Annotated Publication List Anastasiia Tsvietkova

Papers and Preprints

All papers are peer-reviewed. Authors listed in alphabetical order, with contributions of each author being equal. Papers written with postdocs to whom I served a faculty mentor at the time are marked with *.

1. *Standard position for surfaces in link complements in arbitrary 3-manifolds*, with J. S. Purcell, preprint, pending revisions in Algebraic & Geometric Topology, 27 pp., https://arxiv.org/abs/2205.06368

Since the 1980s, it has been known that essential surfaces in alternating link complements can be isotoped into a form that satisfies certain combinatorial restrictions, interacting with the link diagram. This has had applications ranging from the classification of hyperbolic alternating knots, to proving the cabling conjecture, to identifying tangle decompositions, to bounding the count of incompressible surfaces in such link complements. However, the original techniques only apply to classical alternating links projected onto the 2-sphere inside the 3-sphere. In this paper, we prove that many of these properties of surfaces can be extended to a broader class of links, namely weakly generalized alternating links. Such links include all classical prime non-split alternating links, but also many links that are alternating on higher genus surfaces, or lie in manifolds besides the 3-sphere.

2. *Random meander model for links**, with N. Owad, preprint, pending revisions in Discrete & Computational Geometry, 18 pp., https://arxiv.org/abs/2205.03451

We describe a new random model for links based on meanders. Random meander diagrams correspond to matching pairs of parentheses, a well-studied problem in combinatorics. Hence tools from combinatorics can be used to investigate properties of links. We prove that unlinks appear with vanishing probability, no link \$L\$ is obtained with probability 1, and there is a lower bound for the number of non-isotopic knots obtained on every step. Then we give expected twist number of a diagram, and bound expected hyperbolic and simplicial volume of links.

*This work was started when I served as a faculty supervisor for Nicholas Owad, who was a postdoc at OIST.

3. *NP-hard problems naturally arising in knot theory**, with D. Koenig, Transactions of American Math. Soc. (Ser. B 8 2021), 420-441

We prove that certain problems naturally arising in knot theory are NP-hard or NP-complete. These are the problems of obtaining one diagram from another one of a link in a bounded number of Reidemeister moves (unlinking by Reidemesiter moves), determining whether a link has an unlinking or splitting number k, finding a k-component unlink as a sublink, and finding a k-component alternating sublink. These are the first known lower bounds on the complexity of these problems. The bounds are achieved through a series of topological lemmas, followed by Karp reductions of well-known problems from logic and graph theory to modified knot theory problems.

*This work was started when I served as a faculty supervisor for Nicholas Owad, who was a postdoc at OIST.

4. Unlinking, splitting, and some other NP-hard problems in knot theory*, with D. Koenig, Proceedings of the Annual ACM-SIAM Symposium on Discrete Algorithms (SODA), 2021

Paper for a Computer Science conference. According to Wikipedia, SODA is one of the top conferences for research in algorithms. Submissions are peer-reviewed, and only some are selected for publication and presentation. **This work was started when I served as a faculty supervisor for Dale Koenig, who was a postdoc at OIST.*

5. *Tangle decompositions of alternating link complements*, with J. Hass and A. Thompson, to appear in Illinois Journal of Mathematics, https://arxiv.org/abs/1906.06571

The notion of prime tangles was introduced by Kirby and Lickorish; Lickorish proved that by summing prime tangles one obtains a prime link. In a similar spirit, summing two prime alternating tangles will produce a prime alternating link, if summed correctly with respect to the alternating property. We check whether the converse is true, i.e. if a prime alternating link can be decomposed into two prime tangles, each of which is alternating. We refine results of Menasco and Thistlethwaite to show that if such a decomposition exists, either it is visible in an alternating link diagram, or the link is of a particular form, which we call a pseudo-Montesinos link. Our result about visibility such a tangle decomposition (and of the respective genus-2 surfaces) aligns with well-known results of Menasco, who noted that some basic topological properties of alternating link complements can be seen directly in their diagrams: the property of a link being non-split (and the presence of an essential genus-0 surface), and the property of being prime (and the presence of an essential genus-1 surface).

6. *Alternating links have at most polynomially many Seifert surfaces of fixed genus*, with J. Hass and A. Thompson, Indiana Univ. Math. J. 70 (2021), no. 2, 525-534

Let *L* be a non-split prime alternating link with n > 0 crossings. We show that for each fixed *g*, the number of spanning surfaces of fixed Euler characteristic for *L* is bounded by an explicitly given polynomial in *n*. A related problem for closed surfaces was studied by the authors in a previous paper. The proof for spanning surfaces presented here needs to consider difficulties not encountered with closed surfaces. A closed surface is put in standard position, and then decomposed into disks lying above or below the plane. By summing the contributions to the Euler characteristic of the surface of each region, a bound on the number and complexity of the region is obtained. These arguments can be extended, but the resulting regions for surfaces with boundary include cases that contribute zero to the Euler characteristic. This leads to an exponential explosion in the number of possible intersection configurations relative to the number of crossings. However many of these configurations give rise to isotopic surfaces, and we show that the number of surfaces up to isotopy still grows polynomially with the number of crossings.

