3.2. The Derivative as a Function

EXAMPLE 4 **Graph of the derivative** Sketch the graph of f' from the graph of f (Figure 3.18).

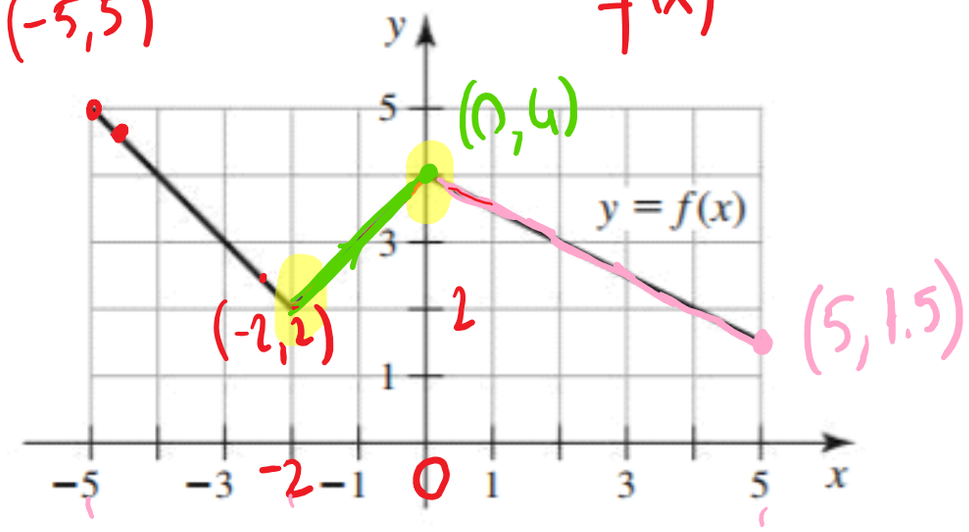


Figure 3.18

f(x)

For $x < -2$; the slope of $f(x)$ is -1.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{5-2}{-5-(-2)} = \frac{3}{-3} = -1$$

$$f'(x) = -1.$$

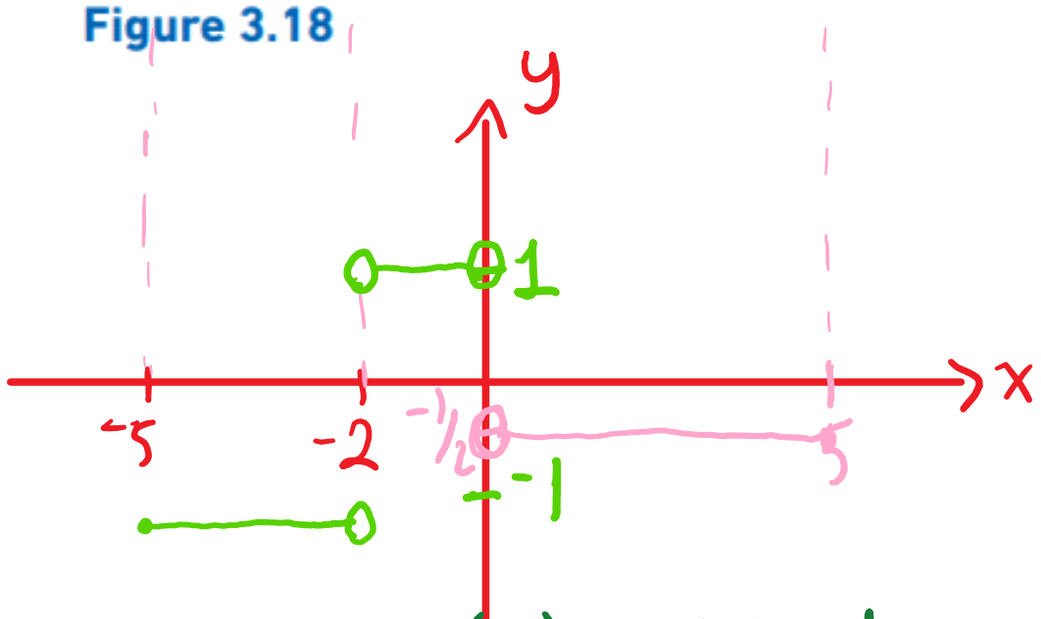
For $-2 < x < 0$; the slope of $f(x)$ is 1.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4-2}{0-(-2)} = \frac{2}{2} = 1.$$

$$f'(x) = 1.$$

For $x > 0$; the slope of $f(x)$ is $\frac{-1}{2}$.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4-1.5}{0-5} = \frac{2.5}{-5} = \frac{-1}{2}.$$



at $x=-2$ $f'(-2)$ undefined

at $x=0$ $f'(0)$ undefined

Note the sharp corners at $x=-2$, and $x=0$.

The slopes of the tangent lines change abruptly at $x=0,-2$.

Therefore; $f'(-2)$ and $f'(0)$ are undefined, thus,

the graph of the derivative function has discontinuity (holes) at these points.