3. Consider the function g below, where a and b are unspecified constants.

$$g(x) = \begin{cases} be^x + a + 1 & x \leq 0 \\ ax^2 + b(x+3) & 0 < x \leq 1 \\ a\cos(\pi x) + 7bx & 1 < x \end{cases}$$

Assume that g is continuous for all x.

(a) What relation must hold between a and b for g to be continuous at $x = 0$? Your answer should be an equation involving a and b.

(b) What relation must hold between a and b for g to be continuous at $x = 1$? Your answer should be an equation involving a and b.

(c) Calculate the value of a.

(d) Calculate the value of b.

a)

1) $g(0) = b \cdot e^0 + a + 1 = b + a + 1$

2) $\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (b \cdot e^x + a + 1) \overset{DSP}{=} b \cdot e^0 + a + 1$
$= b + a + 1$

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (ax^2 + b(x+3)) \overset{DSP}{=} a \cdot 0^2 + b(0+3)$
$= 3b$

$LL = RL \Rightarrow b + a + 1 = 3b$
$\boxed{a + 1 = 2b}$

3) $g(0) = \lim_{x \to 0} g(x) \Rightarrow b + a + 1 = 3b$

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b) 3 Conditions

1) $g(1) = a\cdot 1^2 + b(1+3) = a + b\cdot 4 = \underline{a+4b}$

2) $\lim\limits_{x\to 1^-} g(x) = \lim\limits_{x\to 1^-} \left(ax^2 + b(x+3)\right) \overset{DSP}{=} a\cdot 1^2 + b(1+3) = a+4b$

$\lim\limits_{x\to 1^+} g(x) = \lim\limits_{x\to 1^+} \left(a\cdot\cos(\pi x) + 7bx\right) \overset{DSP}{=} a\cdot\cos(\pi) + 7b$

$= a\cdot(-1) + 7b = -a + 7b$

LL=RL $\Rightarrow$ $\underline{a+4b} = -a+7b$

$\boxed{2a = 3b}$ $\Rightarrow$ $a = \frac{3b}{2}$

3) 1) = 2) $\Rightarrow$ $a+4b = -a+7b \Rightarrow$ $\boxed{2a = 3b}$

c) $\left.\begin{array}{l} a+1 = 2b \\ 2a = 3b \\ a = \frac{3b}{2} \end{array}\right\}$ $\frac{3b}{2} + 1 = 2b \Rightarrow 1 = \frac{2b}{1_{(2)}} - \frac{3b}{2} = \frac{b}{2} = 1$ $\boxed{b = 2}$

$\longrightarrow$ $a = \frac{3\cdot\cancel{2}}{\cancel{2}} = \boxed{3 = a}$