Fall 2020 Exam Question

Consider the following limit.

$$\lim_{x\to\pi/8}\left(\frac{\tan(2x)-1}{x-\pi/8}\right)$$

(a) Use the limit definition of derivative to identify this limit as the derivative of some function $f(x)$ at the point $x=a$. You must explicitly identify f and a.

(b) Use your identifications in part (a) to calculate the given limit. Show all work.

$\lim_{x\to a}\left(\frac{f(x)-f(a)}{x-a}\right)$ template

$a=\frac{\pi}{8}$

$1=f\left(\frac{\pi}{8}\right)$

$f(x)=\tan(2x)$

$f\left(\frac{\pi}{8}\right)=\tan\left(2\cdot\frac{\pi}{8}\right)$

$=\tan\left(\frac{\pi}{4}\right)=1$ ✓

$$f'\left(\frac{\pi}{8}\right)=\lim_{x\to\frac{\pi}{8}}\left(\frac{\tan(2x)-1}{x-\pi/8}\right)$$

$f'(x)=[\tan(2x)]'=\sec^2(2x)\cdot 2=2\cdot\sec^2(2x)$

$f'\left(\frac{\pi}{8}\right)=2\cdot\sec^2\left(2\cdot\frac{\pi}{8}\right)=2\cdot\sec^2\left(\frac{\pi}{4}\right)$ $\boxed{\sec\left(\frac{\pi}{4}\right)=\sqrt{2}}$

$=2\cdot(\sqrt{2})^2$

$=2\cdot 2=4$