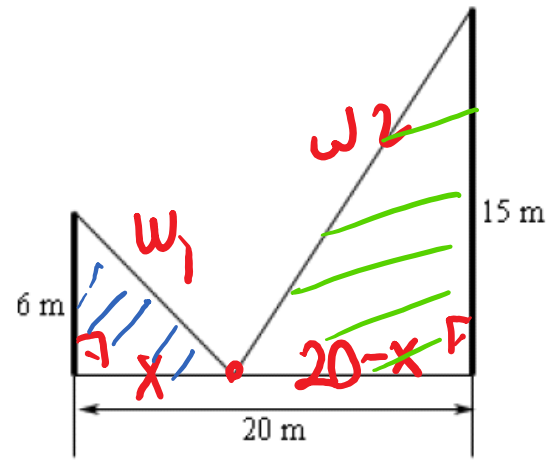
Optimization - Min. Wire Used (From the Coordinator's Website)

4. Two poles, one 6 meters tall and one 15 meters tall, are 20 meters apart. A length of wire is attached to the top of each pole and it is also staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?



Obj: $w_1 + w_2$ ↓

Constraints?

w → wire

$w_1^2 = 6^2 + x^2 \Rightarrow w_1 = \sqrt{36+x^2}$

$w_2^2 = 15^2 + (20-x)^2$

x w_1+w_2 ↓

$w_2 = \sqrt{15^2+(20-x)^2}$

↳ wire

Obj. F: $w_1 + w_2 = W(x)$

$$W(x) = \sqrt{36+x^2} + \sqrt{225+400-40x+x^2} \quad \text{min.}$$

$$W(x) = \sqrt{36+x^2} + \sqrt{625-40x+x^2}$$

$W'(x) = 0$ or DNE cri. P. → sign chart

local ~~→

global ~~.

$$W'(x) = \left[(36+x^2)^{1/2}\right]' + \left[(625-40x+x^2)^{1/2}\right]'$$

$$= \frac{1}{2}(36+x^2)^{-1/2} \cdot 2x + \frac{1}{2}(625-40x+x^2)^{-1/2} \cdot (2x-40)$$

$$2(x-20)$$

$$= \frac{x}{\sqrt{36+x^2}} + \frac{x-20}{\sqrt{625-40x+x^2}}$$

$$W'(x) = \frac{x}{\sqrt{36+x^2}} + \frac{x-20}{\sqrt{625-40x+x^2}} = 0 \quad \text{or DNE}$$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$b^2-4ac < 0 \Rightarrow$ no real sol.

$\sqrt{36+x^2} \rightarrow \infty$ real #

$x^2+36=0$

$\sqrt{625-40x+x^2} \rightarrow x^2-40x+625$

$\Delta = 1600 - 4 \cdot 1 \cdot 625$

$\Delta < 0$

$W'(x)=0 \Rightarrow \left(\frac{x}{\sqrt{36+x^2}}\right)^2 = \left(-\frac{(x-20)}{\sqrt{625-40x+x^2}}\right)^2$ no real #

$$\frac{x^2}{36+x^2} = \frac{(x-20)^2}{625-40x+x^2}$$

$x^2(625-40x+x^2) = (36+x^2)(x-20)^2$

$9(3x+40)(7x-40)=0$

$x = -\frac{40}{3}$, $x = \frac{40}{7}$ critical p.

global min?

endP. $\rightarrow$? $[0,20]$

Global min/max Procedure in 4.1

$W(x)$ $W(0)$, $W(20)$, $W\left(\frac{40}{7}\right)$

$$w(x) = \sqrt{36+x^2} + \sqrt{625-40x+x^2} \quad \text{min?}$$

$$w(0) = \sqrt{36} + \sqrt{625} = 6+25 = 31$$

$$w(20) \approx 35.8805$$

$$w\left(\frac{40}{7}\right) \approx 29 \leftarrow \text{min.}$$

It should be staked $\frac{40}{7}$ meters away from the shorter pole.