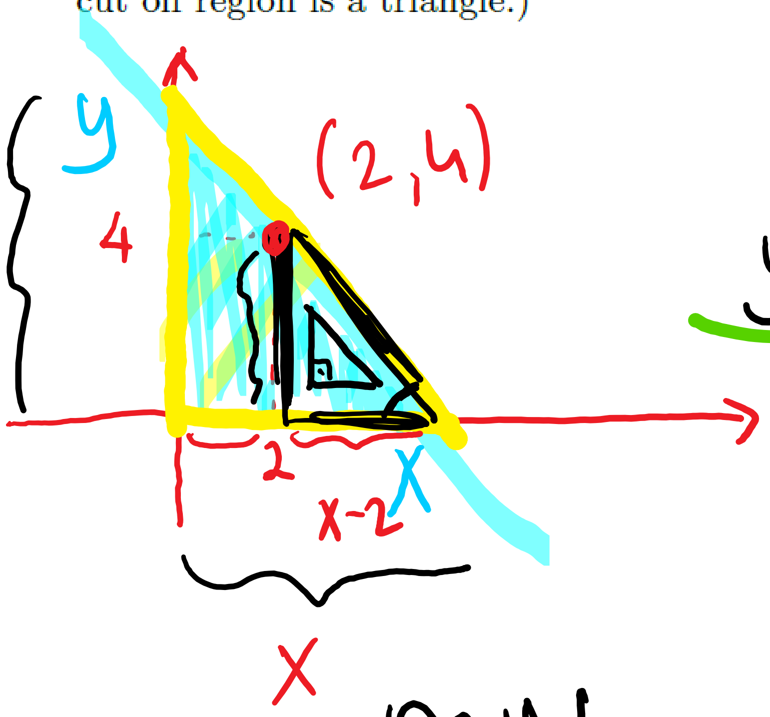
Optimization - Eq. that gives the Min. Area (Coordinator's Website - Fall 2020)

x is in $(0, \infty)$

3. Find the equation of the line through $(2, 4)$ that cuts off the least area from the first quadrant. (Observe that this cut off region is a triangle.)



Area$_\Delta$ $\downarrow$ Obj. F.

$y \to x?$, $x \to y?$ constraint?

$$\text{Area}_\Delta = \frac{x \cdot y}{2}$$

MIN.

$$A(x,y) = \frac{x \cdot y}{2}$$ Obj. F

Construct a similarity statement 4 (Δ with base $x-2$) and y (Δ with base x)

$$\frac{\text{Base}}{\text{Height}} \qquad \frac{x-2}{4} = \frac{x}{y} \Rightarrow \frac{4 \cdot x}{x-2} = \frac{y(x-2)}{x-2} \Rightarrow y = \frac{4x}{x-2}$$

constraint

Re-write the Obj. F. by using the constraint to eliminate a var.

$$A(x,y) \to A(x) \qquad A(x) = \frac{x \cdot \left(\frac{4x}{x-2}\right)}{2} \qquad \text{MIN.}$$

MIN.

global min?

$$A(x) = \frac{\cancel{4}^{2} x^2}{\cancel{2}_{1}(x-2)} = \frac{2x^2}{x-2}$$

$$A(x) = \frac{2x^2}{x-2} \qquad \text{Obj. } \quad \text{MIN.}$$

$$A'(x) = \frac{4x(x-2) - 2x^2 \cdot 1}{(x-2)^2} = \frac{4x^2 - 8x - 2x^2}{(x-2)^2}$$

$$A'(x) = \frac{2x^2 - 8x}{(x-2)^2} = \frac{2x(x-4)}{\cancel{(x-2)^2}}$$

$A'(1) < 0$
$A'(3) < 0$
$A'(6) > 0$

critical P. of $A'(x)$ are $A'(x) = 0$ or DNE

$$\frac{2x \cdot (x-4)}{(x-2)^2} \begin{array}{l} \to x = \cancel{0},\ 4 \\ \to x = 2 \end{array}$$

