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THE MEASUREMENT PROBLEM: SOME "SOLUTIONS"

In 'Interpreting the Many Worlds Interpretation'[1] we discussed two ways of understanding the Everett–Wheeler many worlds interpretation of quantum theory.[2] One way, adopted by most philosophical and popular discussions, claims that in certain interactions the universe literally splits into many universes; hence the name "the splitting worlds view". We argued that the splitting worlds view does not yield an adequate interpretation of quantum theory. The other way, which we called "the many minds view", is novel, although elements of it can be found in Everett's original work. It claims that there is only one quantum mechanically described world and associates with each sentient being a set of minds determined by that being's quantum mechanical description. Since we were aware that some would consider the many minds to be "far out" we were careful to motivate the introduction of minds into an interpretation and to mark the account's virtues. We didn't argue that it is the uniquely correct interpretation but we did claim that the foundational probems in quantum theory are so vexing and the virtues of the many minds view so attractive as to render it worthy of serious consideration.

Dennis Dieks is unconvinced by our discussion. In his 'On Some Alleged Difficulties in the Interpretation of Quantum Mechanics'[3] he claims that our criticisms of the splitting worlds view and our arguments for the many minds view are defective. He also proposes his own interpretation which he claims provides a solution to the measurement problem without introducing exotic ontology. We are grateful to Dieks for his criticisms, but as we will shortly show, they are based on misunderstandings. Dieks's own interpretation is a variant of a proposal which has been developed by Kochen, van Fraassen, and Healey among others. If it were viable then it would indeed appear to be superior to the many minds view since it seems to interpret quantum theory without going beyond the physical ontology of the theory. But we will show later in this paper that intepretations like Dieks's are irremediably flawed. Prior to that we will set the stage by discussing Dieks's criticisms of our previous paper.

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Dieks begins his discussion with a review of the semantic account of theories. He correctly points out that a given mathematical formalism can be given various interpretations where an interpretation is a mapping of aspects of the mathematical formalism to features of physical reality which validates the statements of the theory. All of Dieks's criticisms of our arguments stem from his belief that we failed to observe the distinction between the formal mathematical structure of quantum theory and an interpretation of the theory. He seems to think that we argued that the formalism of quantum theory all by itself determines an interpretation and specifically that the formalism excludes the splitting worlds view and that it entails many minds view. We should say at once that we are perfectly aware of the distinction between a mathematical formalism and its interpretations and we are astonished that Dieks could read our paper in such a way as to think otherwise. We did not argue that the quantum mechanical formalism *by itself* excludes the splitting worlds view or that it entails the many minds view. As we said in our paper, by "quantum mechanics" we understand not just the mathematical formalism but also the usual correspondences between operators and physical quantities, states and vectors, and the following principles connecting states with physical quantities and their measurement:

(E) If system S is in state \$ which is represented by vector $|\$\rangle$ which is an eigenvector of operator O corresponding to physical quantity Q and the eigenvalue of O is b then the value of Q for S is b.

(P) If the state of S is represented by $|\$\rangle$ and $|O = b_k\rangle$ is an eigenstate of O in which Q possesses the value b_k and Q is measured on S then the probability that the measurement yields result b_k is $|\langle\$|O = b_k\rangle|^2$.

While the meanings of (E) and (P) are not completely clear (in particular "measurement" needs to be defined) every interpretation of quantum theory (including the many worlds interpretation and the interpretation sketched by Dieks in his paper) endorses these principles under some understanding of them. We argued in our paper that the splitting worlds view is incompatible with the quantum mechanical formalism, the usual association of physical quantities with operators, etc., and these principles. We also argued that the many minds view provides an interpretation of the formalism which validates these principles while solving the main difficulty in the foundations of quantum theory, the measurement problem.

