

# Time-dependent density functional theory

**E.K.U. Gross**

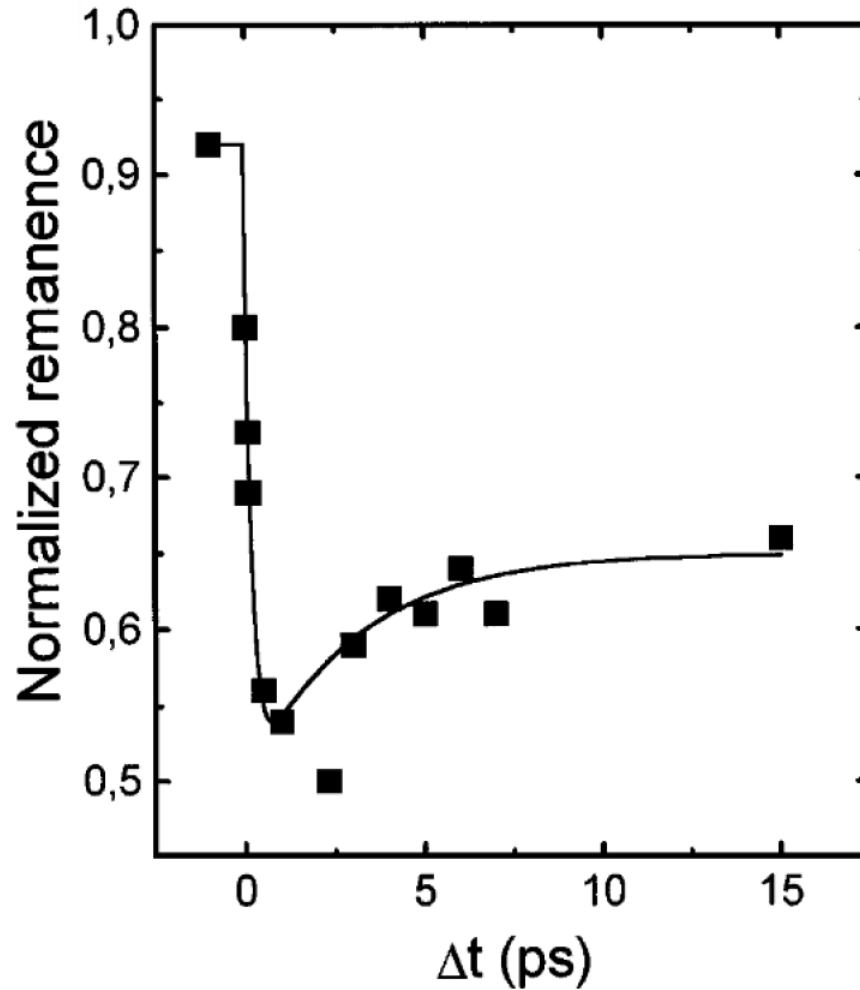
**Fritz Haber Center for Molecular Dynamics**

האוניברסיטה העברית בירושלים  
THE HEBREW UNIVERSITY OF JERUSALEM  
الجامعة العبرية في اورشليم القدس



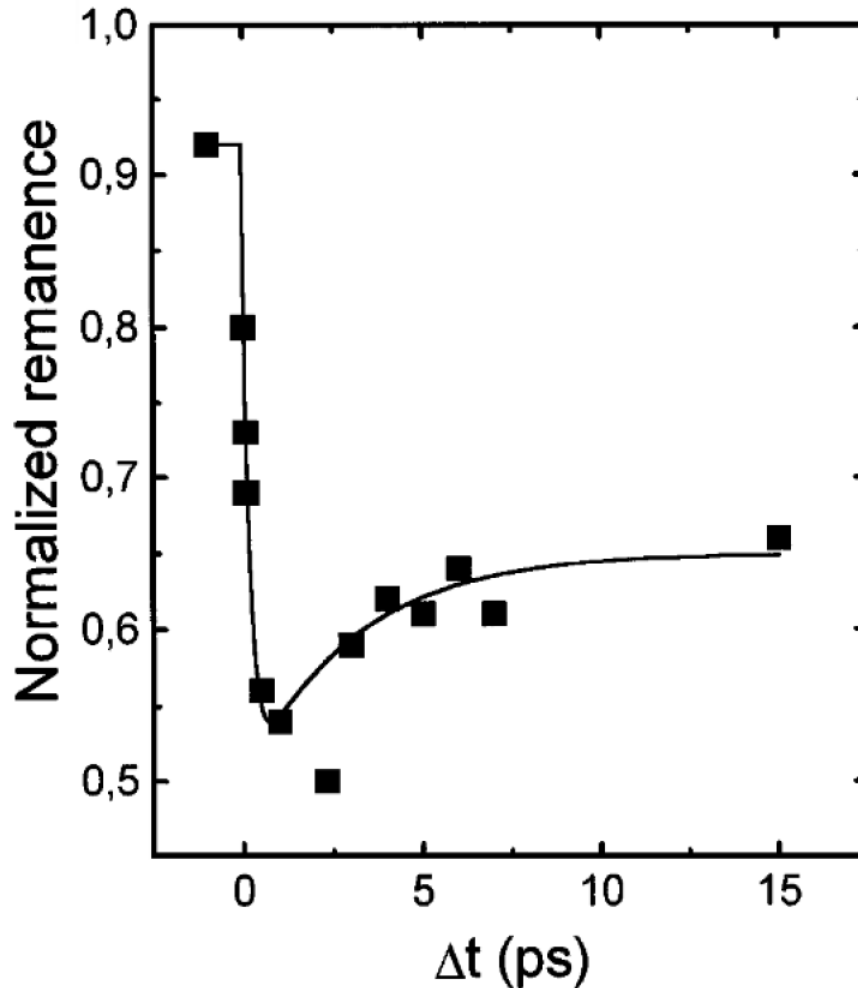
**DAY 2: Real-time TDDFT beyond the linear regime:  
Ultrafast laser-driven spin dynamics in solids**

# First experiment on ultrafast laser induced demagnetization



Beaurepaire et al, PRL 76, 4250 (1996)

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Beaurepaire et al, PRL 76, 4250 (1996)

**Demagnetization in less than 100 fs has been demonstrated experimentally**

# Non-collinear-Spin TDDFT with Spin-Orbit-Coupling (weakly relativistic limit of relativistic TDDFT)

$$i \frac{\partial}{\partial t} \varphi_k(r, t) = \left[ \frac{1}{2} \left( -i\nabla - A_{laser}(t) \right)^2 + v_S[\rho, \mathbf{m}](r, t) - \mu_B \boldsymbol{\sigma} \cdot B_S[\rho, \mathbf{m}](r, t) \right. \\ \left. + \frac{\mu_B}{2c} \boldsymbol{\sigma} \cdot \left( \nabla v_S[\rho, \mathbf{m}](r, t) \right) \times (-i\nabla) \right] \varphi_k(r, t)$$

$$v_S[\rho, \mathbf{m}](r, t) = v_{lattice}(r) + \int \frac{\rho(r', t)}{|r - r'|} d^3 r' + v_{xc}[\rho, \mathbf{m}](r, t)$$

$$B_S[\rho, \mathbf{m}](r, t) = B_{external}(r, t) + B_{xc}[\rho, \mathbf{m}](r, t)$$

where  $\varphi_k(r, t)$  are Pauli spinors

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**Universal  
functionals  
of  $\rho$  and  $\mathbf{m}$**

where  $\varphi_k(r, t)$  are Pauli spinors

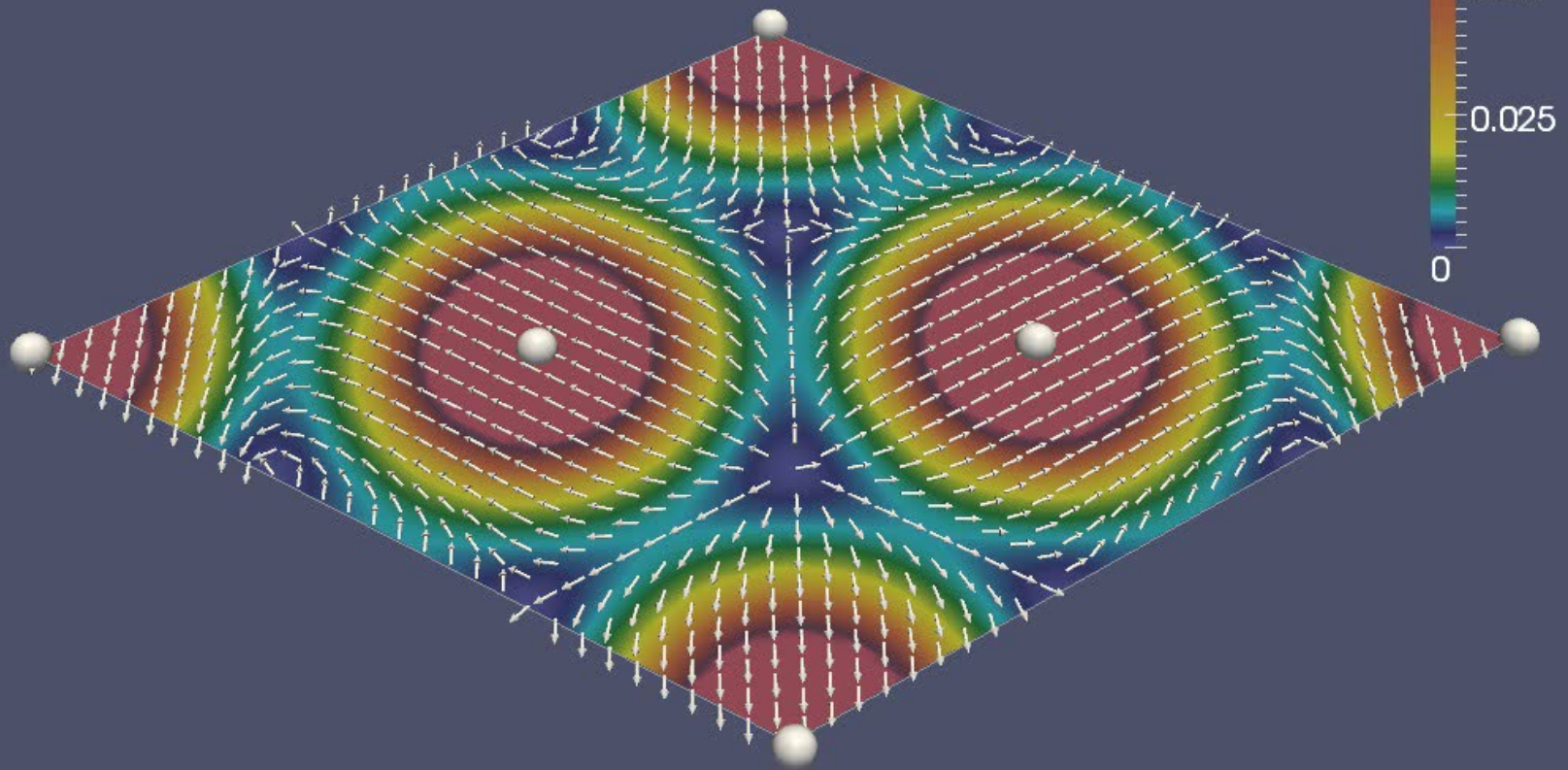
$$n(\mathbf{r}, t) = \sum_{j=1}^N \varphi_j^\dagger(\mathbf{r}, t) \varphi_j(\mathbf{r}, t)$$

$$\vec{\mathbf{m}}(\mathbf{r}, t) = \sum_{j=1}^N \varphi_j^\dagger(\mathbf{r}, t) \vec{\sigma} \varphi_j(\mathbf{r}, t)$$

**Quantity of prime interest:**  
**vector field of spin magnetization**



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**Cr monolayer in ground state**

## Aspects of the numerical implementation

- Wave length of laser in the visible regime  
(very large compared to unit cell)
  - ⇒ Dipole approximation is made  
(i.e. electric field of laser is assumed to be spatially constant)
  - ⇒ Laser can be described by a purely time-dependent vector potential
- **Periodicity of the TDKS Hamiltonian is preserved!**
- **Implementation in ELK code (FLAPW) (<http://elk.sourceforge.net/>)**