x is in $(0, \infty)$
$[0, \infty)$

sign chart

	(0	1	2	3	4	6	∞)
$A'(x)$		$-$		$-$	0	$+$	
$A(x)$			VA		local min		

MIN

$\hookrightarrow$ Global min

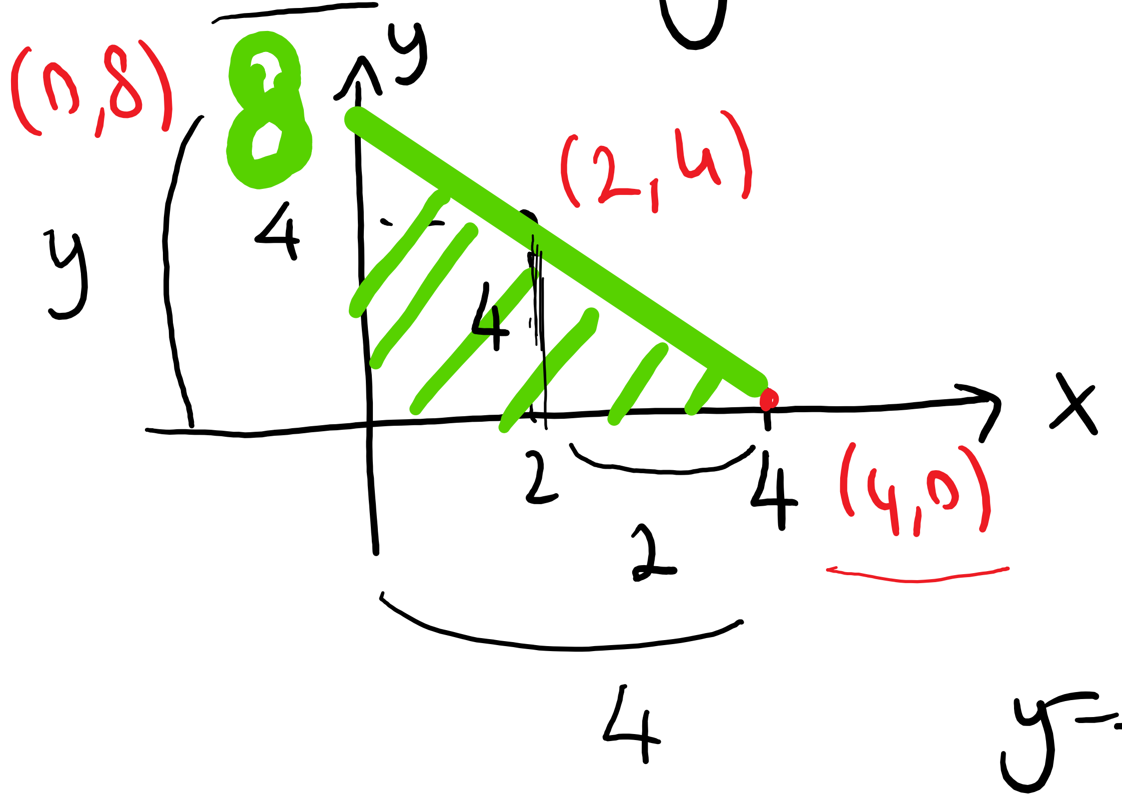
$$A(x) = \frac{2x^2}{x-2} \to \text{VA}$$

1 local min
therefore, global min.

$x=4$ is the global min.

$$A(4)=\frac{\cancel{2}\cdot 4^2}{\cancel{4-2}}=16$$

MIN.



$$\frac{\text{base}}{\text{height}} \rightarrow \frac{2}{4}=\frac{4}{y}$$

$$y=\frac{4x}{x-2}=\frac{4\cdot 4}{4-2}=\frac{16}{2}$$

$$\boxed{y=8}$$

$m=-2$

$(4,0)$

Eq. $y-y_1=m(x-x_1)$

$y-0=-2(x-4)$

$$\boxed{y=-2x+8}$$