The measurement problem arises from an apparent conflict among (E), (P) and the principle (which is part of the quantum mechanical formalism) that the evolution of the state of an isolated system is described by a linear equation (Schrödinger's equation). We will illustrate this problem by considering a perfect measurement of the x-spin of an electron 'e' by a device M. We suppose that M can be in three orthogonal states $|R\rangle$, $|U\rangle$, and $|D\rangle$ where $|R\rangle$ is (the vector representing) M's ready state, $|U\rangle$ corresponds do a pointer registering "Up" and $|D\rangle$ to a pointer registering "Down" and if M starts in its ready state and 'e' has spin up (down) then at the conclusion of the measurement M will be in state $|U\rangle$ ($|D\rangle$). It follows from the linearity of Schrödinger's equation that if prior to measurement the state of 'e'(+)M is $[c_1||\uparrow\rangle + c_2|\downarrow\rangle]|R\rangle$ (i.e., 'e' is not in an eignestate of x-spin) then the post measurement state of M(+)'e' will be

MEAS $\quad c_1|U\rangle|\uparrow\rangle + c_2|D\rangle|\downarrow\rangle$

MEAS is neither an eigenstate of x-spin up nor an eigenstate x-spin down. So (E) by itself does not assign a physical value corresponding to either spin value. On the other hand (P) says that the probability that the measurement will yield the value "up" is c_1^2. How can this be right if MEAS does not assign a value to x-spin? Even more puzzling, as Schrödinger long ago pointed out with his famous cat thought experiment, MEAS is not an eigenstate of the observable corresponding to the pointer's position (pointing to "up" or pointing to "down") either. If (E) specifies all of a system's physical quantities then it follows that in MEAS the pointer does not possess a particular position. Unsurprisingly, this consequence strikes many as manifestly absurd. The measurement problem is the problem of reconciling this apparent conflict.

There is a standard resolution of the measurement problem due to von Neuman which denies that measurement interactions are correctly described by Schrödinger's equation. Instead, it is proposed that the final state of M + 'e' is one of the states $|U\rangle\uparrow\rangle$ or $|D\rangle\downarrow\rangle$ with probabilities c_1^2 and c_2^2. A measurement is said to produce a "collapse" of a state like MEAS into an eigenstate of the pointer's position. This is a very ingenious solution since it allows one to maintain that (E) is the *only* connection between states and values while validating (P). There are, however, well known objections to collapse accounts of measurement interactions. First, all interactions which have been investigated by physicists have been found to conform to Schrödinger's equation. Since these interactions involve microscopic phenomena it can be maintained that interactions involving macroscopic measuring devices conform to the collapse dynamics rather than the Schrödinger equation. But this seems very implausible since macroscopic measuring devices are composed out of microscopic particles and the two kinds of dynamics are radically different.[4] Since Dieks's view as well as ours are "non-collapse" accounts we will not discuss such views any further.

Everett's motivation in developing the many worlds interpretation was precisely to see how far one could take the idea that state evolution is always linear (in accordance with Schrödinger's equation) while maintaining that the quantum state provides the complete description of physical reality. The most immediate problem confronting this idea is finding a way to square it with our conviction that, for example, when we look at M(+)S we find the pointer either at the D or the U position. Everett noted that if we include the observer in the state evolution then, assuming perfect accuracy of observation, the state will evolve into

$$\text{OBS} \qquad c_1|\text{B}_\text{U}\rangle|\text{U}\rangle|\uparrow\rangle + c_2|\text{B}_\text{D}\rangle|\text{D}\rangle|\downarrow\rangle.$$

($|\text{B}_\text{U}\rangle$ is a vector which represents a state in which the observer "sees" the pointer in the up position.) The two branches in this representation of OBS suggest the existence of two "worlds"; one in which the spin is up, the device registers "up" and the observer sees "up", etc. According to advocates of the splitting worlds view the evolution from the pre-measurement state to OBS as involving a splitting of the universe into two universes, one containing an observer, a measuring device and an electron in state $|\text{B}_\text{U}\rangle|\text{U}\rangle|\uparrow\rangle$ and the other containing a measuring device and electron in state $|\text{B}_\text{D}\rangle|\text{D}\rangle|\downarrow\rangle$. In each universe the x-spin of the electron and the pointer possesses definite values and so the observer always sees the pointer at a definite position.