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Kay Dewhurst

ELK = Electrons in K-Space  
or  
Electrons in Kay's Space



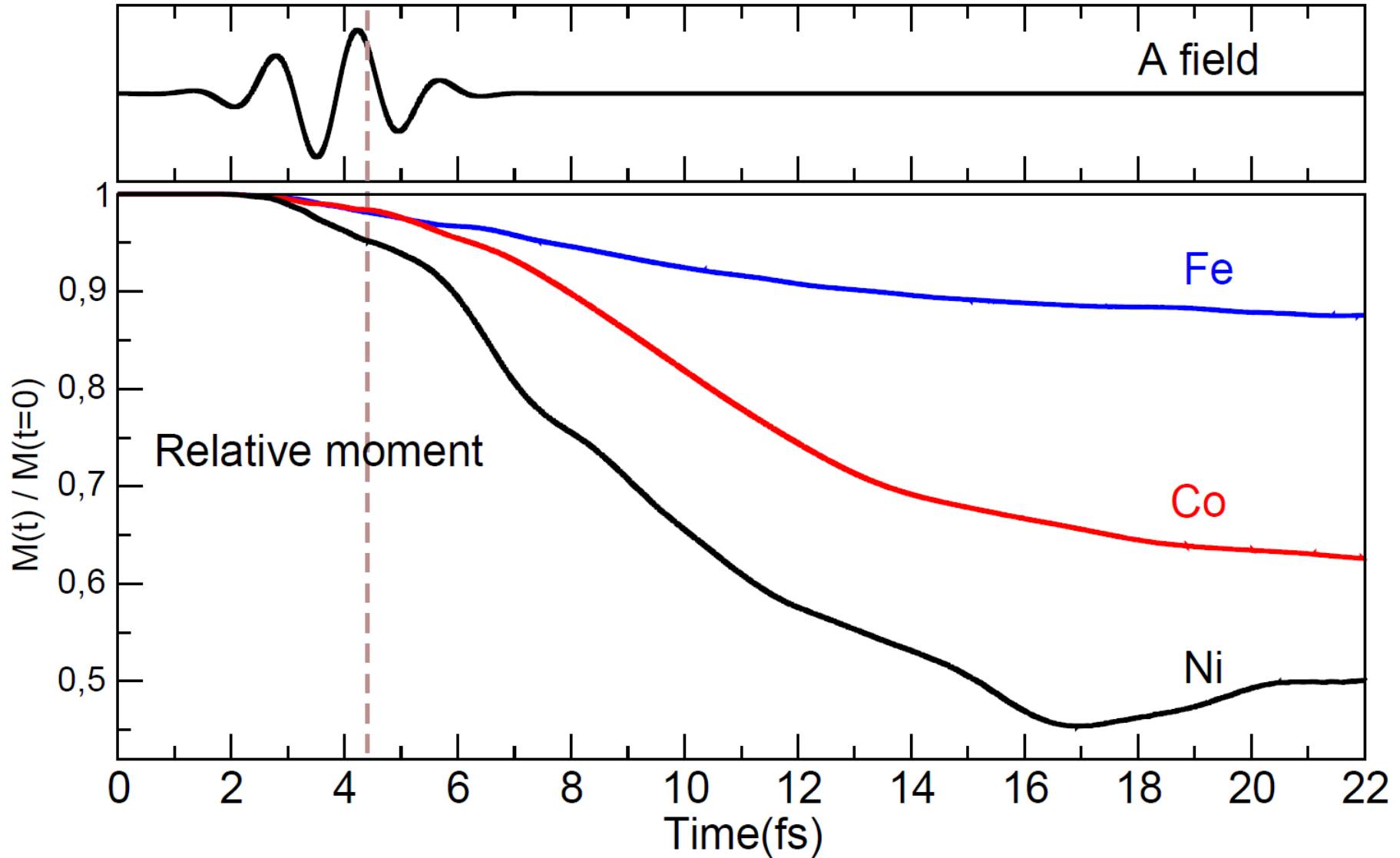
Sangeeta Sharma

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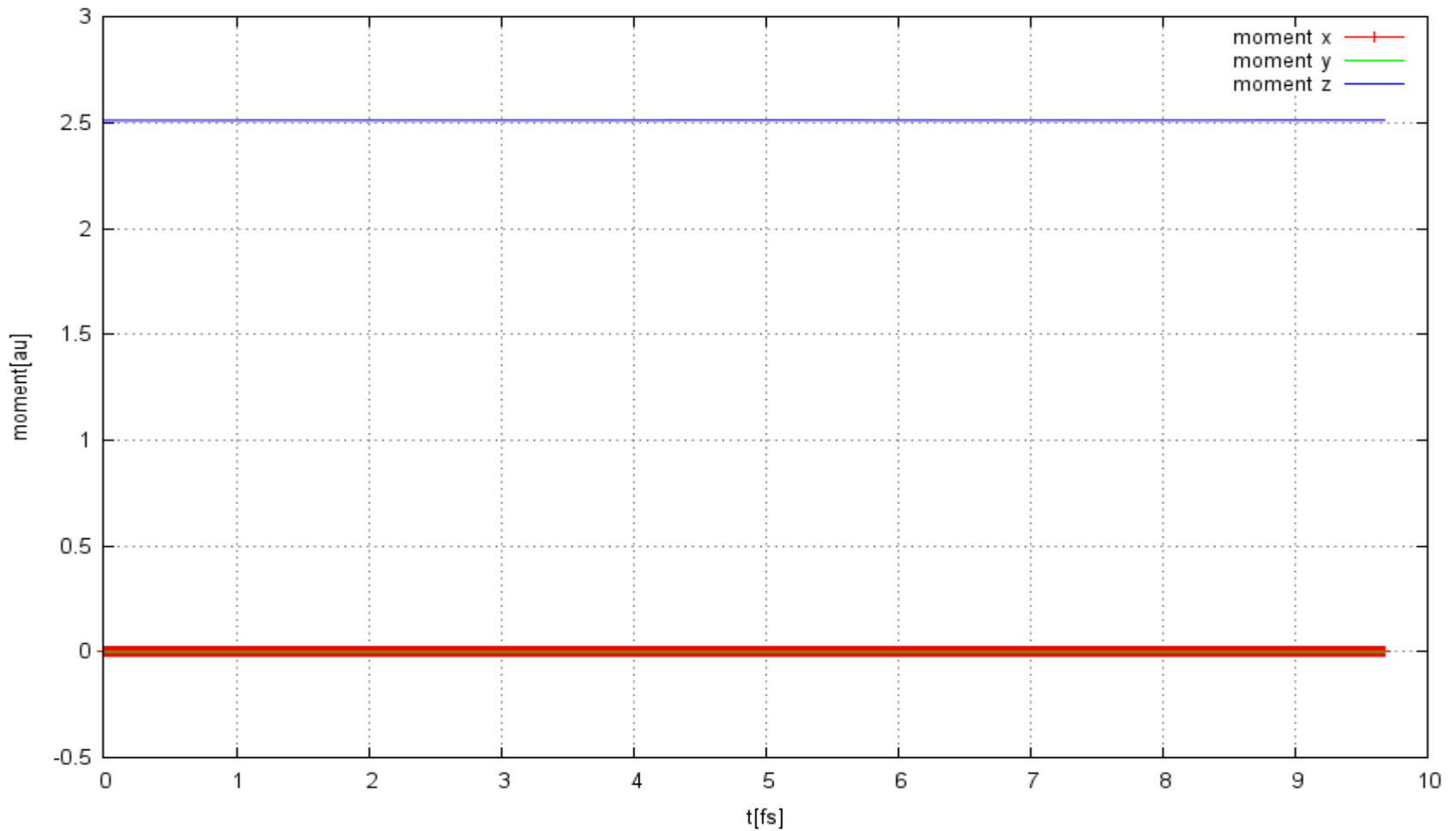
# Demagnetisation in Fe, Co and Ni



# **Analysis of the results**

# Calculation without spin-orbit coupling

components of spin moment



## Exact equation of motion

$$\begin{aligned}\frac{\partial}{\partial t} m_z(\mathbf{r}, t) &= \frac{i}{\hbar} \left\langle \Phi_{\text{KS}}^{\text{det}}(t) \left| \left[ \hat{H}_{\text{KS}}, \hat{m}_z(\mathbf{r}) \right] \right| \Phi_{\text{KS}}^{\text{det}}(t) \right\rangle \\ &= \left\{ m_x(\mathbf{r}, t) B_{xc,y}(\mathbf{r}, t) - m_y(\mathbf{r}, t) B_{xc,x}(\mathbf{r}, t) \right\} \\ &\quad + \frac{1}{2c^2} \left\{ \hat{x} \cdot \left[ \nabla v_s(\mathbf{r}, t) \times \mathbf{j}_y(\mathbf{r}, t) \right] - \hat{y} \cdot \left[ \nabla v_s(\mathbf{r}, t) \times \mathbf{j}_z(\mathbf{r}, t) \right] \right\} \\ &\quad - \nabla \cdot \mathbf{j}_z(\mathbf{r}, t)\end{aligned}$$

$$\vec{\mathbf{j}}(\mathbf{r}, t) = \left\langle \hat{\sigma} \otimes \hat{\mathbf{j}}(\mathbf{r}) \right\rangle \quad \text{spin current tensor}$$



# Exact equation of motion

$$\begin{aligned} \frac{\partial}{\partial t} m_z(\mathbf{r}, t) &= \frac{i}{\hbar} \left\langle \Phi_{\text{KS}}^{\text{det}}(t) \left| \left[ \hat{H}_{\text{KS}}, \hat{m}_z(\mathbf{r}) \right] \right| \Phi_{\text{KS}}^{\text{det}}(t) \right\rangle && \text{local spin torque} \\ & && (\mathbf{m} \times \mathbf{B}_{\text{xc}})_z \\ &= \left\{ m_x(\mathbf{r}, t) B_{\text{xc},y}(\mathbf{r}, t) - m_y(\mathbf{r}, t) B_{\text{xc},x}(\mathbf{r}, t) \right\} && \text{---} \uparrow \\ &+ \frac{1}{2c^2} \left\{ \hat{x} \cdot \left[ \nabla v_s(\mathbf{r}, t) \times \mathbf{j}_y(\mathbf{r}, t) \right] - \hat{y} \cdot \left[ \nabla v_s(\mathbf{r}, t) \times \mathbf{j}_z(\mathbf{r}, t) \right] \right\} && \text{SOC} \\ &- \nabla \cdot \mathbf{j}_z(\mathbf{r}, t) && \text{"continuity"} \end{aligned}$$

$$\vec{\mathbf{j}}(\mathbf{r}, t) = \left\langle \hat{\sigma} \otimes \hat{\mathbf{j}}(\mathbf{r}) \right\rangle \quad \text{spin current tensor}$$



## Exact equation of motion for total moment

$$\begin{aligned}\frac{\partial}{\partial t} M_z(t) &= \frac{i}{\hbar} \int d^3r \left\langle \Phi_{\text{KS}}^{\text{det}}(t) \left| \left[ \hat{H}_{\text{KS}}, \hat{m}_z(\mathbf{r}) \right] \right| \Phi_{\text{KS}}^{\text{det}}(t) \right\rangle \\ &= \int d^3r \left\{ m_x(\mathbf{r}, t) B_{xc,y}(\mathbf{r}, t) - m_y(\mathbf{r}, t) B_{xc,x}(\mathbf{r}, t) \right\} \\ &+ \int d^3r \frac{1}{2c^2} \left\{ \hat{x} \cdot \left[ \nabla v_s(\mathbf{r}, t) \times \mathbf{j}_y(\mathbf{r}, t) \right] - \hat{y} \cdot \left[ \nabla v_s(\mathbf{r}, t) \times \mathbf{j}_z(\mathbf{r}, t) \right] \right\} \text{ SOC} \\ &- \int d^3r \left\{ \nabla \cdot \mathbf{j}_z(\mathbf{r}, t) \right\} = \mathbf{0}\end{aligned}$$

$$\vec{\mathbf{j}}(\mathbf{r}, t) = \left\langle \hat{\sigma} \otimes \hat{\mathbf{j}}(\mathbf{r}) \right\rangle \quad \text{spin current tensor}$$

