DEFINITION Higher-Order Derivatives

Assuming $y = f(x)$ can be differentiated as often as necessary, the **second derivative** of f is

$$f''(x) = \frac{d}{dx}(f'(x)).$$

For integers $n \geq 1$, the ***n*th derivative** of f is

$$f^{(n)}(x) = \frac{d}{dx}(f^{(n-1)}(x)).$$

➤ Parentheses are placed around n to distinguish a derivative from a power. Therefore, $f^{(n)}$ is the nth derivative of f, and f^n is the function f raised to the nth power. By convention, $f^{(0)}$ is the function f itself.

➤ The notation $\frac{d^2f}{dx^2}$ comes from $\frac{d}{dx}\left(\frac{df}{dx}\right)$ and is read d 2 f dx squared.

Other common notations for the second derivative of $y = f(x)$ include $\frac{d^2y}{dx^2}$ and $\frac{d^2f}{dx^2}$; the notations $\frac{d^ny}{dx^n}$, $\frac{d^nf}{dx^n}$, and $y^{(n)}$ are used for the nth derivative of f.

EXAMPLE 6 **Finding higher-order derivatives** Find the third derivative of the following functions.

a. $f(x) = 3x^3 - 5x + 12$

b. $y = 3t + 2e^t$

a. $f'(x) = 3\cdot 3\cdot x^2 - 5\cdot 1 + 0 = 9x^2 - 5$

$f''(x) = \frac{d^2f}{dx^2} = \frac{d}{dx}\left(9x^2 - 5\right) = 9\cdot 2\cdot x^1 = 18x$

$f'''(x) = \frac{d^3f}{dx^3} = \frac{d}{dx}\left(18x\right) = \boxed{18}$

b. $\frac{dy}{dt} = \frac{d}{dt}(3t + 2e^t) = \underbrace{3 + 2\cdot e^t}$

$\frac{d^2y}{t^2} = (3+2e^t)' = 0 + \underbrace{2e^t}$

$\frac{d^3y}{dt^3} = \frac{d}{dt}(2e^t) = \boxed{2e^t}$