In our paper we pointed out a number of defects in the splitting worlds view. We will briefly repeat one of our points since Dieks seems to have it in mind when he accuses us of failing to observe a distinction between the quantum mechanical formalism and its physical interpretation. Our objection is that, contrary to its advocates' intentions, the splitting worlds view violates the conjunction of (E) and Schrödinger's linear dynamics. The reason is that prior to measurement the state of

M(+)'e' is an eigenstate of mass with some eigenvalue k and the post measurement state is also an eigenstate of mass with the same eigenvalue. But on the splitting worlds view the total mass of the reality described by MEAS has doubled by the end of the measurement since that reality has split into two universes. It should be clear that this argument does not attempt to demonstrate that the splitting world view is inconsistent with the QM formalism by itself but rather that it is inconsistent with the formalism and the usual interpretational principles of the theory ((E) and the association of an operator with mass). We hope this convinces Dieks. In any case, since he has no more sympathy for the splitting worlds view than we do (he brings it up merely to instruct us on the distinction between a formalism and its physical interpretation) we will turn to the many minds view.

The many minds view attempts to maintain (E) as the only principle connecting states with the values of physical quantities and Schrödinger's equation as describing the dynamics. At the same time it attempts to account for how it is that an observer in a state like $|OBS\rangle$ has an experience of "seeing" the pointer at either the Up or Down position. Here is how it works. We assume that there are certain quantum mechanical states of the brains of sentient beings (and their environments) on which mental states supervene. For example, there is a state B_U such that whenever a system S is in this state it contains a human being who is having the impression of seeing a pointer in the "U" position. We call these states "eigenstates of mentality". A complete eigenstate of mentality characterizes all of a person's mental features and so characterizes a mind. These states form a complete basis (of course some of the states in the basis will entail that no mental states or activities are present). For human beings we may suppose that such states are enormously complicated, involving trillions of particles, but their exact nature is an empirical matter to be discovered, if they ever are discovered, by future neuro-physicists.

The central claim of the many minds view is that associated with a state like OBS are two kinds of minds; one experiencing the pointer at "U", the other experiencing the pointer at "D". In general, to find the minds associated with a system in state \$ write $|\$\rangle$ as a sum of complete eigenstates of mentality. Each element in the sum (with non-zero amplitude) determines a complete set of mental attributes; i.e., a kind of mind. These are the minds associated with the system. In the measurement described above the observer begins with all her minds in the mental state corresponding to a ready state (i.e., ready to perform the experiment) and ends with some "seeing" the pointer at "U" and some "seeing" the pointer at "D". The important feature is that mental states are always definite, e.g., involve experiences of pointers at particular positions, even when the brain supporting these mental states is characterized by a superposition of eigenstates of mentality.

The many minds view is dualist to the extent that it postulates minds and mental states which are not described by the quantum mechanical formalism. However, it is physicalist to the extent that the quantum mechanical state of a system determines the set of minds associated with the system. According to the view we explored, each eigenstate of mentality in the superposition is associated with an infinite set of minds. As a state evolves, e.g., into OBS a mind has a chance = c^2 of evolving into possession of mental states associated with a branch of the superposition whose amplitude is c (for details see our papers).

On the many minds view the pointer described by MEAS is not in an eigenstate of position and so does not possess a definite position. However, a sentient being observing the pointer (or a cat in a superposition of alive and dead states) will be in a state like OBS with which are associated minds, all of which experience the pointer at a definite location, etc. The linearity of the dynamics guarantees that as long as an observer makes good measurements the subsequent measurements will agree with the first one. This entails that a mind will experience measurement results which agree with her first measurement, so although a mind's belief that the pointer is at "U" is not strictly true, it possesses a kind of pragmatic validity.

