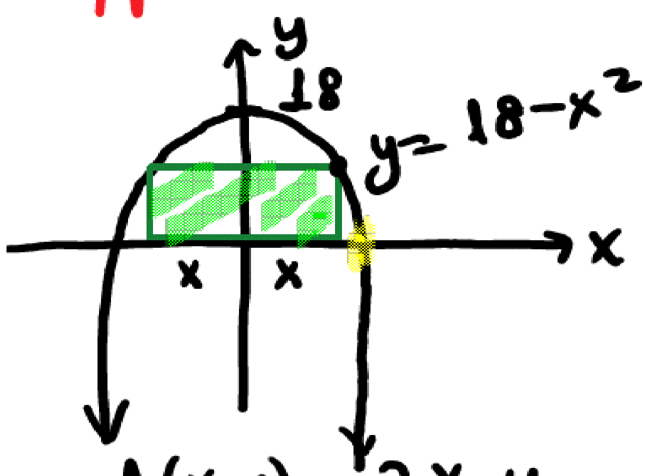
Q8) Find the length and width of the rectangle with the largest area whose lower two vertices lie on the x-axis and whose upper two vertices lie on the graph of $y = 18 - x^2$?



$A_{\text{rectangle}} = l \cdot w$

Let $2x$ be the length

y be the width

$(y = 18 - x^2)$ → constraint

$A(x,y) = 2x \cdot y$

$A(x) = 2x \cdot (18 - x^2) = 36x - 2x^3$ Obj. Function

$A'(x) = 0$ or DNE

$A'(x) = 36 - 6x^2 = 6\underbrace{(6 - x^2)}_{0} = 0$ or ~~DNE~~ polynomial

$6 - x^2 = 0 \Rightarrow x = \pm\sqrt{6}$ (x is a dim. of rect. can not be negative)

$x = \sqrt{6}$

Interval: x could be as wide as the x-int. of y

x-int: $y = 0 \Rightarrow y = 0 = 18 - x^2 \Rightarrow x = \pm\sqrt{18} \Rightarrow \underbrace{x = 3\sqrt{2}}_{\text{pos. only}}$

sign chart

	(0	2	$\sqrt{6}$	3	$3\sqrt{2}$)
$A'(x)$		+	0	−	
$A(x)$			local MAX		

$\underbrace{(0, 3\sqrt{2})}_{\text{interval}}$

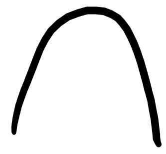
$A'(x) = 6(6 - x^2)$

To justify $x=\sqrt{6}$ really produces the MAX area:

$A'(x) = 6(6-x^2)$

$A''(x) = -12x$

Always negative



concave down

$A(x)$ has only 1 local max, therefore, it's also the global max.

$x=\sqrt{6}$ is actually global max.

$l = 2x = \boxed{2\cdot\sqrt{6}}$, $w = 18 - x^2 \Rightarrow w = 18 - (\sqrt{6})^2 = 18 - 6 = \boxed{12}$