7. Simplicial volume of links from link diagrams, with O. Dasbach, Mathematical Proceedings Cambridge Philosophical Society 166 (2019), no. 1, 75–81

The hyperbolic volume of a link complement is known to be unchanged when a half-twist is added to a link diagram, and a suitable 3-punctured sphere is present in the complement. We generalize this to the simplicial volume of link complements by analyzing the corresponding toroidal decompositions. We then use it to prove a refined upper bound for the volume in terms of twists of various lengths for links.

8. Determining isotopy classes of crossing arcs in alternating links, Asian Journal of Mathematics, Vol. 22, No. 6 (2018), 1005-1024

Given a reduced alternating diagram for a link, we obtain conditions that guarantee that the link complement has a complete hyperbolic structure, crossing arcs are the edges of an ideal geodesic triangulation, and every crossing arc is isotopic to a simple geodesic. The latter was conjectured by Sakuma and Weeks in 1995. As an example, we provide infinite families of closed braids for which our conditions hold. Previously, the conjecture was only proved for hyperbolic 2-bridge links and a few families of alternating links with simple and highly symmetric diagrams.

9. *The number of surfaces of fixed genus in an alternating link complement,* with J. Hass and A. Thompson, International Mathematics Research Notices 6 (2017), 1611-1622

Let L be a prime alternating link with n crossings. We show that for each fixed g, the number of genus g closed incompressible surfaces in the complement of L is bounded by a polynomial in n. Previous bounds were exponential in n. The theorem also applies to many closed manifolds obtained as Dehn fillings of hyperbolic alternating links.

10. Intercusp geodesics and the invariant trace field of hyperbolic 3-manifolds, with W. Neumann, Proceedings of the American Mathematical Society 14 (2016), No. 2, 887-896

Given a cusped hyperbolic 3-manifold with finite volume, we define two types of complex parameters which capture geometric information about the preimages of geodesic arcs traveling between cusp cross-sections. We prove that these parameters are elements of the invariant trace field of the manifold, providing a connection between the intrinsic geometry of a 3-manifold and its number-theoretic invariants. Further, we explore the question of choosing a minimal collection of arcs and associated parameters to generate the field. We prove that for a tunnel number k manifold it is enough to choose 3k specific parameters. For many hyperbolic link complements, this approach allows one to compute the field from a link diagram. We also give examples of infinite families of links where a single parameter can be chosen to generate the field, and the polynomial for it can be constructed from the link diagram as well.

11. *A refined upper bound for the hyperbolic volume of alternating links and the colored Jones polynomial*, with O. Dasbach, Mathematical Research Letters 22 (2015), No. 4, 1047-1060

We give a refined upper bound for the hyperbolic volume of an alternating link in terms of the first three and the last three coefficients of its colored Jones polynomial, showing. Previous bounds involved only two the first two and the last two coefficients which, unlike the third coefficient, do not change from classical Jones polynomial to colored Jones polynomial.

 Exact volume of hyperbolic 2-bridge links, Communications in Analysis and Geometry 22 (2014), No. 5, 881-896

W. Thurston suggested a method for computing hyperbolic volume of hyperbolic 3-manifolds, based on a triangulation of the manifold. The method was implemented by J. Weeks in the program SnapPea, which produces a decimal approximation as a result. For hyperbolic 2-bridge links, we give formulae that allow one to find the exact volume, i.e. to construct a polynomial and to find volume as an analytic function of one of its roots. The computation is performed directly from a reduced, alternating link diagram.

13. An alternative approach to hyperbolic structures on link complements, with M. Thistlethwaite, Algebraic & Geometric Topology 14 (2014), 1307-1337

An alternative method is described for determining the hyperbolic structure on a link complement, and some of its elementary consequences are examined. The method does not use any triangulation or polyhedral decomposition of a link complement, and allows to obtain geometric information directly from link diagrams, often by hand. It can be applied to any link that has a diagram satisfying a few mild restrictions. In particular, every reduced alternating diagram is a suitable diagram.

