

Exp) Graph a rational function

$$f(x) = \frac{3x-5}{x-2}, \quad f'(x) = \frac{-1}{(x-2)^2}, \quad f''(x) = \frac{2}{(x-2)^3}$$

Step 1)  $f(x)$

x-int:  $y = f(x) = 0 = \frac{3x-5}{x-2} \quad x = \frac{5}{3} \quad \left(\frac{5}{3}, 0\right)$

y-int:  $x = 0 \Rightarrow f(0) = \frac{-5}{-2} = \frac{5}{2} \quad \left(0, \frac{5}{2}\right)$

\*  $\rightarrow$  include  $x=2$  in sign charts

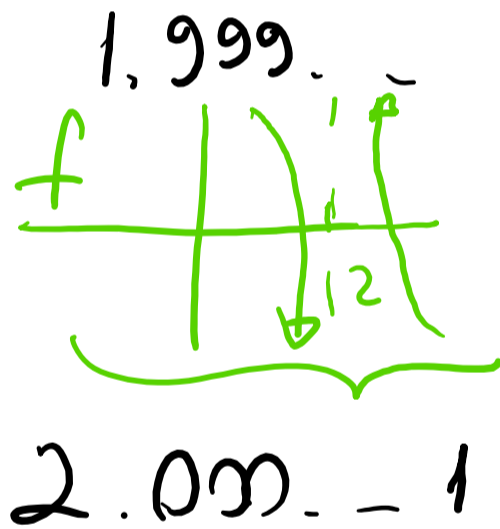
VA:  $x=2$

$$\lim_{x \rightarrow 2^-} \left( \frac{3x-5}{x-2} \right)$$

DSP " "  $\frac{\oplus}{0^-} \rightarrow -\infty$

$$\lim_{x \rightarrow 2^+} \left( \frac{3x-5}{x-2} \right)$$

DSP " "  $\frac{\oplus}{0^+} \rightarrow +\infty$



HA:  $y=3$

$$\lim_{x \rightarrow \pm\infty} \left( \frac{3x-5}{x-2} \right)$$

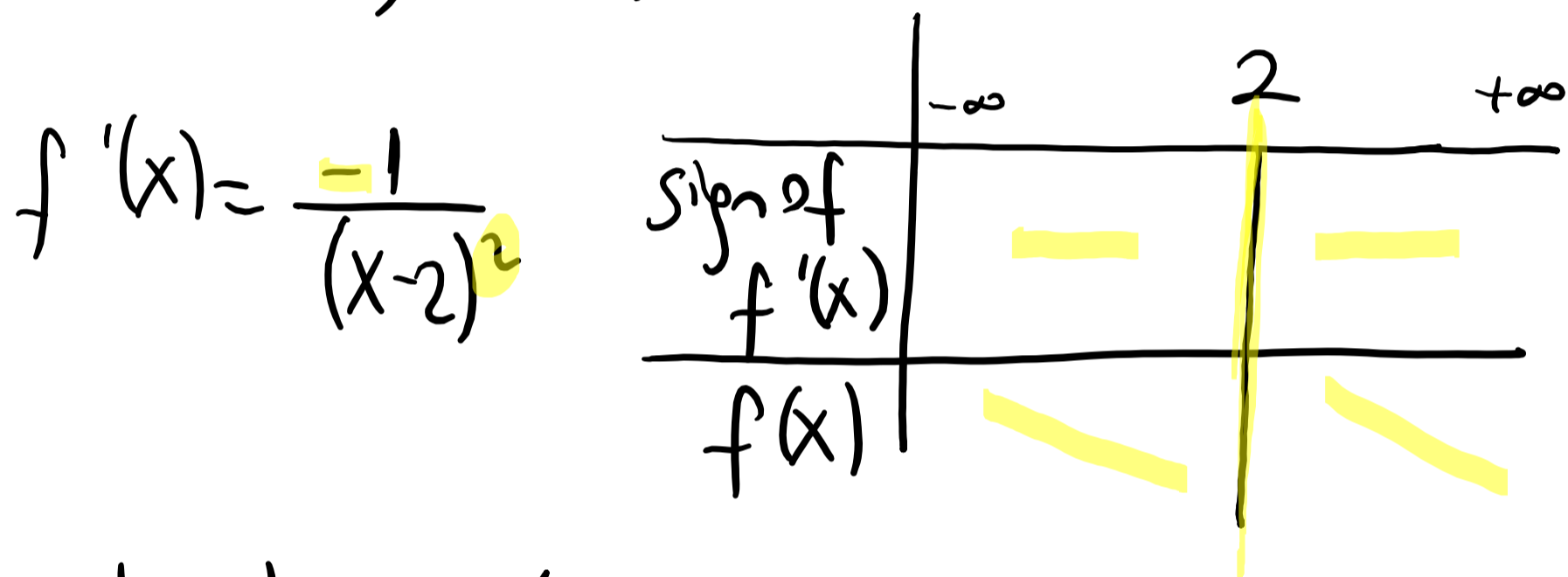
DSP " "  $\frac{\infty}{\infty} \Rightarrow \lim_{x \rightarrow \pm\infty} \left( \frac{3}{1} \right) = 3$

Step 2)  $f'(x) = \frac{-1}{(x-2)^2}$

Critical P:  $f'(x) = 0$  or DNE  
X X

$x=2$  is VA

Since  $x=2$  is VA, therefore, no first-order critical P.



