Determine where the following function is continuous. Write your answer using interval notation. You must use proper notation and calculus methods to justify your answer.

$$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & x < 3 \\ 0 & x = 3 \\ 5x-9 & 3 < x < 4 \\ 11 & x = 4 \\ 27 - x^2 & x > 4 \end{cases}$$

1 2 3 4 5

a) $(-\infty, 4), (4, \infty)$

b) $(-\infty, 3), (3, \infty)$

c) $(-\infty, 3), (3, 4), (4, \infty)$

d) $(-\infty, \infty)$

check cont. for each piece

$\dfrac{x^2-9}{x-3} = (x+3)$ $\quad x < 3$

$x - 3 \to x \neq 3$

ALL function pieces are cont.

@ $x=3$ transition P.

1) $f(3) = 0$

2) $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} \left(\dfrac{x^2-9}{x-3}\right) = \lim\limits_{x \to 3^-} (x+3) = 6$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (5x-9) = 5 \cdot 3 - 9 = 6$

$\lim\limits_{x \to 3} f(x) = 6$

3) $f(3) = \lim\limits_{x \to 3} f(x)$ $\quad 0 \neq 6$

$f$ is NOT cont. at $x=3$

Determine where the following function is continuous. Write your answer using interval notation. You must use proper notation and calculus methods to justify your answer.

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@ $x=4$

1) $f(4) = 11$

2) $\lim\limits_{x\to 4^-} f(x) = \lim\limits_{x\to 4^-} (5x-9) \overset{DSP}{=} 5\cdot 4 - 9 = 11$

$\lim\limits_{x\to 4^+} f(x) = \lim\limits_{x\to 4^+} (27-x^2) \overset{DSP}{=} 27-4^2 = 27-16 = 11$

$\lim\limits_{x\to 4} f(x) = 11$

3) $f(4) = \lim\limits_{x\to 4} f(x) = 11$ ✓

$f$ is Cont. at $x=4$

$f$ is NOT Cont. at $x=3$ Exclude 3

Interval for Cont: $(-\infty, 3), (3, \infty)$