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 &= \int d^3r \left\{ m_x(r, t) B_{xc,y}(rt) - m_y(r, t) B_{xc,x}(rt) \right\} \\
 &+ \int d^3r \frac{1}{2c^2} \left\{ \hat{x} \cdot \left[ \nabla v_s(r, t) \times j_y(r, t) \right] - \hat{y} \cdot \left[ \nabla v_s(r, t) \times j_z(r, t) \right] \right\} \\
 &- \int d^3r \left\{ \nabla \cdot j_z(r, t) \right\} = 0
 \end{aligned}$$

**Global torque  
exerted by  $B_{xc}$   
= 0 (zero  
torque  
theorem)**



**SOC**

$$\vec{j}(r, t) = \left\langle \hat{\sigma} \otimes \hat{j}(r) \right\rangle \quad \text{spin current tensor}$$

# Exact equation of motion for total moment

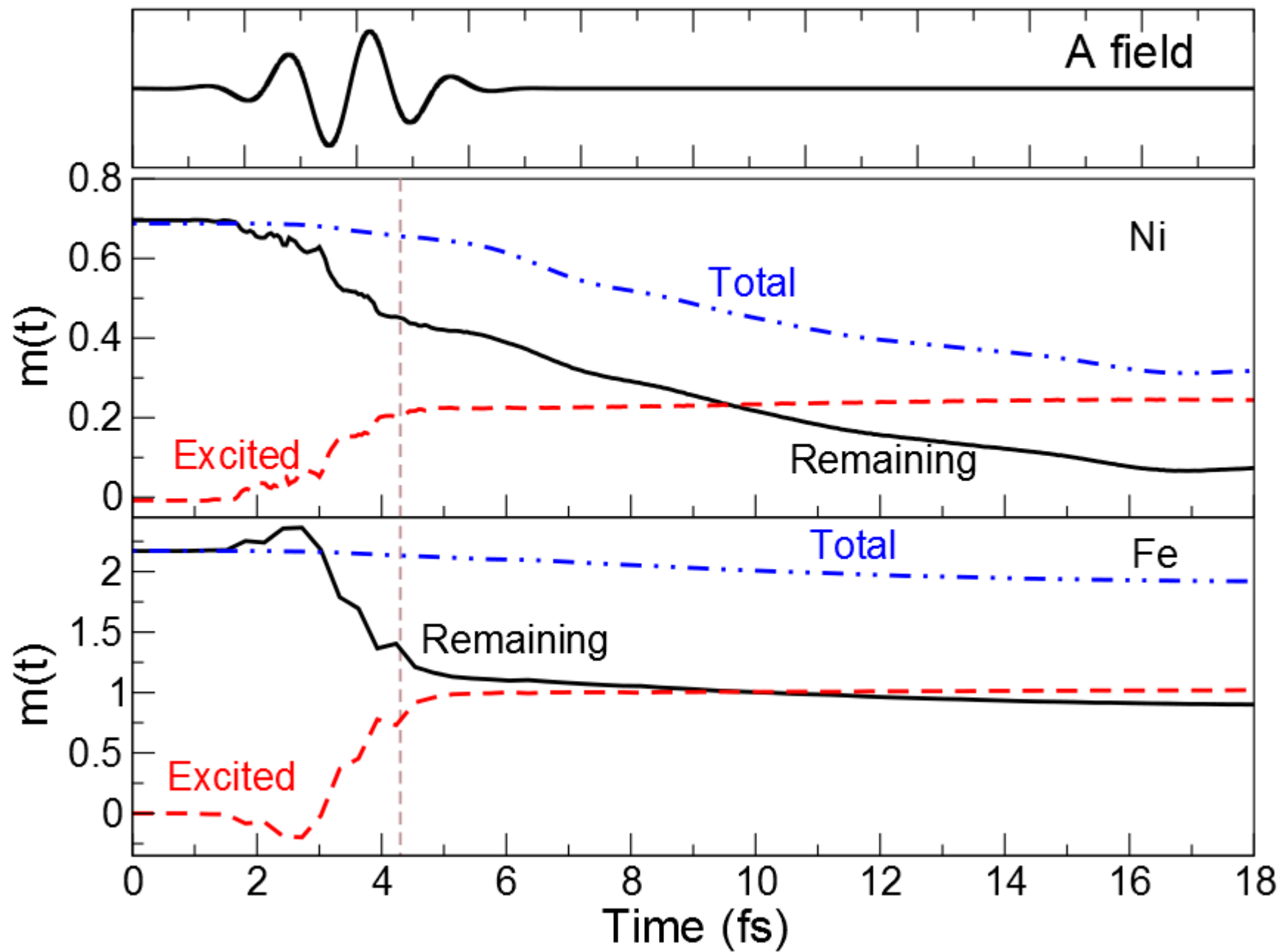
$$\begin{aligned}
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 \end{aligned}$$

**Global torque  
exerted by  $B_{xc}$   
= 0 (zero  
torque  
theorem)**

**SOC**

$$\vec{\mathbf{j}}(\mathbf{r}, t) = \left\langle \hat{\sigma} \otimes \hat{\mathbf{j}}(\mathbf{r}) \right\rangle \quad \text{spin current tensor}$$

**SOC is the only term which can change the total moment!**



## Demagnetization occurs in two steps:

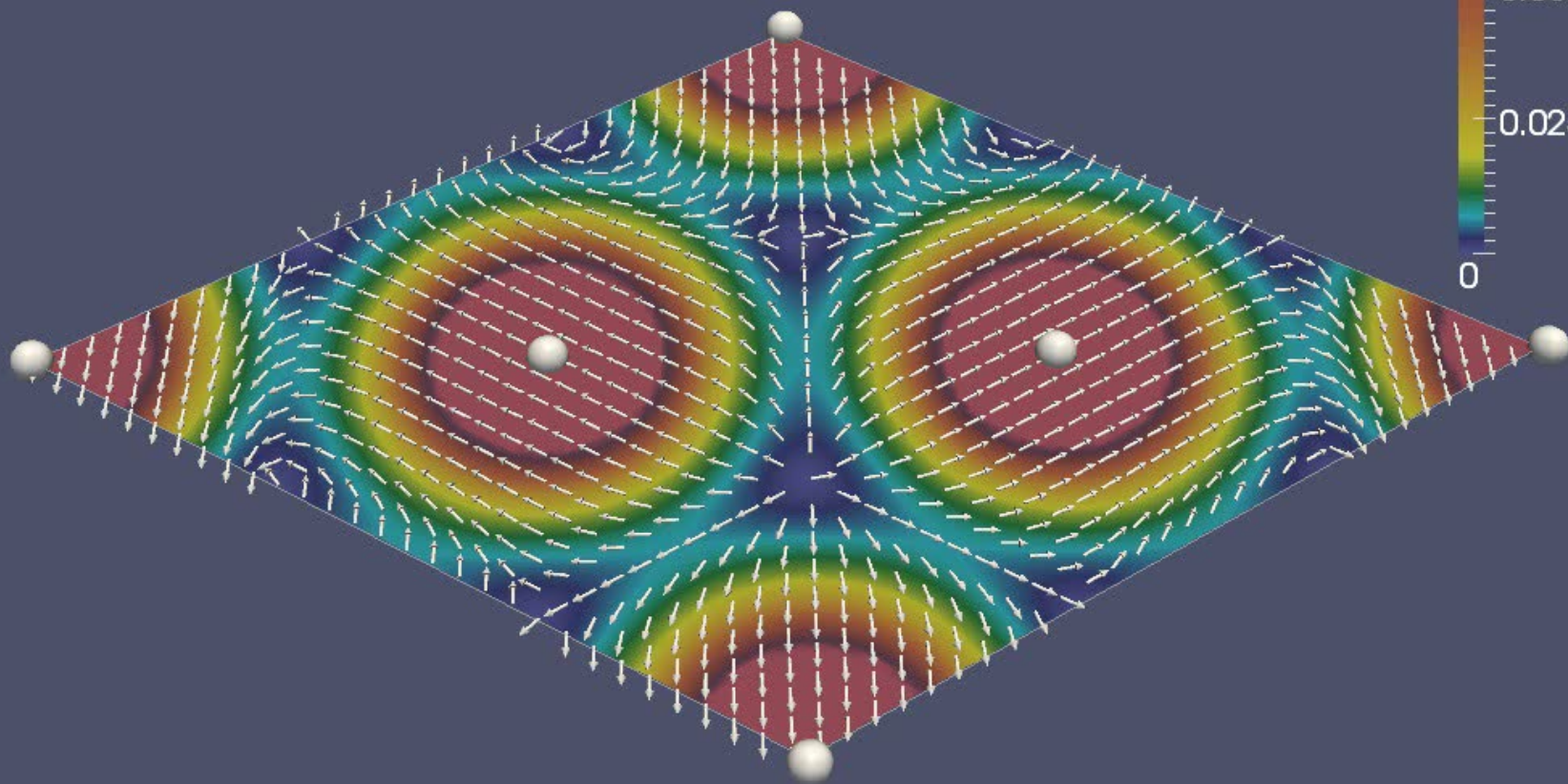
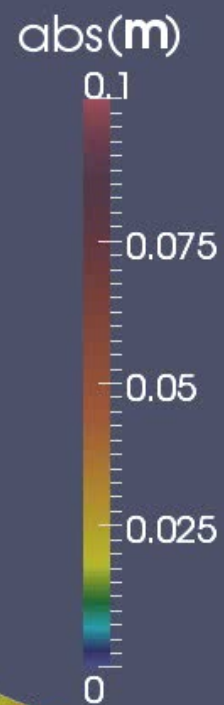
- Initial excitation by laser *moves* magnetization from atomic region into interstitial region. Total Moment is basically conserved during this phase.
- Spin-Orbit term drives demagnetization of the more localized electrons until stabilization at lower moment is achieved
- This is a local mechanism, hence occurs in this form in essentially all systems, e.g. magnetic clusters (Sanvito group, Dublin) or magnetic mono-layer / few-layer systems
- **K. Krieger, J.K. Dewhurst, P. Elliott, S. Sharma, E.K.U. Gross, JCTC 11, 4870 (2015).**
- **K. Krieger, P. Elliott, T. Müller, N. Singh, J. K. Dewhurst, E.K.U. Gross, S. Sharma, J. Phys. Cond. Matter 29, 224001 (2017).**
- **V. Shokeen, M. Sanchez Piaia, J.Y. Bigot, T. Mueller, P. Elliott, J.K. Dewhurst, S. Sharma, E.K.U. Gross, Phys. Rev. Lett. 119, 107203 (2017).**