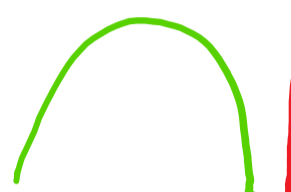
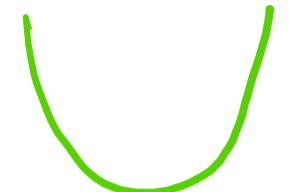
There's no local min/max

$f$  is decreasing on  $(-\infty, 2), (2, \infty)$

Step 3)  $f''(x) = \frac{2}{(x-2)^3}$

VA  
x=2

sign chart

	1	2	3
$f''$	-		+
$f$			

POI?

$f''(x) = 0$  or DNE  
X X

no second-order critical P.

Is  $x=2$  POI?

NO!

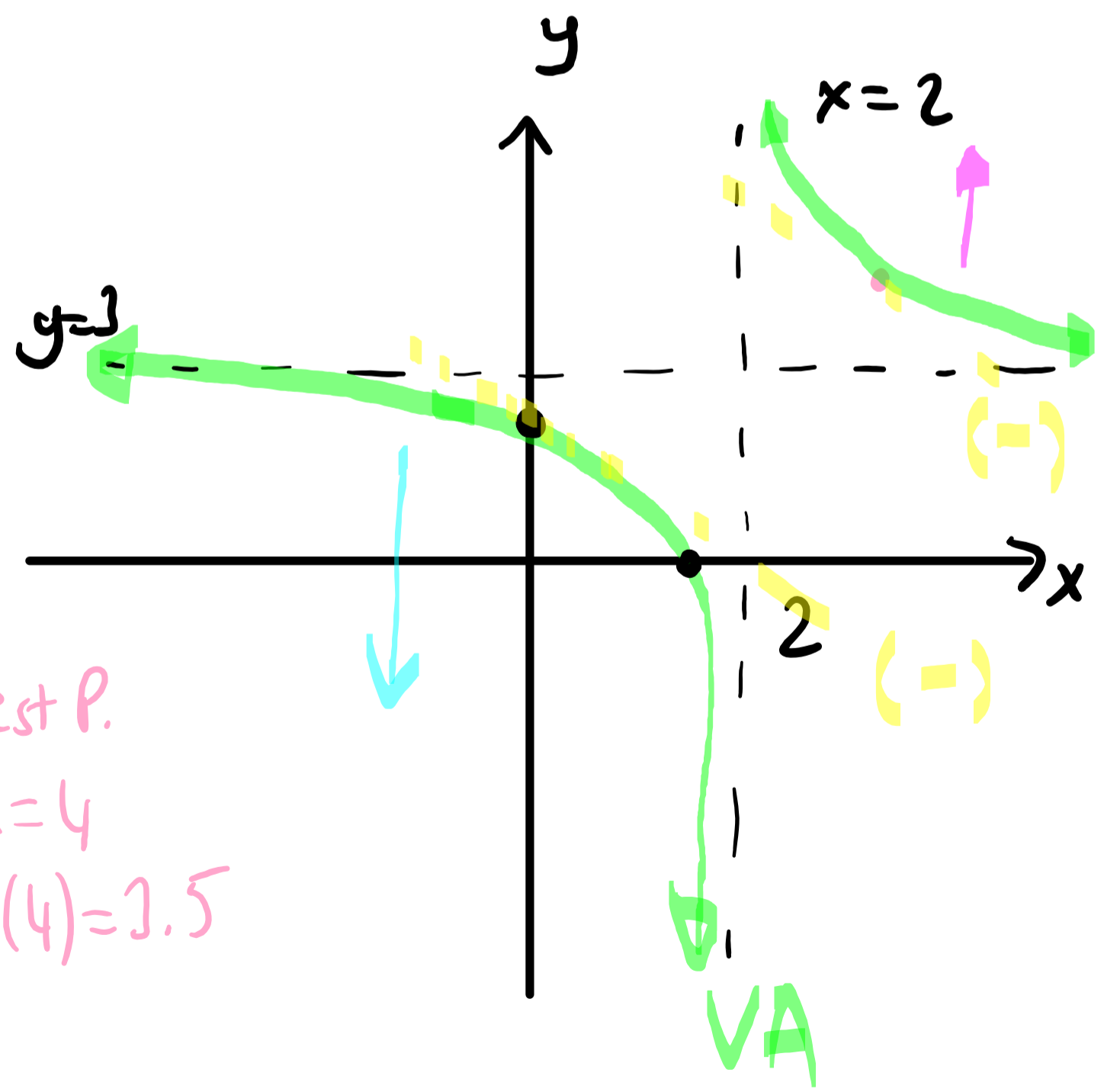
No, because  $x=2$  is VA  
 $f$  is concave down on  $(-\infty, 2)$   
 $f$  is concave up on  $(2, \infty)$

Step 4) Sketch  $f(x)$

VA  $x=2$

HA  $y=3$

$(0, \frac{5}{2}), (\frac{5}{3}, 0)$



test P.  
 $x=4$   
 $f(4) = 3.5$

	$-\infty$	2	$+\infty$
$f'$	-		-
$f''$	-		+
$f$	decr.		decr.