14. Decomposition of Cellular Balleans, with. I. Protasov, Topology Proceedings 36 (2010), 77-83

A ballean is a set endowed with some family of its subsets which are called the balls. We postulate the properties of the family of balls in such a way that the balleans can be considered as the asymptotic counterparts of the uniform topological spaces. This is reminiscent of the coarse spaces introduced by John Roe. The isomorphisms in the category of balleans are called asymorphisms. Every metric space can be considered as a ballean. The ultrametric spaces are prototypes for the cellular balleans. We prove a general theorem about decomposition of a homogeneous cellular ballean in a direct product of a pointed family of sets. As a corollary we show that the balleans of two uncountable groups of the same regular cardinality are asymorphic.

15. *Asymptotic Rays*, with O. Kuchaiev, International J. of Pure and Applied Math. 56, No. 3 (2009), 353-358 (undergraduate paper)

We prove that a graph G is asymptotically isomorphic to the ray if and only if G is uniformly spherically bounded and is of bounded local degrees. This problem arose from interactions between combinatorics (the study of colorings of graphs and sets) and asymptotic topology and was formulated as a question by Banakh and Protasov in [3].

PhD Thesis

16. *Hyperbolic Structures from Link Diagrams*, University of Tennessee (2012), 77 pp., https://trace.tennessee.edu/cgi/viewcontent.cgi?referer=&httpsredir=1&article=2503&context=utk_graddiss

In Preparation

17. Universal bounds for surfaces in weakly generalized alternating links, with J. S. Purcell, preprint, 20 pp.

Weakly generalized alternating links have an alternating diagram on some closed surface in a compact irreducible 3manifold. They include all classical prime, non-split alternating links, but also many links that are not alternating links or are not links in the 3-sphere, including virtual links. In this paper, we show that for many such links, there is a polynomial bound on the number of surfaces in the link complement that have a fixed genus and a fixed number of boundary components; the bound is polynomial in the number of crossings of a diagram. As a corollary, this gives a polynomial bound on the number of essential surfaces whose boundary forms any boundary slope in the complement of a classical alternating link.

18. PSL(2,C)-representations of knot groups by knot diagrams, with K. Petersen, preprint, 50 pp.

We describe a new method of producing equations for representation and character varieties of the canonical component of a knot group into PSL(2, C). Unlike known methods, this does not involve any decomposition of the link complement, and uses only a link diagram satisfying a few restrictions. This results in a simple algorithm that can often be performed by hand, and in many cases, for an infinite family of knots at once.

19. Polynomially many surfaces in terms of hyperbolic volume in a 3-manifold, with M. Lackenby, preprint, 18 pp.

We give a universal upper bound for the number of non-isotopic surfaces embedded in a hyperbolic 3-manifold polynomial in hyperbolic volume. The 3-manifolds may be closed or cusped, with finite volume, and the surfaces are all closed essential surfaces or essential surfaces with boundary. Previously, surface count was known to be either at most exponential or almost quasi-polynomial in terms of genus for various classes of 3-manifolds, and the bounds depended on the 3-manifold. Universal polynomial bounds were known only for alternating links in 3-sphere, in terms of their crossing number.

20. On bit-computability and its extensions of DISCRETE-FREE SUBGROUP of PSL(2, R) problem, with J. Gilman, preprint, 12 pp.

Which theoretical model of computation is most suitable for a mathematical problem with real input? In this paper, we briefly review several existing models and seek a reasonable generalization for one of them, with a certain mathematical problem in mind. In particular, we consider the Gilman-Maskit \$PSL(2,R)\$ two-generator discreteness algorithm, and the issues pertaining to the algorithm with respect to bit-computability. While the algorithm is not originally suited for this model of computation, we generalize the model as two oracle upper and lower computability, and show that the discreteness problem without parabolics (so that the corresponding quotient has no cusps) is semi-decidable.

Selected Software

21. Hyperbolic structures from alternating link diagrams.

Implementation of an alternative method for computing hyperbolic structures of alternating links with small regions. Based on paper #13 above. Written in C++.

https://sites.rutgers.edu/anastasiia-tsvietkova/research/

22. Hyperbolic structures from alternating link diagrams, with D. Koenig and A. Lowen.

A more general version of the above, for links with any region size, including non-alternating links. Written in Python, can be integrated with SnapPy. Now in the final stages of testing, to be posted on CompuTop.org.

23. Computing invariant trace field from a link diagram, with no approximation involved.

Worksheet that gives the polynomial for the invariant trace field of a hyperbolic 2-bridge link. Based on paper # 12. Written in Mathematica.

https://sites.rutgers.edu/anastasiia-tsvietkova/research/

24. Footstep, with O. Kuchaiev, P. Protsyk, P. Shelyazhenko.

Software system written as part of Microsoft Imagine Cup competition, placed first in Ukraine in 2006