The many minds view possesses a number of virtues. First, and perhaps most important, it is completely explicit. At any moment the real physical state of an isolated system (and the whole universe) is its quantum state. The minds, if any, associated with a system at a time are completely determined by the system's quantum state. As we have seen, this interpretation solves the measurement problem since it accounts for the definiteness of our experience and, as appropriately understood, it validates (E) and (P). Further, according to it all state evolution is described by Schrödinger's equation and so is local. Measuring the x-spin of one electron of an EPR pair does not produce any kind of change in the other electron. This doesn't contradict Bell's theorem since among its premises is the assumption that measurements yield unique outcomes. This is denied by the many minds view.

It should be clear, contra Dieks, that we did not argue that the quantum mechanical formalism entails the many minds view. Rather, we claimed that it solves the measurement problem and in a way that is faithful to Everett's original proposal in that it maintains that the quantum mechanical state is the complete physical state and that all state evolution is linear. We certainly do not think that the many minds view is the only way of solving the measurement problem. Certain so-called "hidden variables" views, for example Bohm's interpretation, also resolve the meaurement problem.[5] The same can not be said for the Dieks proposal, to which we now turn.

As we mentioned, Dieks's interpretation is similar to proposals made by Kochen, Healey, and van Fraassen.[6] The basic idea of these interpretations is that the component systems of a complex system, e.g., M + 'e', possesses values for certain dynamical quantities even though these component systems are not in eigenstates of these quantities. Dieks's particular account works as follows. The state MEAS can be represented in various bases but the particular representation used above has a special feature. As long as $c \neq c_2$ it is the *unique* representation of the form SUM $c_i|a\rangle_i|b\rangle_i$ where the $|a\rangle_i$ are orthoganol and the $|b\rangle_i$ are orthogonal. Such representations are called "bi-orthonormal". Dieks's interpretation stipulates that when MEAS is the state of M(+)'e' system 'e' (the measured system) taken by itself possesses one of the values of the observable associated with $|\uparrow\rangle$ and $|\downarrow\rangle$. The probabilities of these values are c_1^2 and c_2^2. If 'e' possesses the value "up" ("down") then the observable characterizing the position of the pointer possesses the value "U" ("D"). More generally, the rules which determine the defined observables of subsystems S_1 and S_2 of system S are:

(R1) if SUM $c_i|a\rangle_i|b\rangle_i\rangle_{S2}$ is the biorthonormal representation of $|\$\rangle$ and the $|a\rangle_i$ are eigenstates of observable A and the $|b\rangle_i$ are eigenstates of observable B then A is defined (has a value) on system S_1 and B is defined on S_2.

(R2) If S in the state referred to in R1 the probability that A has the value a_i and B has value b_i is c_i^2.

The crucial claim of Diek's interpretation is the denial that (E) is the only rule connecting the quantum state with the values of physical quantities. (E) assigns no values of observables to M or to 'e' when the state of M + 'e' is MEAS. But (R1) assigns to 'e' a value for x-spin and to M a (correlated) value to the pointer position. Further, if an observer looks at the pointer and the state evolves into OBS (R1) will assign a correlated belief to the observer. All this is very nice. At first sight it must seem superior to the many minds view. It apparently solves the measurement problem. It appears to have the result that in measurement situations pointers end up in definite positions and observers have definite beliefs about these positions. Further, it validates (E), (P), and Schrödinger evolution of state without the spectre of sentient beings possessing an infinity of non-physical minds.

Before showing how Dieks's account fails we want to point out two respects in which the account as so far described is incomplete. The first is that the prescription for determining the values of defined observables on a subsystem is underspecified. A many-particle system can be decomposed in many ways into subsystems with each way possessing a biorthonormal state representation. Rules need to be introduced to guarantee that the values of observables which are defined according to one such decomposition do not conflict with values defined in another. It is likely that this can be done in more than one way and so whatever rules are chosen may appear to be arbitrary.[7] The second, and more serious, lacuna is that Dieks does not discuss the dynamics of the physical quantities defined in accord with (R1). As the state $(t) of a composite system evolves in accordance with Schrödinger's equation the vectors in the biorthonormal representation of $(t) will typically change. In consequence the observables defined on the system's subsystems in accord with (R1) will change. The question that the dynamics needs to answer is how is the value of an observable defined at $t+$ related to the value defined at t? If there is no relation then R1 will have the effect that values will jump around. For example if position is well defined at t and equals x, and is not well defined at $t+$, but defined again at $t++$ the position at $t++$ can be far from x. We leave it to Dieks and other advocates of this view to work out the dynamics of physical quantities.

