Review of Appl. of Optimization in Business

Goal #1: Max. Profit

$$P(x) = R(x) - C(x) = p(x)\cdot x - C(x)$$

$P'(x) = 0$ $\quad x \to$

Goal #2: Min. Average Cost

$$AC(x) = \frac{C(x)}{x} \qquad \left[AC(x)\right]' = 0 \quad x \to$$

Fall 2019 Exam#2 Q

Exp) If x units of a certain product are produced, $C(x) = 5x^2 + 104x + 80$. Find the level of production ? which minimizes the average cost per unit.

$$AC(x) = \frac{C(x)}{x} = \frac{5x^2+104x+80}{x} = 5x + 104 + \frac{80}{x}$$

$$\rightarrow \left(5x + 104 + 80\cdot x^{-1}\right)' = 5 + 0 + 80\cdot(-1)\cdot x^{-2} = 0$$

$$\left(5 - \frac{80}{x^2}\right) = 0 \Rightarrow \frac{5}{1} = \frac{80}{x^2} \Rightarrow 5x^2 = 80$$

$$x^2 = 16$$

$$x = \pm 4$$

$x = 4$ ✓

$$\left(AC(x)\right)'' = \left(5 - 80\cdot x^{-2}\right)'$$

AC ∪

$$= -80\cdot(-2)\cdot x^{-3} = \frac{160}{x^3} > 0$$

reject $x = -4$ ($x > 0$)