffee when the price was p dollars per pound. That is, D is the weekly demand of the consumers.

   (c) Calculate $D'(7)$ and explain its precise meaning in the given context.

   (d) At what rate was the weekly demand for coffee changing with respect to time exactly ten weeks after data collection began?

2. A stone is dropped off the edge of a 54-ft cliff on Mars, where the acceleration due to gravity is about 12 ft/s^2. The height (in feet) of the stone above the ground t seconds after it is dropped is $s(t) = -6t^2 + 54$. Find the velocity and speed of the stone when it hits the ground.

3.8: implicit differentiation and 3.9: log. differentiation

1. Find the slope of the tangent line to the graph of $5x^2y - \pi \cdot cos(y) = 6\pi$ at $P(1, \pi)$.

2. The number of cell phones produced when x dollars is spent on labor and y dollars is spent on capital invested by a manufacturer can be modeled by the equation $60x^{3/4}y^{1/4} = 3240$.

   (a) Find $\frac{dy}{dx}$ and evaluate at the point $(81, 16)$.

   (b) Interpret the result of your answer in part a.

3. Evaluate $f'(x)$ for $f(x) = (1 + x^2)^{\sin x}$

3.11: related rates and applications

1. At noon on a certain day, a truck is 250 miles due east of a car. The truck is traveling west at a constant speed of 25 mph, while the car is traveling north at 50 mph.

   (a) At what rate is the distance between them changing at time t?

   (b) At what time is the distance between the car and the truck neither increasing nor decreasing?

2. Water is falling on a surface, wetting a circular area that is expanding at a rate of 18π mm^2/s. How fast is the radius of the wetted area expanding when the radius is 12 mm?

4.6: linear approximation, marginal analysis

1. The selling price per unit of a certain product is modeled by $p(x) = \dfrac{800}{x+5}$ if x units are already being produced. Use marginal analysis to estimate the revenue derived from producing the 16th unit. Your answer should be an integer or a simplified fraction of integers.

2. In a healthy person of height x in., the average pulse rate in beats per minute is modeled by the formula:
$$P(x) = \frac{596}{\sqrt{x}}$$
Estimate the change in pulse rate that corresponds to a height change from 59 to 60 in.

4.7: l'Hôpital's rule

1. **True or False**

   L'Hopital's rule is the only method to evaluate the following limits:

   (a) $\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x^3+2x+1}\right)$

   (b) $\lim\limits_{x\to 2}\left(\dfrac{x^3-3x^2+2x}{x-2}\right)$

   (c) $\lim\limits_{x\to 0}\left(\dfrac{e^x-x-1}{5x^2}\right)$

2. Evaluate the limit or determine that it does not exist. If the limit is infinite, then your answer should be ∞ or -∞ (as appropriate), instead of "does not exist". If you use the l'Hôpital's Rule, justify the use of it, state the indeteterminate form used.

   (a) $\lim\limits_{x\to 0}\left(\dfrac{x\cdot\tan x}{\sin 3x}\right)$

   (b) $\lim\limits_{x\to \infty}\left(\dfrac{3+\ln x}{x^2+7}\right)$