

Homework: Homework: Section 5.3

Score: 3 of 3 pts

◀ 14 of 14 (1 cor

✔ EXTRA 5.105

Consider the function below and its graph shown to the right.

$$f(t) = \begin{cases} \frac{1}{3}t & \text{if } -4 \leq t < 0 \\ \frac{t^2}{3} & \text{if } 0 \leq t \leq 4 \end{cases}$$

Let $F(x) = \int_{-2}^x f(t) dt$ and $G(x) = \int_{-4}^x f(t) dt$. Complete parts a. through f.

$$\begin{aligned} \text{a. } F(-4) &= \int_{-2}^{-4} f(t) dt = - \int_{-4}^{-2} f(t) dt = - \int_{-4}^{-2} \frac{1}{3}t dt = - \left. \frac{t^2}{2 \cdot 3} \right|_{-4}^{-2} \\ &= - \left(\frac{(-2)^2}{6} - \frac{(-4)^2}{6} \right) = - \left(\frac{4}{6} - \frac{16}{6} \right) = - \left(-\frac{12}{6} \right) = \boxed{2} \end{aligned}$$

$$\begin{aligned} F(4) &= \int_{-2}^4 f(t) dt = \int_{-2}^0 \frac{1}{3}t dt + \int_0^4 \frac{t^2}{3} dt = \left. \frac{t^2}{6} \right|_{-2}^0 + \left. \frac{t^3}{9} \right|_0^4 \\ &= \left(\frac{0}{6} - \frac{(-2)^2}{6} \right) + \left(\frac{4^3}{9} - 0 \right) = \frac{-4}{6} + \frac{64}{9} = \frac{-12+128}{18} = \frac{58}{9} \end{aligned}$$

(3) (2)

b) $F'(x)$ for $-4 \leq x < 0$

$$F'(x) = \frac{d}{dx} \int_{-2}^x f(t) dt = f(x) = \frac{x}{3}$$

recall: $t \rightarrow$ dummy var.

c) $F'(x)$ for $0 \leq x \leq 4$

$$F'(x) = \frac{d}{dx} \int_{-2}^x f(t) dt = f(x) = \frac{x^2}{3}$$

$$d) F'(-2) = \left. \frac{x}{3} \right|_{x=-2} = \frac{-2}{3} \quad F'(2) = \left. \frac{x^2}{3} \right|_{x=2} = \frac{4}{3}$$

The value of $F'(-2)$ is the value of $f(t_1)$, where $t_1 = -2$

The value of $F'(2)$ is the value of $f(t_1)$, where $t_1 = 2$

$$e) F''(-2) = \left(\frac{x}{3} \right)' \Big|_{x=-2} = \frac{1}{3}$$

$$F''(2) = \left(\frac{x^2}{3} \right)' \Big|_{x=2} = \left(\frac{2x}{3} \right) \Big|_{x=2} = \frac{4}{3}$$

f) Find a constant C such that $F(x) = G(x) + C$

$$\left. \int_{-2}^x f(t) dt = \int_{-4}^x f(t) dt + C = \int_{-4}^{-2} f(t) dt + \int_{-2}^x f(t) dt + C \right\} 0 = \int_{-4}^{-2} f(t) dt + C$$

$$0 = \int_{-4}^{-2} \frac{t}{3} dt + C \Rightarrow 0 = \left. \frac{t^2}{6} \right|_{-4}^{-2} + C \Rightarrow 0 = \frac{(-2)^2}{6} - \frac{(-4)^2}{6} + C \Rightarrow 0 = \frac{4}{6} - \frac{16}{6} + C$$

$$0 = -2 + C \Rightarrow C = 2$$