# **Beyond 3D bulk**

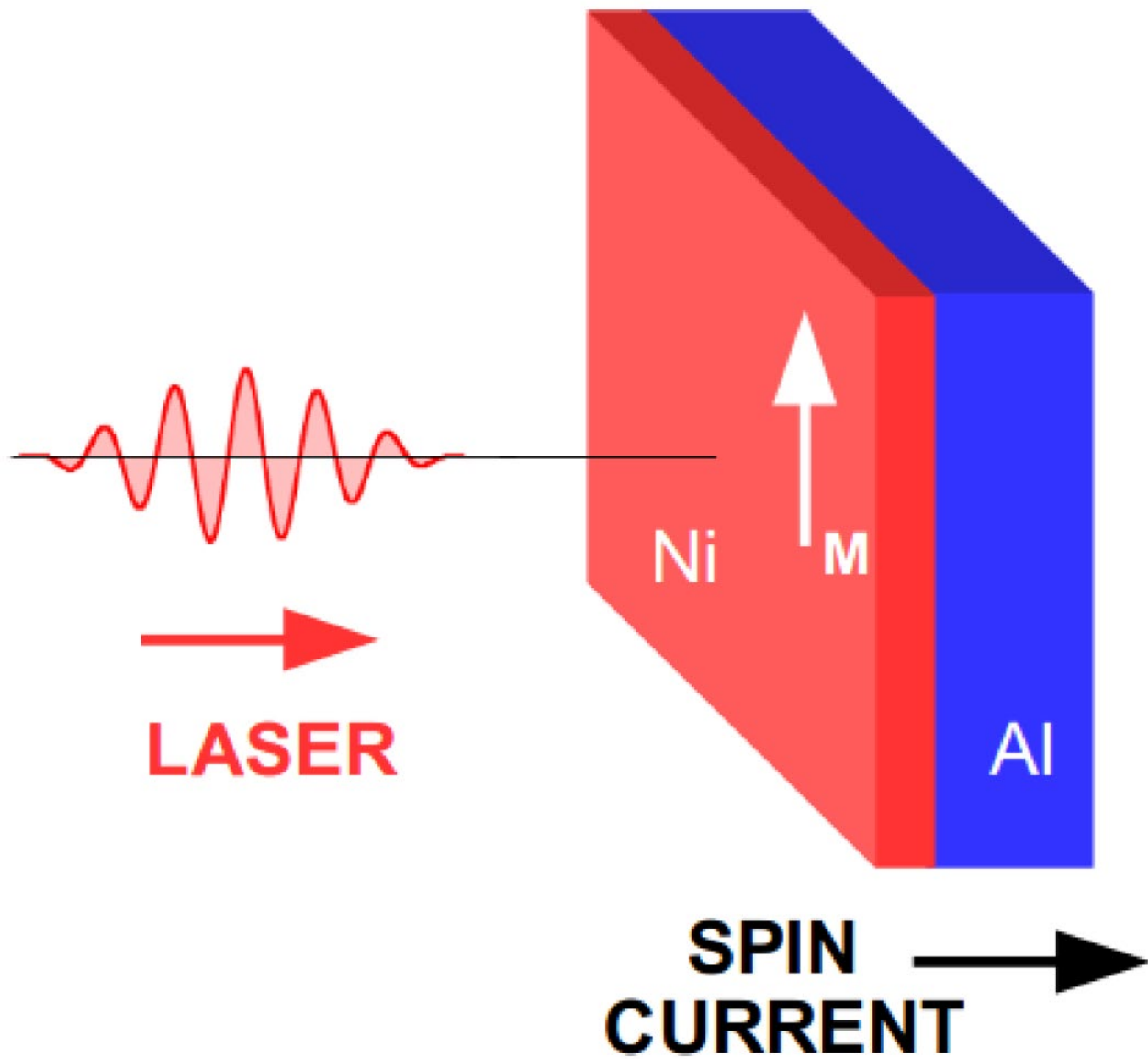


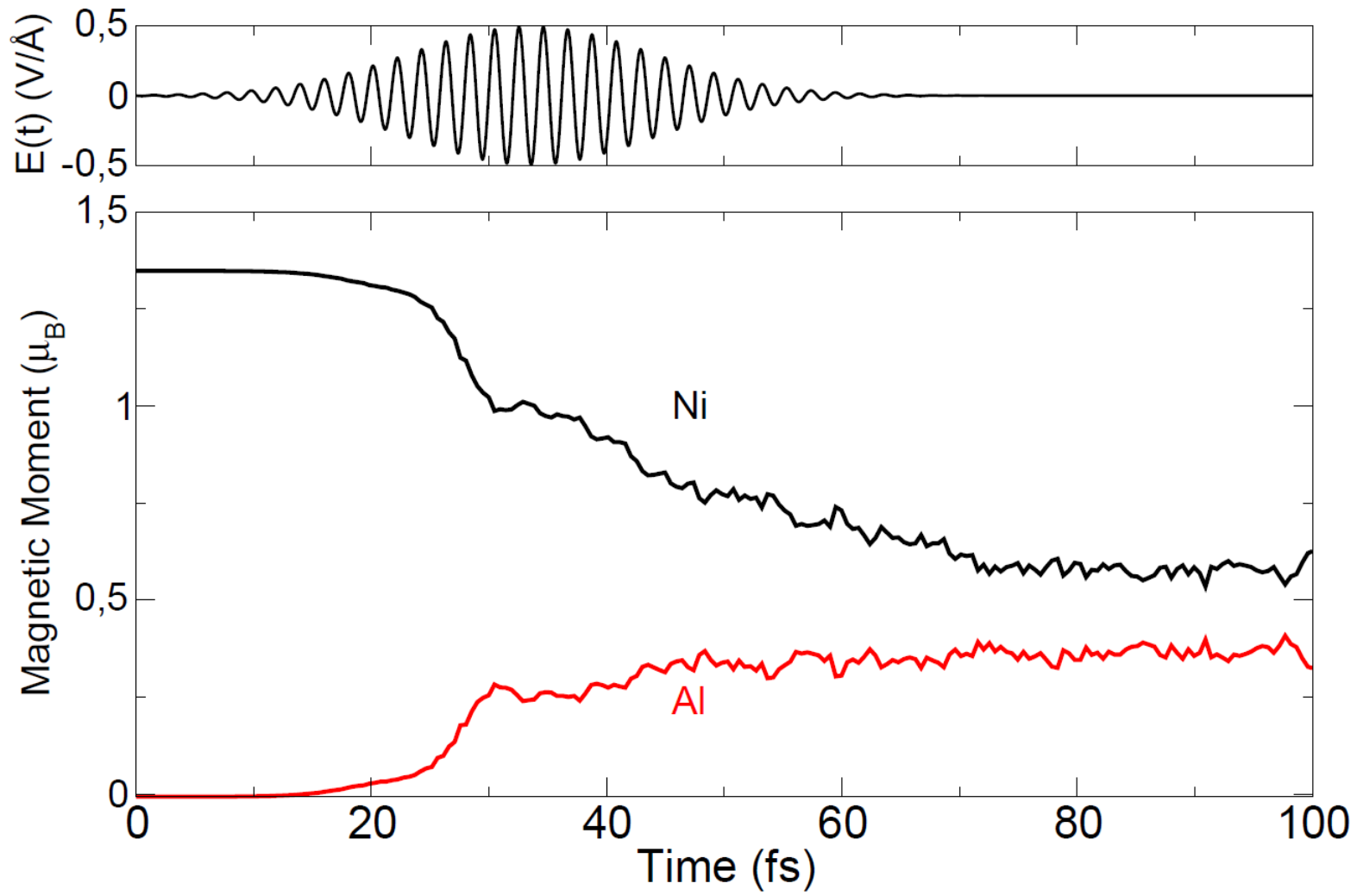
Time: 0.0 fs

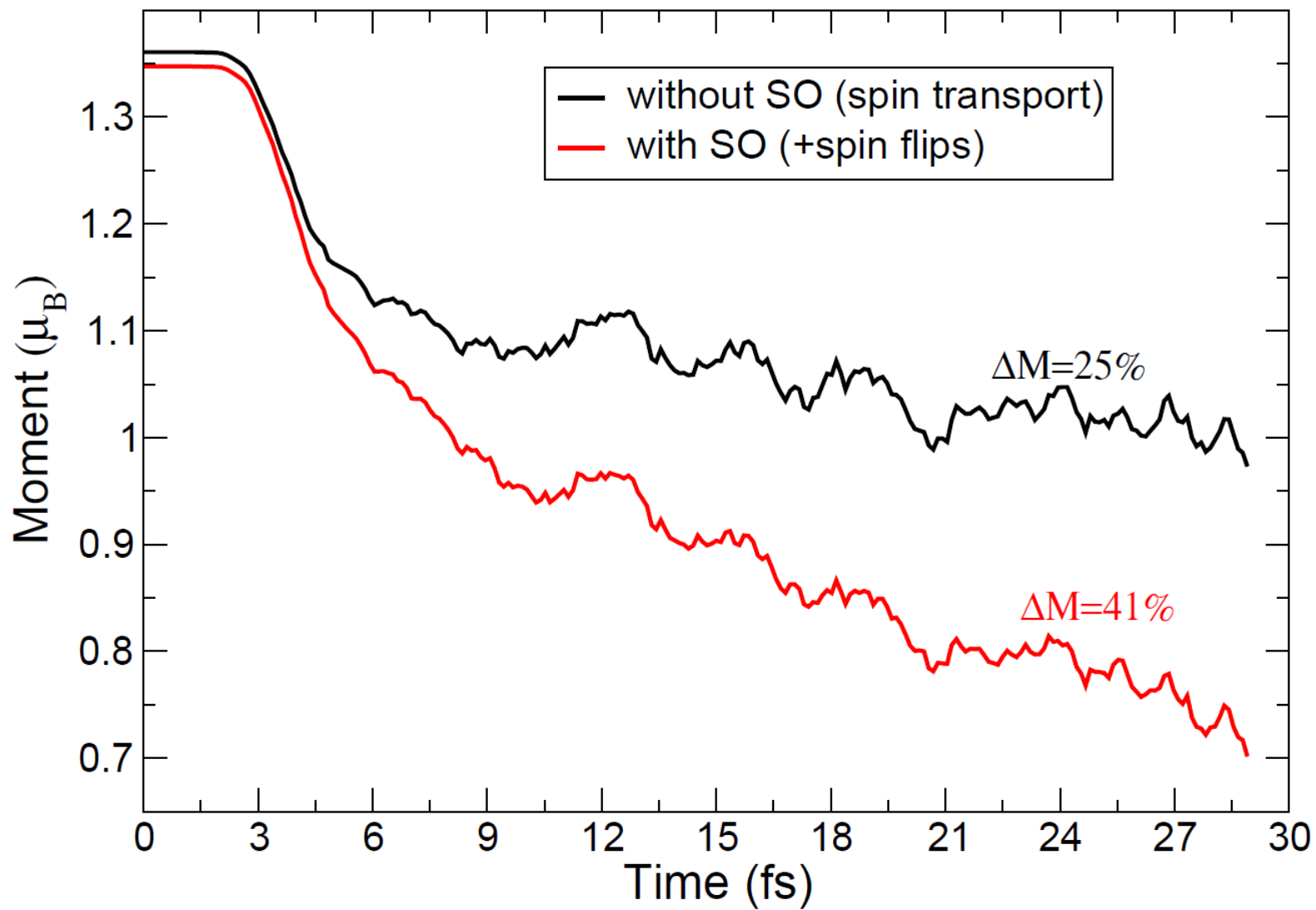
E-field



**Cr monolayer**







## Experimental confirmation:

*Spin Flips versus Spin Transport in Nonthermal Electrons Excited by Ultrashort Optical Pulses in Transition Metals,*