We now come to an objection which is, in our view, fatal to Dieks's proposal. It is that, contrary to the initial appearances, it fails to solve the measurement problem. Specifically, we will show that in the course of ordinary measurements Dieks's interpretation fails to assign definite values to observables which are supposed to record the result of the measurement; e.g., it fails to assign a definite position to the pointer which is supposed to register the measurement outcome. Even worse, it fails to assign a definite perception (or any mental state at all) to an observer reading M.

To see why this is so let us recall how Dieks's interpretation works in the case of perfect measurement. A perfect measurement of observable A with values a_i requires a measuring device with orthogonal states $|B = b_i\rangle$ and $|R\rangle$ such that if prior to the measurement S + M is in the state $|A = a_k\rangle|R\rangle$ then after a time interval t (when the measurement is completed) the state of S + M is $|A = a_k\rangle|B = b_k\rangle$. It follows then that if S + M is in state SUM $c_i|A = a_i\rangle|R\rangle$ at t then at $t + x$ it will be in state SUM $c_i|A = a_i\rangle|B = b_i\rangle$. Since the $|B_i\rangle$ are orthogonal it follows from (R1) that M possesses some value of B (which one determined by (R2)). However, real measurements are almost never perfect in this sense. In a real measurement there is always some probability of the measuring device making an error. For this reason in a realistic measurement the post-measurement state of M + 'e' will be

MEAS* $c_3|\uparrow\rangle|U\rangle + c_4|\downarrow\rangle|D\rangle + c_5|\downarrow\rangle|U\rangle + c_6|\uparrow\rangle|D\rangle$.

The components $|\downarrow\rangle|U\rangle$ and $|\uparrow\rangle|D\rangle$ represent errors. If the measurement is a good one then the amplitudes of these error components will be small but, as we have noted, in realistic measurements they will inevitably be non-zero. MEAS* is obviously not a biorthonormal representation. The biorthonormal representation of this state is

BMEAS* $c_7|\uparrow @\rangle|G\rangle + c_8|\downarrow @\rangle|H\rangle$

where $|\uparrow @\rangle$ represents the state in which spin in direction @ (not x-spin) is up and $|G\rangle$ represents a state of the pointer. (R1) does not assign a definite x-spin to the electron in BMEAS* although it does assign a definite value to the spin in the @ direction. This is a bit dismaying since M is measuring x-spin. But the real problem is that $|G\rangle$ and $|H\rangle$ are not eigenstates of the observable corresponding to the pointer's position. That is, (R) fails to assign a definite position to the pointer. If all M's observables which possess values are determined by (R1), then according to Dieks's account, there simply isn't going to be a matter of fact when MEAS* obtains about whether the pointer is pointing to 'up' or to 'down'!

The same point applies to the measurement of the pointer's position by an observer's perceptual state. This measurement is also not perfect and so (R1) will (almost always) fail to assign any particular definite perception to the observer. And as a matter of fact it will follow from

Dieks's account that the observables that pick out particular mental states will almost never have values, under any circumstances; and so it will follow from that account that actual people will almost never have perceptions or beliefs.[8]

Dieks seems to be aware that in realistic measurements the states corresponding to the measurement outcomes are not strictly orthogonal. This apparently doesn't disturb him since he says that the component states in his representation are orthogonal "for all practical purposes".[9] Perhaps what he means by this is that, e.g., in MEAS* the amplitudes associated with errors are very small. But this is no comfort whatsoever since (R2) applies to the *biorthonormal* representation, and not a representation which is almost or approximately biorthonormal!

