Experimental Comparison of Quadrangulation Algorithms for Sets of Points

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Abstract

The classical problem in scattered bivariate data interpolation can be stated as follows. Given a set $V = (v_1, v_2, \ldots, v_n)$ of n points in the plane along with an elevation $z_i, i = 1, 2, \ldots, n$ associated with each point v_i, determine a function f such that $f(v_i) = z_i$, for all i. There is a large body of literature on this subject describing a variety of methods that yield functions with different properties. These methods usually start with a triangulation of V which is subsequently refined in some way. Since a triangulation of a set of points in the plane always exists, there is never a problem starting in this way. Recently, Lai and Schumaker [LS94] showed that there are computational advantages when starting with a quadrangulation of V rather than a triangulation. In a quadrangulation of V the basic elements are quadrangles (quadrilaterals) rather than triangles. Furthermore, for this application, the quadrangle need not be convex although it is desirable that they be *fat*. However, a set of points does not always admit a quadrangulation. This problem can be "solved" in two ways: (1) by adding additional points (Steiner points) to the data, although these points should be kept to a minimum since the data is considered too "sacred" to allow such contamination and (s) by just leaving some remaining triangles in the final "quadrangulation" which are then handled with the classical interpolation theory. Motivated b this application, Bose and Toussaint [BT95] first characterized the sets of points that admit a quadrangulation. One such characterization is surprisingly simple: a set V admits a quadrangulation if, and only if, the convex hull of V contains an *even* number of extreme points. This characterization leads to several algorithms that quadrangulate a set of points using at most *one* Steiner point in optimal $O(n \log n)$ time. Although these methods are optimal with respect to both the time complexity and use of few

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Steiner points they yield quadrangles which are not fat and which are frequently non-convex. A different approach is suggested by another characterization: a set V admits a quadrangulation if, and only if, the set admits a triangulation whose dual graph admits a perfect matching [RRT95]. This characterization suggests algorithms that start with a "good" triangulation such as the Delaunay triangulation and then apply either maximum cardinality or maximum weighted matching algorithms to the dual graph of the triangulation. Some of these algorithms yield very "beautiful" quadrangulations with as many as 98% of the quadrangles convex and fat , and very few remaining triangles on the average. On the other hand, here there is no worst-case guarantee that no more than one Steiner point may be required and the time complexity of the algorithms is far from optimal. In this study we present an empirical comparison of several quadrangulation algorithms and compare them with respect to three objective measures of fatness, the number of convex quadrangles they yield, the number of Steiner points they require and also the subjective beauty of the resulting quadrangulations. The results suggest that there appears to be an inherent trade-off between beauty (fatness) and the number of Steiner points needed and it is an open challenge to find efficient algorithms for obtaining "nice" quadrangulations that use at most one Steiner point in the worst case.

References

[BT95] P. Bose and G. T. Toussaint. No quadrangulation is extremely odd. In *Proc. of the International Symposium on Algorithms and Computation*, pages 372–281, Cairns, Australia, December 4-6 1995.

[LS94] M. J. Lai and L. L. Schumaker. Scattered data interpolation using C^2 piecewise polynomials of degree six. In *Third Workshop on Proximity Graphs*, Starkville, Mississippi, December 1-3 1994. Mississippi State University.

[RRT95] S. Ramaswami, P. Ramos, and G. T. Toussaint. Converting triangulations to quadrangulations. In *Proc. 7th Canadian Conference on Computational Geometry*, pages 297–302, Laval University, Quebec, August 10-13 1995.