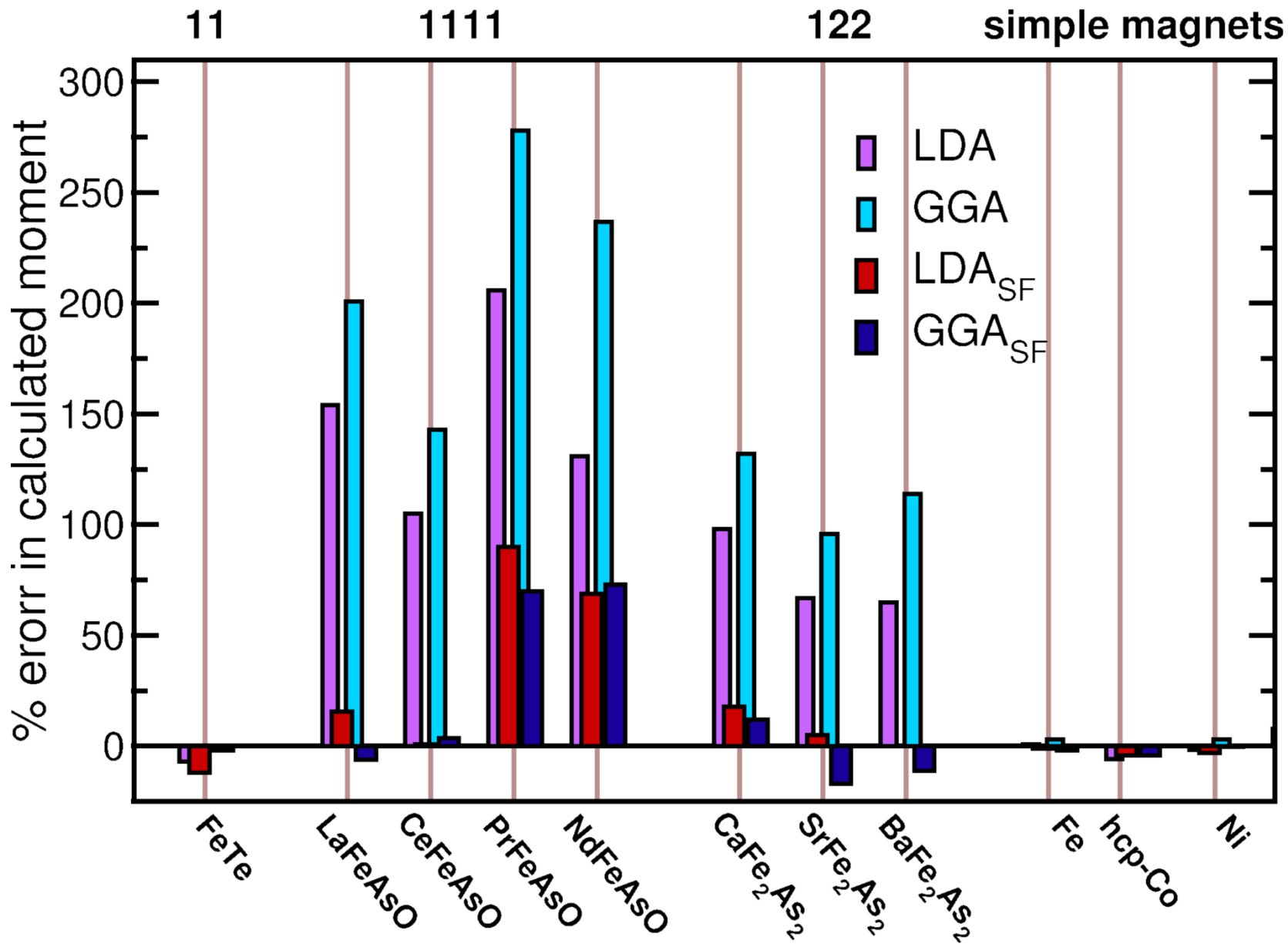
V. Shokeen, M. Sanchez Piaia, J.-Y. Bigot, T. Mueller, P. Elliott, J.K. Dewhurst, S. Sharma, E. K. U. Gross, *Phys. Rev. Lett.* **119**, 107203 (2017).

## Review article:

*Time-Dependent Density Functional Theory for Spin Dynamics,*

P. Elliott, M. Stamenova, J. Simoni, S. Sharma, S. Sanvito, and E.K.U. Gross, in: *Handbook of Materials Modeling*, W. Andreoni, S. Yip eds, Springer (2020), p. 841

# **xc-functional in SDFT**



# Construction of a novel xc functional for non-collinear magnetism

Enforce property of the exact xc functional:

$$\mathbf{B}_{xc}^{exact}(\mathbf{r}) = \nabla \times \mathbf{A}_{xc}^{exact}(\mathbf{r})$$

**K. Capelle, E.K.U. Gross, PRL 78, 1872 (1997)**

By virtue of Helmholtz' theorem, any vector field can be decomposed as:

$$\mathbf{B}_{xc}^{GGA}(\mathbf{r}) = \nabla \times \mathbf{A}_{xc}(\mathbf{r}) + \nabla \phi(r)$$

Enforce exact property by subtracting source term!



## Explicit construction:

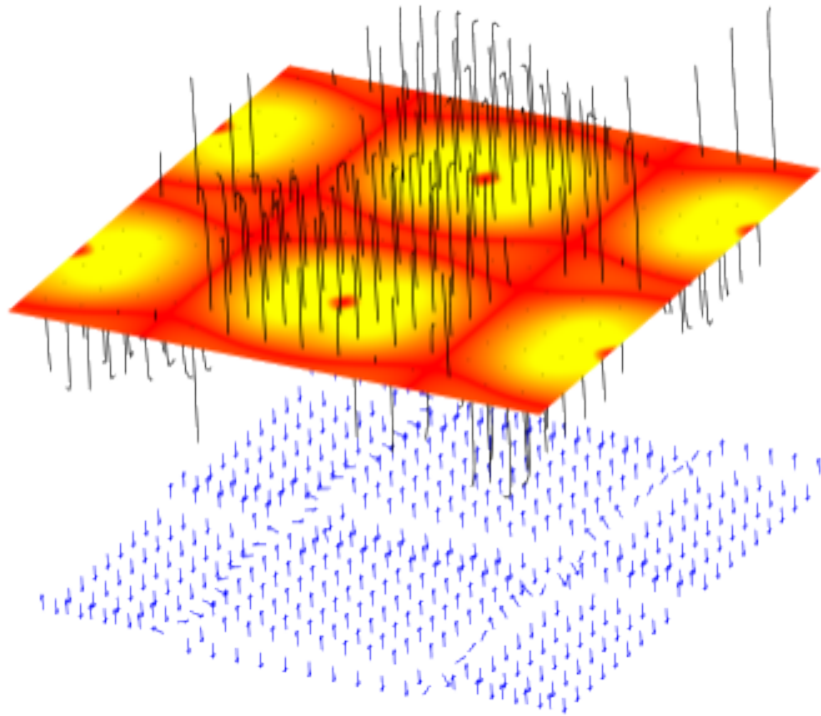
S. Sharma, E.K.U. Gross, A. Sanna, K. Dewhurst, JCTC14, 1247 (2018)

$$\nabla^2 \phi(\mathbf{r}) = 4\pi \nabla \cdot B_{xc}^{GGA}(\mathbf{r})$$

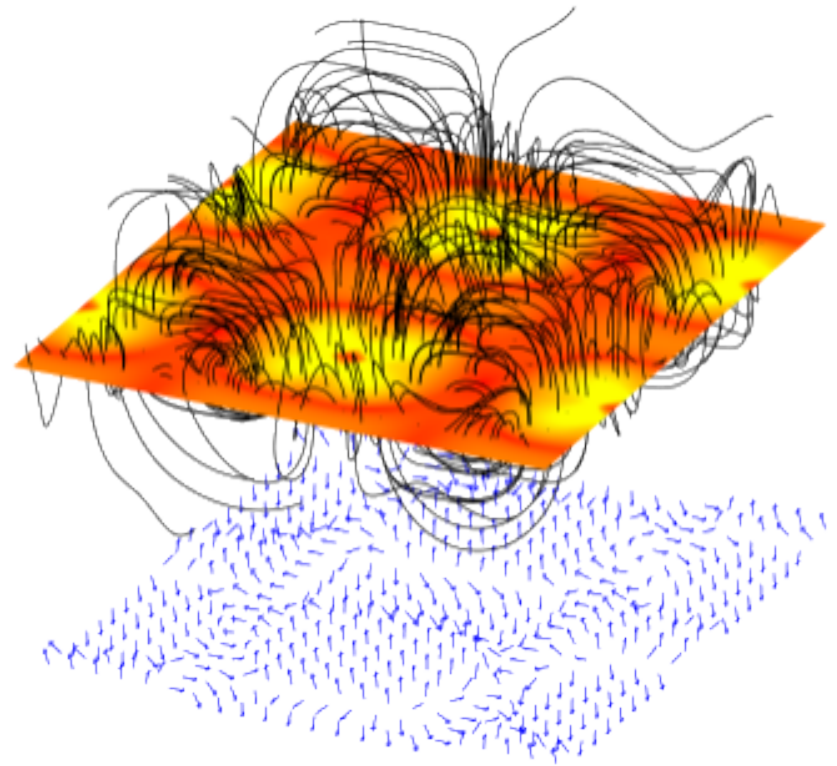
$$\tilde{B}_{xc}(\mathbf{r}) \cong B_{xc}^{GGA}(\mathbf{r}) - \frac{1}{4\pi} \nabla \phi(\mathbf{r})$$

$$B_{xc}^{SF}(\mathbf{r}) = s \tilde{B}_{xc}(\mathbf{r})$$

Scaling factor, s, only depends on underlying functional (GGA/LSDA), nothing else. (s = 1.14 for GGAs)