Or perhaps what he means is that even though his prescription won't exactly pick out the positions of the pointers as well-defined, nonetheless it will pick out observables that are "close" to those position observables as well-defined; i.e., it will pick out observables that nearly commute with the pointer position observable. But this suggestion wouldn't be of any help either, even if it were true. It wouldn't help because we want the position of the observable to be well-defined, not some other quantity. And it happens to be false. In the event (for example) that c_1 is approximately equal to c_2 (of course, if they are equal then Dieks's prescription doesn't apply since the biorthonormal representation is not unique) it will frequently turn out, even if the measuring device involved is extremely accurate, that the pointer observable that gets picked out by BMEAS* as the one that is well-defined will be an observable that almost maximally fails to commute with the pointer position observable.[10]

It is worth noting that imperfect measurements create no special problems for the many minds view. For example, if an observer's perception of the pointer's position is not quite accurate then the state of the observer + M + 'e' will be

OBS $\quad a_1|B_U\rangle|U\rangle|\uparrow\rangle + a_2 B_U|D\rangle|\downarrow\rangle + \cdots$

While this representation is not in biorthonormal form it does specify, according to the many minds view, which minds exist. In this state some of the minds will have perceptions (and beliefs) which are not even pragmatically true. That is, on subsequent measurements (perceptions of the measuring device) a mind associated with $|B_U\rangle|D\rangle|\downarrow\rangle$ will have a probability of experiencing a result which differs from the original one. But this is just what we expect from imperfect measurements (observations). The important point is that definite perceptions (perceptions of the pointer at definite locations) are produced in the observer and that is what is required of a solution of the measurement problem.

To sum up, Dieks's interpretation does not solve the measurement problem since it provides no guarantee that measurements will possess definite outcomes (that the physical quantities corresponding to definite outcomes will possess values) and there are many reasons to think that in fact the states of measurement devices at the conclusion of actual measurements are almost always ones to which (R1) does not assign appropriate physical quantities. Perhaps even more serious (since we might learn to live with the pointer's position being undefined) is that the mental state of an observer who looks at the pointer will, on Dieks's view, typically not be well defined either. This contrasts sharply with the many minds view which is designed to supply definite states of perception and belief even to brains which have evolved into superpositions of eigenstates of mentality. We hope that Dennis Dieks will forgive us if we adopt a phrase of his and conclude by noting that his dismissal of the many minds view and his assertion that he has solved the measurement problem are "much too rash".

NOTES

[1] Albert, D. and B. Loewer: 1988, 'Interpreting the Many Worlds Interpretation', *Synthese* **77**, 195–213.

[2] The classic papers concerning the many worlds interpretation are contained in DeWitt and Graham: 1973, *The Many Worlds Interpretation of Quantum Mechanics*, Princeton University Press, Princeton.

[3] Dieks, D.: 'On Some Alleged Difficulties in Interpretation of Quantum Mechanics', *Synthese* this issue.

[4] An interesting proposal has recently been made by Ghirardi, Rimini, and Weber which avoids the problem of there being two dynamics. For a discussion see Bell, J.: 1988. *The Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, Cambridge.

[5] See Bohm, D.: 'A Suggested Interpretation of Quantum Theory in Terms of "Hidden Variables": Part I', *Physical Review* **85**, 166–79, Bell, *J. op. cit.*, and Albert D., and L. Loewer, 'Two No-collapse Interpretations of Quantum Mechanics', *Nous* **23**, 169–86.

[6] Kochen, S.: 1985, 'A New Interpretation of Quantum Mechanics', in Lahti and Mittelstaedt (eds): *Symposium on the Foundations of Modern Physics*, World Scientific Publishing Co., 151–70; and Healey, R.: 1989, *The Philosophy of Quantum Mechanics*, Cambridge University Press, New York, and van Fraassen, B.: *Quantum Mechanics: an Empiricist View* (unpublished).

[7] See Healey *op. cit.*
[8] Even if (R2) does assign a particular belief there is no guarantee that the dynamics will result in those beliefs persisting.
[9] See Dieks's 'Resolution of the Measurement Problem Through Decoherence of the Quantum State' (unpublished).
[10] We owe this point to Yakir Aharanov.

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