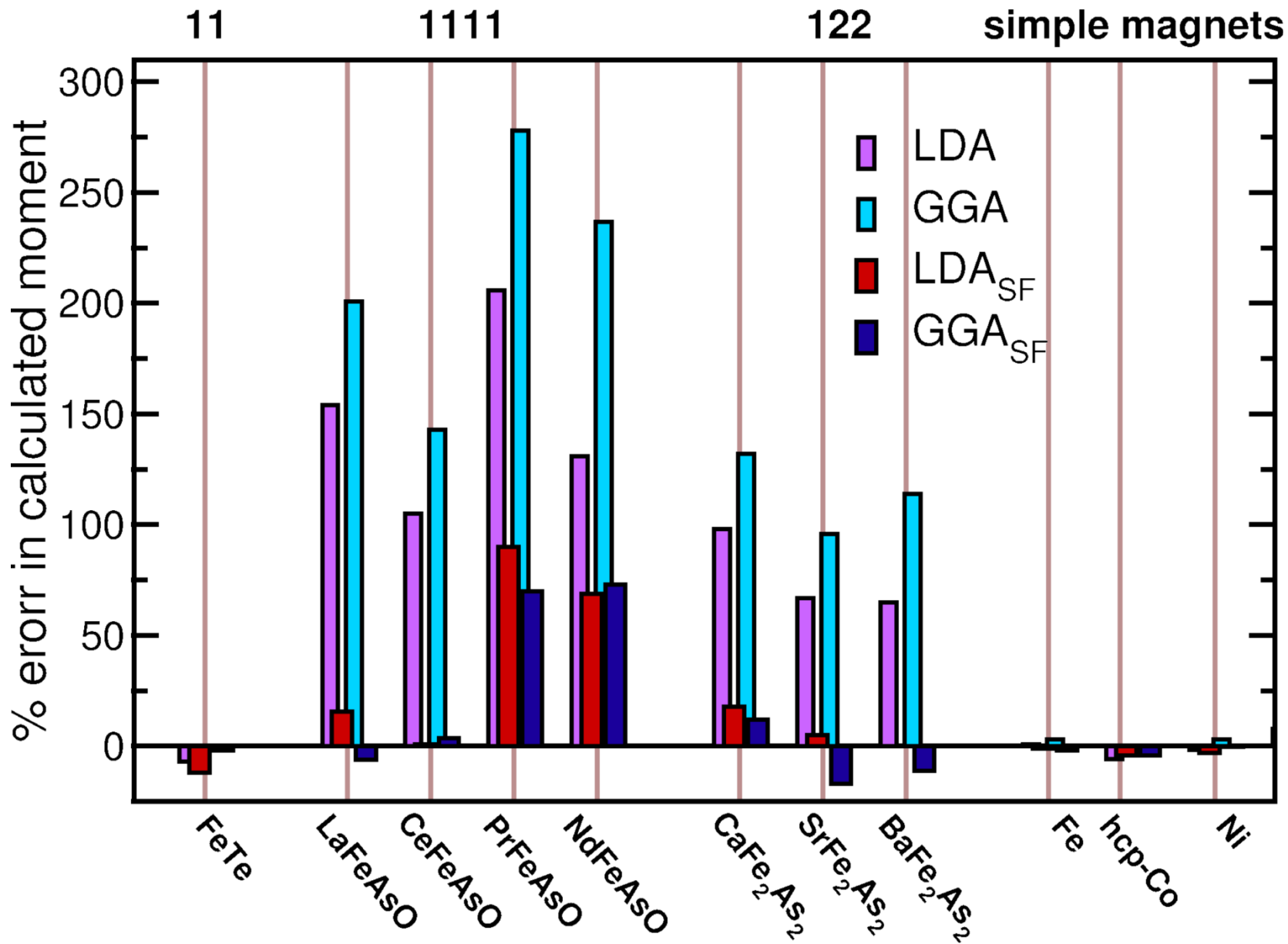


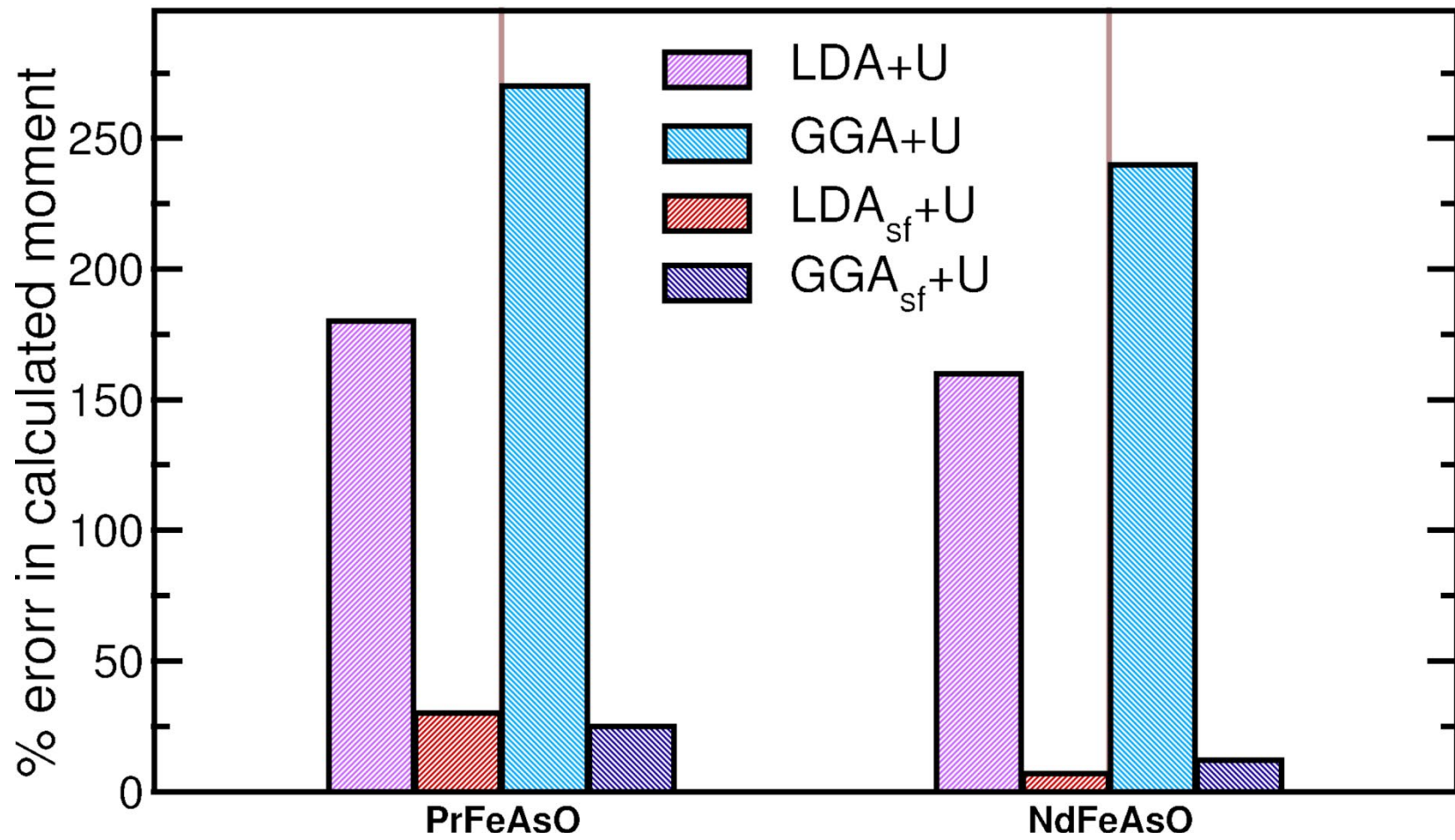
**LSDA**

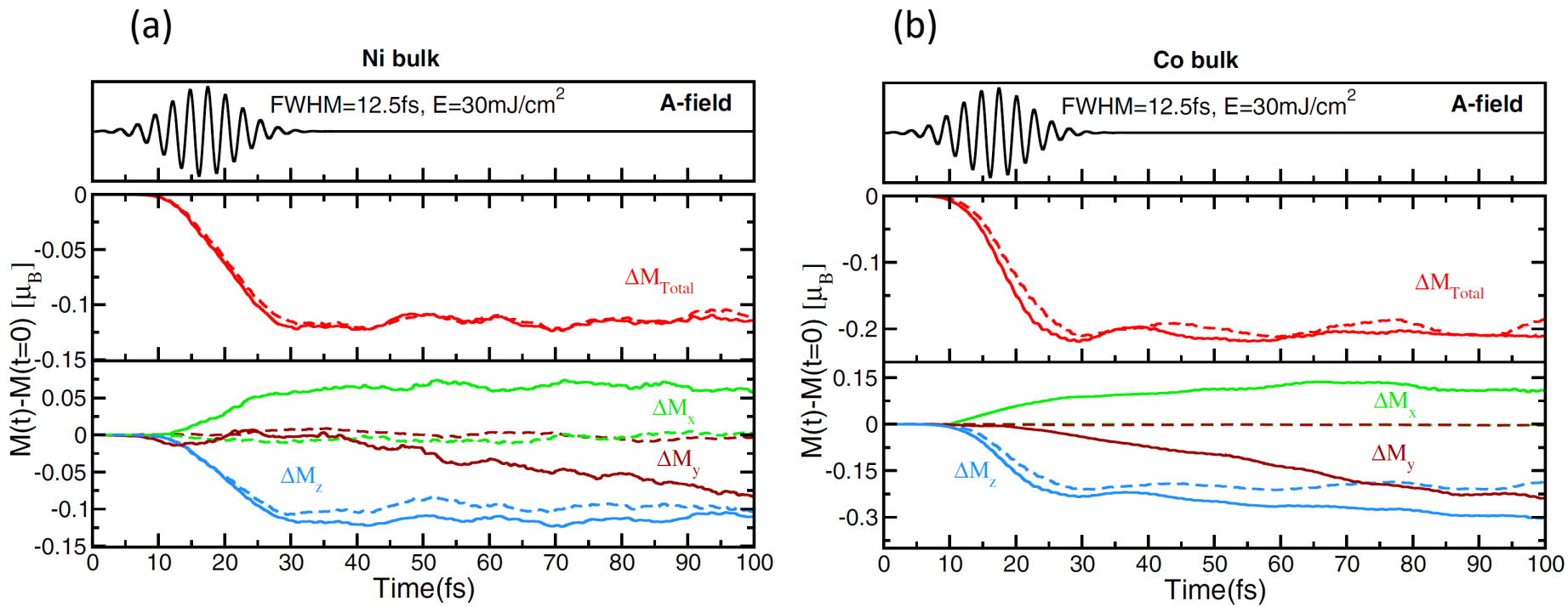


**Source-free LSDA**

Vector field  $B_{xc}$  for  $BaFe_2As_2$  on a plane containing the Fe atoms. Colors indicate the magnitude of  $B_{xc}$  while arrows show the direction. The black field lines originate from a regular grid in the plane and follow the vector field  $B_{xc}$ . The removal of the source term leads to an enhancement of non-collinearity.

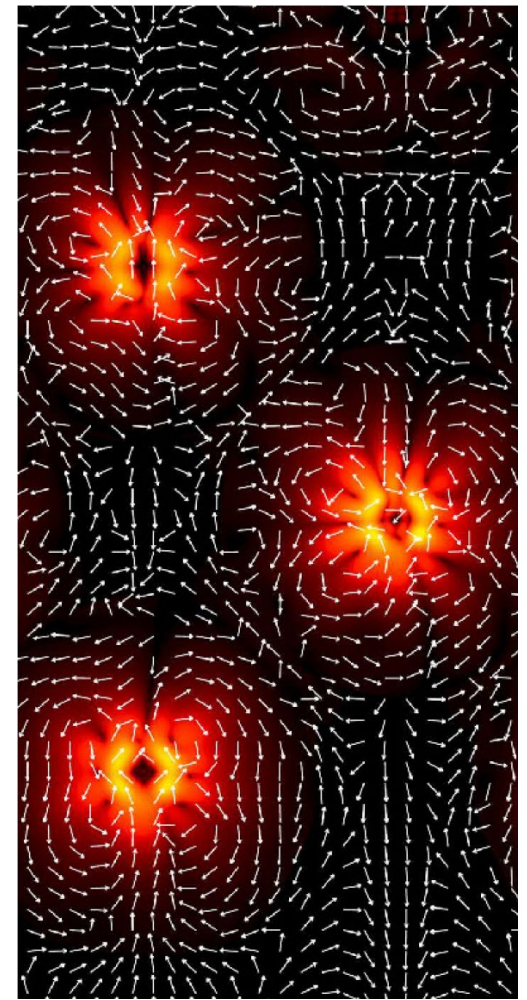
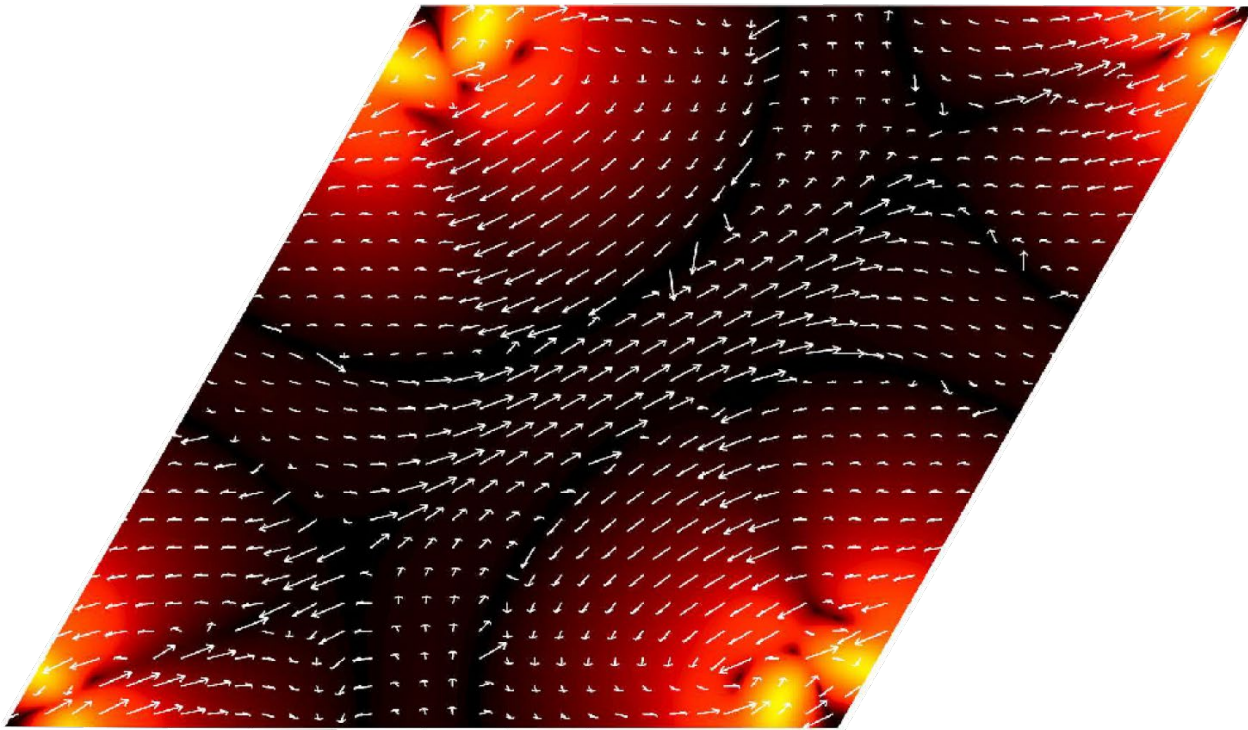






(a) Middle panel shows the total moment (red) and the bottom panel x (green), y (brown) and z (blue) projected moments for bulk Ni as a function of time. Dashed lines are the results obtained using the ALSDA and full lines the results obtained using the source-free functional. (b) The same as (a) but for bulk Co.





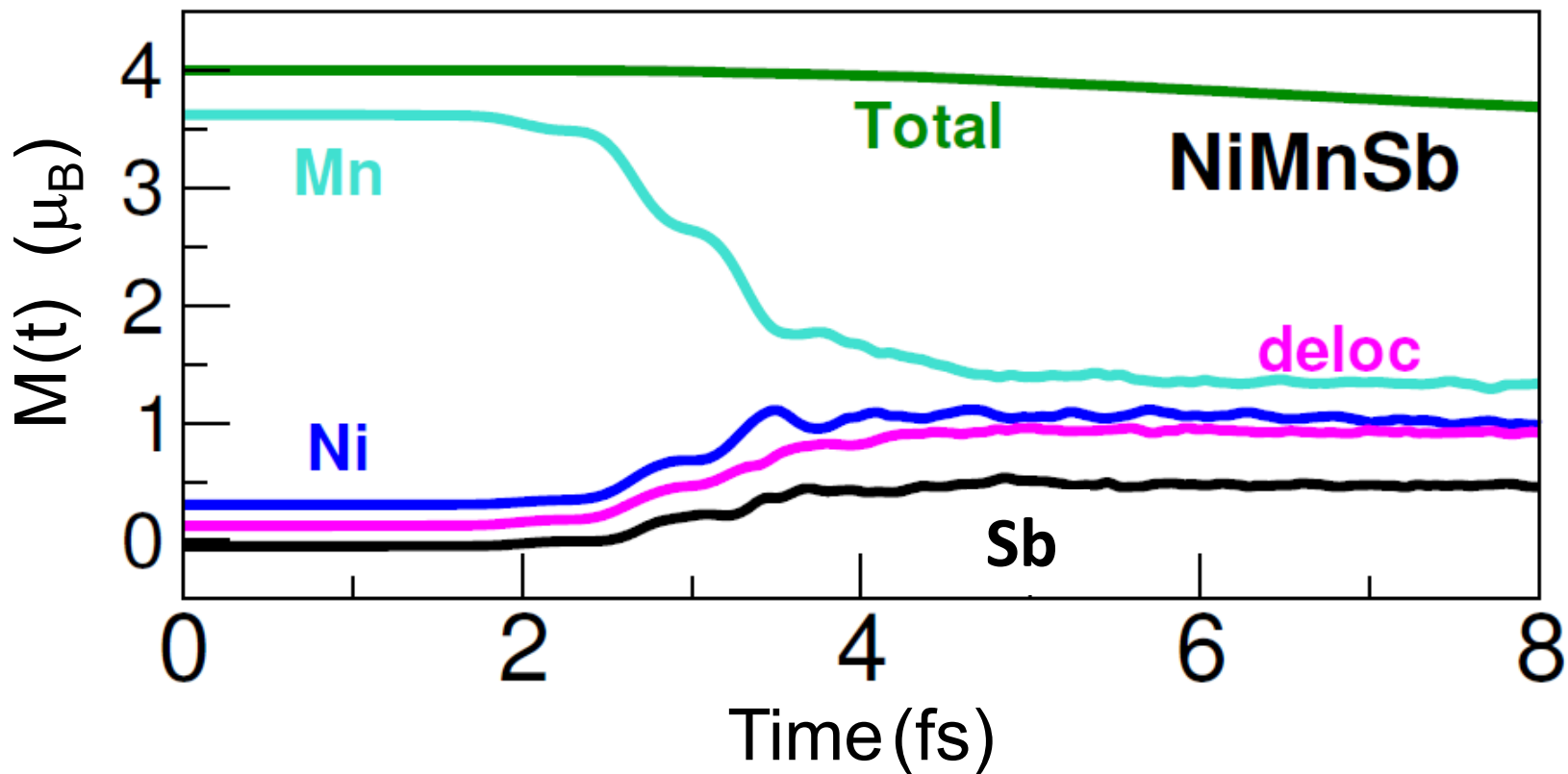
Left panel: Local xc torque for bulk Ni in (111) plane. Right panel: Local xc torque for 3ML Ni@5ML Pt in the (110) plane. The arrows indicate the direction and colors the magnitude.

# Optical intersublattice spin transfer (OISTR)

P. Elliott, T. Mueller, K. Dewhurst, S. Sharma, E.K.U.Gross,  
Scientific Reports 6, 38911 (2016)

K. Dewhurst, P. Elliott, S. Shallcross, E.K.U. Gross, S. Sharma,  
Nano Lett. 18, 1842 (2018)

OISTR was first predicted with TDDFT and later found experimentally  
(Aeschlimann group, Kaiserslautern, 2018)

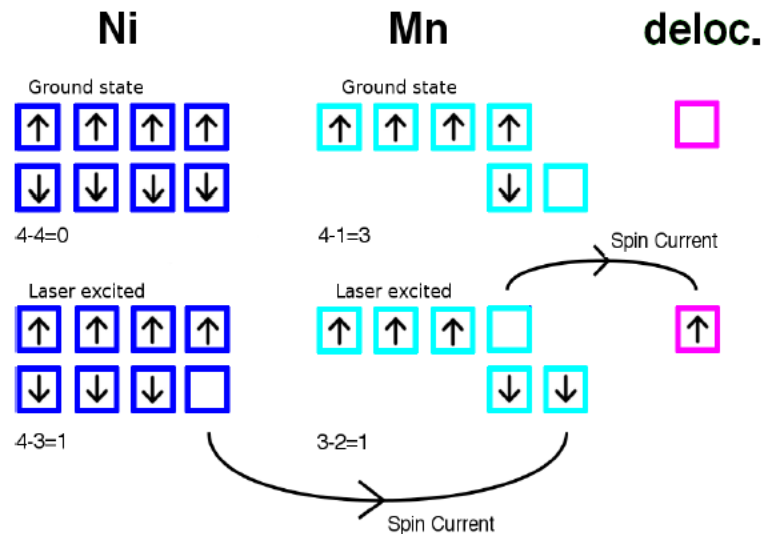
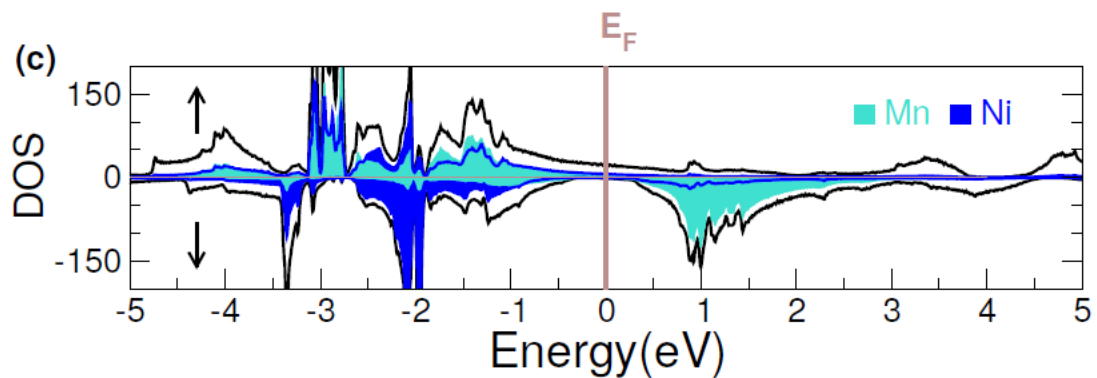
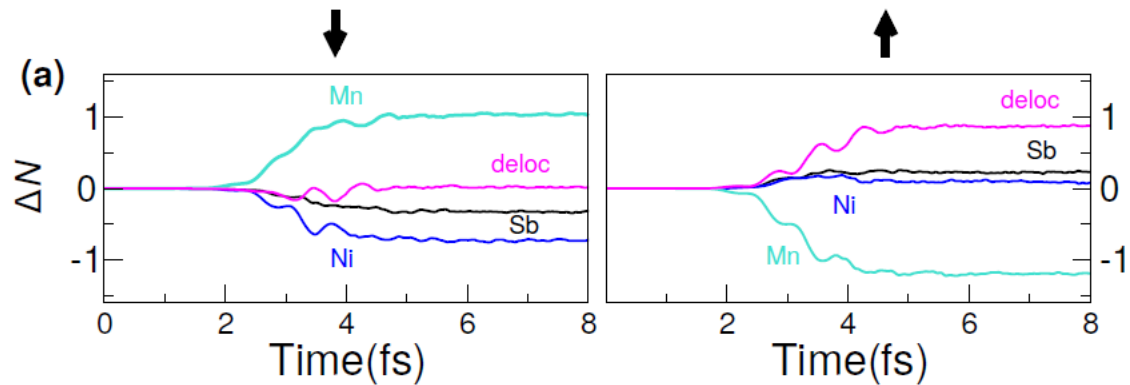


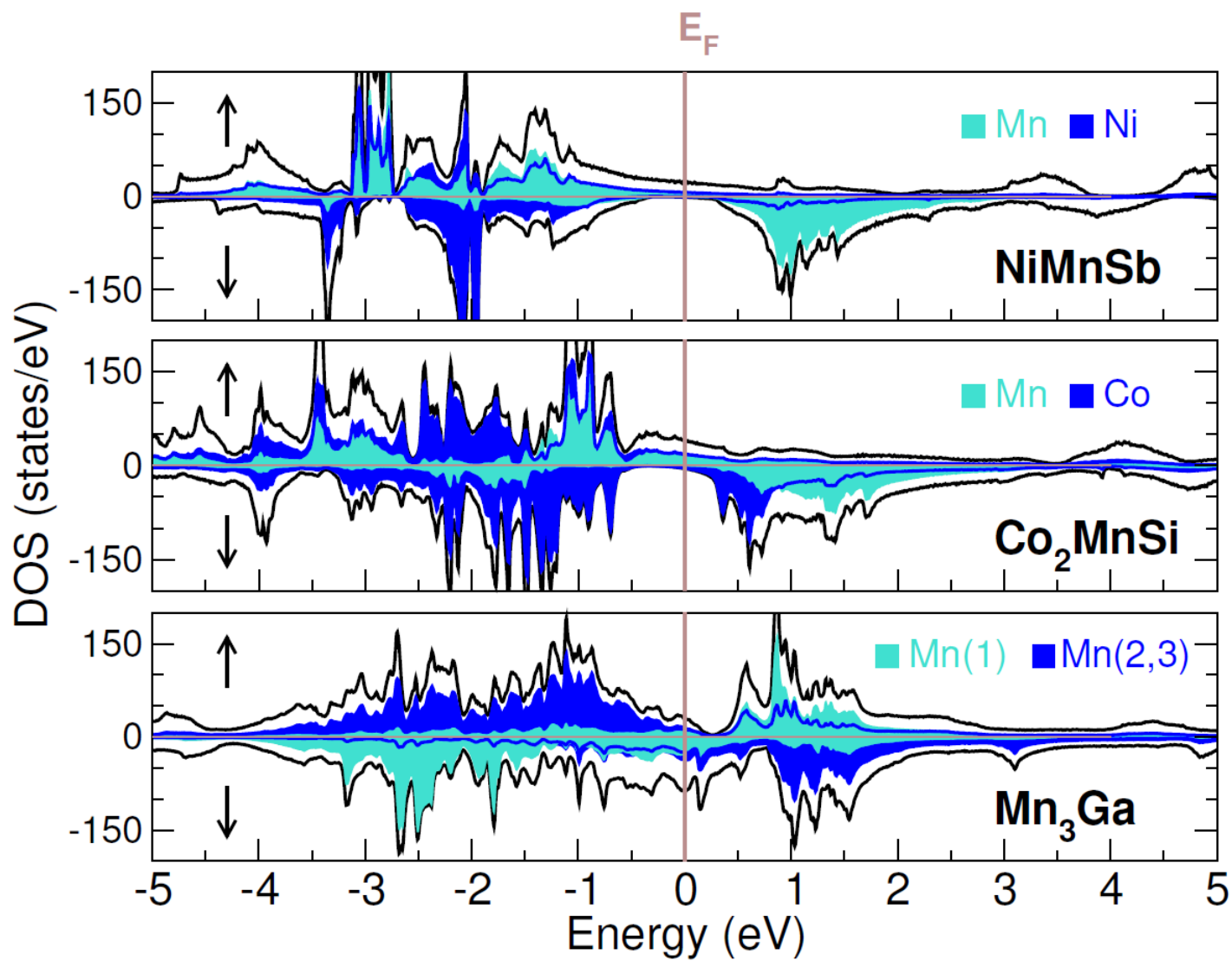
**Global moment  $|M(t)|$  nearly preserved. Local moments change.**

Laser parameters:  $\omega = 2.72$  eV, a FWHM of 2.42 fs, and fluence of 93.5 mJ/cm<sup>2</sup>, giving a peak intensity of  $1 \times 10^{14}$  W/cm<sup>2</sup>.

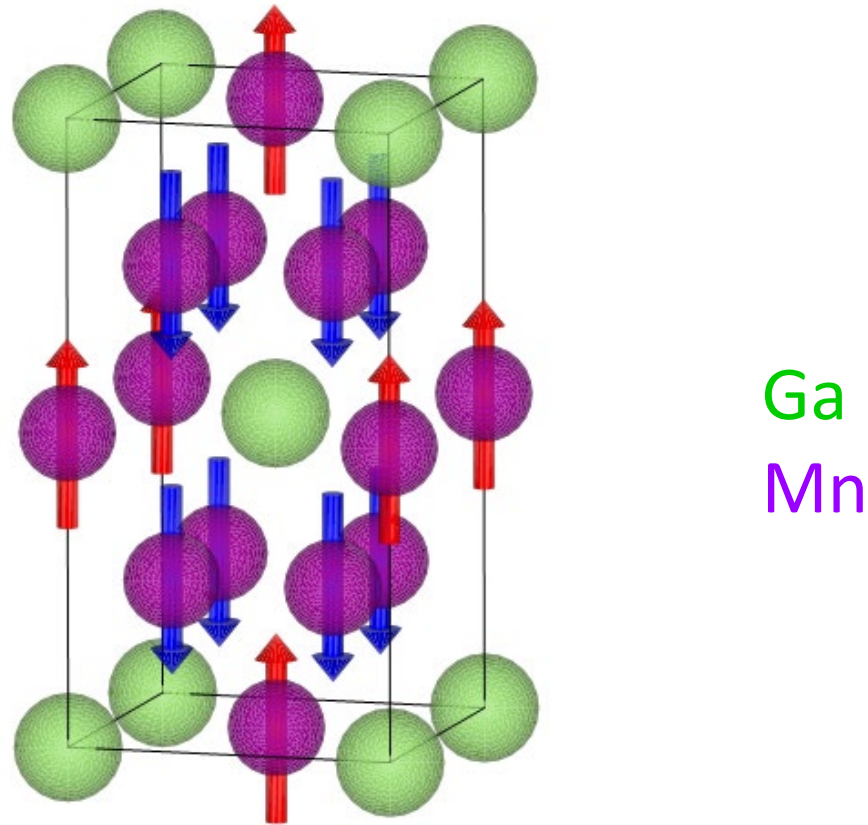


# NiMnSb





# Mn<sub>3</sub>Ga (ferri-magnet)



**TDDFT prediction for Mn<sub>3</sub>Ga: ferri → ferro transition within 4 fs**

## PHYSICS

# Ultrafast optically induced spin transfer in ferromagnetic alloys

M. Hofherr<sup>1,2</sup>, S. Häuser<sup>1</sup>, J. K. Dewhurst<sup>3</sup>, P. Tengdin<sup>4</sup>, S. Sakshath<sup>1</sup>, H. T. Nembach<sup>4,5</sup>, S. T. Weber<sup>1</sup>, J. M. Shaw<sup>5</sup>, T. J. Silva<sup>5</sup>, H. C. Kapteyn<sup>4</sup>, M. Cinchetti<sup>6</sup>, B. Rethfeld<sup>1</sup>, M. M. Murnane<sup>4</sup>, D. Steil<sup>7</sup>, B. Stadtmüller<sup>1,2</sup>, S. Sharma<sup>8</sup>, M. Aeschlimann<sup>1</sup>, S. Mathias<sup>7\*</sup>

The vision of using light to manipulate electronic and spin excitations in materials on their fundamental time and length scales requires new approaches in experiment and theory to observe and understand these excitations. The ultimate speed limit for all-optical manipulation requires control schemes for which the electronic or magnetic subsystems of the materials are coherently manipulated on the time scale of the laser excitation pulse. In our work, we provide experimental evidence of such a direct, ultrafast, and coherent spin transfer between two magnetic subsystems of an alloy of Fe and Ni. Our experimental findings are fully supported by time-dependent density functional theory simulations and, hence, suggest the possibility of coherently controlling spin dynamics on subfemtosecond time scales, i.e., the birth of the research area of attomagnetism.

## INTRODUCTION

Next-generation quantum materials will make it possible to surpass the speed and efficiency limits of current devices to generate faster, smaller, and more energy-efficient technological implementations (1–8). A promising approach to enhance data processing speed is to use ever shorter external stimuli for the manipulation and control of the state of matter. In this context, light represents the fastest means to alter the state of a material since laser pulses can now be generated with extremely short temporal duration down to a few tens of attoseconds. Visible lasers can deliver pulses with few-femtosecond durations that can be used to excite matter, while attosecond pulses can be generated in the extreme ultraviolet (EUV) and soft x-ray regions to probe the resulting dynamics (9–12). When combined with advanced spectroscopies, these new light sources are enabling comprehensive views into

material results in a nonequilibrium hot charge distribution, which subsequently triggers a series of cascaded incoherent secondary processes including transport, (spin-flip) scattering, and quasiparticle generation, ultimately leading to macroscopic demagnetization of the magnetic material within <500 fs (18–22).

The fastest manipulation of the magnetic state should occur, however, through the direct (possibly coherent) interaction between the spin system of the material and the light field itself (23–25). While the first experiments have provided indications that such a direct manipulation scheme might be possible (26–29), to our knowledge, only one experimental study on magnetic metallic systems to date has focused on this challenging aspect of coherent ultrafast magnetism induced by femtosecond laser pulses (24). One particularly interesting and previously unknown scheme for the ultrafast

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## PHYSICS

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The vision of using light to manipulate electronic and spin excitations in materials on their fundamental time and length scales requires new approaches in experiment and theory to observe and understand these excitations. The ultimate speed limit for all-optical manipulation requires control schemes for which the electronic or magnetic subsystems of the materials are coherently manipulated on the time scale of the laser excitation pulse. In our work, we provide experimental evidence of such a direct, ultrafast, and coherent spin transfer between two magnetic subsystems of an alloy of Fe and Ni. Our experimental findings are fully supported by time-dependent density functional theory simulations and, hence, suggest the possibility of coherently controlling spin dynamics on subfemtosecond time scales, i.e., the **birth of the research area of attomagnetism.**

## INTRODUCTION

Next-generation quantum materials will make it possible to surpass the speed and efficiency limits of current devices to generate faster, smaller, and more energy-efficient technological implementations (1–8). A promising approach to enhance data processing speed is to use ever shorter external stimuli for the manipulation and control of the state of matter. In this context, light represents the fastest means to alter the state of a material since laser pulses can now be generated with extremely short temporal duration down to a few tens of attoseconds. Visible lasers can deliver pulses with few-femtosecond durations that can be used to excite matter, while attosecond pulses can be generated in the extreme ultraviolet (EUV) and soft x-ray regions to probe the resulting dynamics (9–12). When combined with advanced spectroscopies, these new light sources are enabling comprehensive views into

material results in a nonequilibrium hot charge distribution, which subsequently triggers a series of cascaded incoherent secondary processes including transport, (spin-flip) scattering, and quasiparticle generation, ultimately leading to macroscopic demagnetization of the magnetic material within <500 fs (18–22).

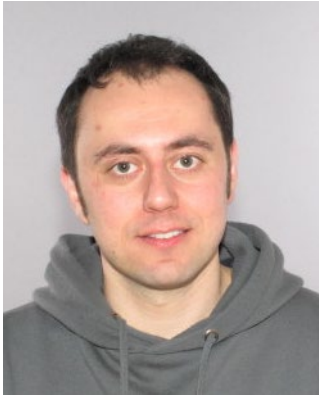
The fastest manipulation of the magnetic state should occur, however, through the direct (possibly coherent) interaction between the spin system of the material and the light field itself (23–25). While the first experiments have provided indications that such a direct manipulation scheme might be possible (26–29), to our knowledge, only one experimental study on magnetic metallic systems to date has focused on this challenging aspect of coherent ultrafast magnetism induced by femtosecond laser pulses (24). One particularly interesting and previously unknown scheme for the ultrafast

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## What's missing:

- **Include relaxation processes due to el-el scattering**
  - in principle contained in TDDFT,
  - but not with adiabatic xc functionals
  - need xc functional approximations with memory or CDFT functional
- **Include relaxation processes due to electron-nuclear interaction**
  - decoherence
- **Include relaxation due to radiative effects**
  - simultaneous propagation of TDKS and Maxwell equations
- **Include dipole-dipole interaction to describe motion of domains**
  - construct approximate xc functionals associated with dipole dipole interaction

# Thanks!



Kevin Krieger



Sangeeta Sharma



Florian Eich

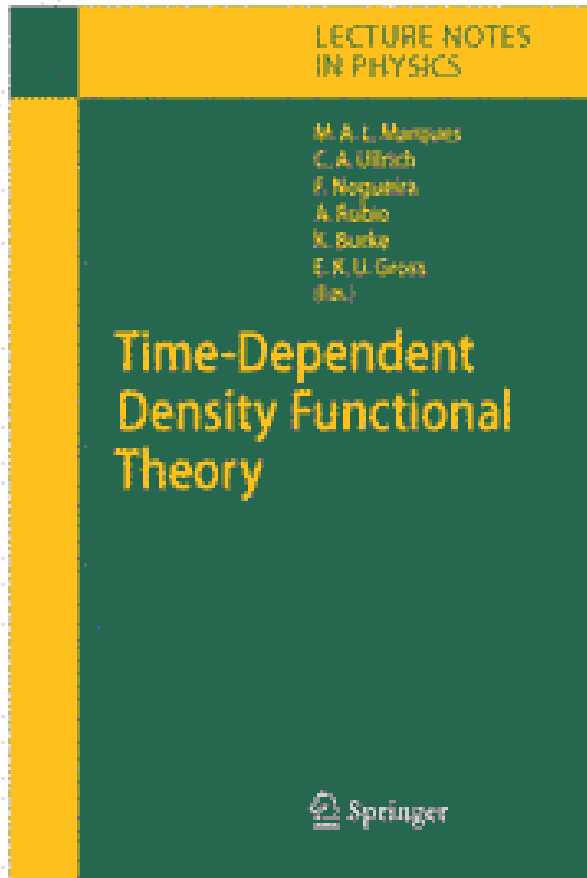


Kay Dewhurst

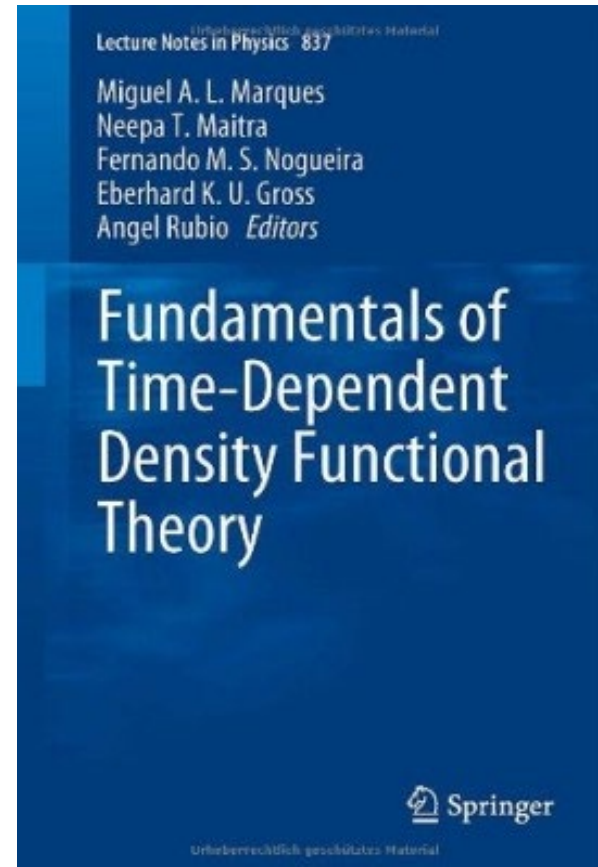


Peter Elliott





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