

DYADIC DEONTIC DETACHMENT

1.

Paradoxes thrive in deontic logic. Perhaps the most interesting and significant is Chisholm's 'contrary to duty paradox'. It is important because it has been taken to show that the formal representation of deontic reasoning requires a dyadic conditional-obligation operator  $O(B/A)$ , it ought to be the case that  $B$  given  $A$ . We begin our own addition to *l'histoire d'O* with a review of the paradox and the reasoning which leads to the introduction of the dyadic operator. We will see that there have been two quite different kinds of conditional-obligation operators suggested to cope with the paradox. Each one has its virtues but captures only a part of  $O$ 's personality. We argue that neither is able to resolve the paradox. To accomplish that we will need to introduce considerations of tense, and affirm a distinction between conditional and actual obligation. The system of deontic logic we develop, 3-D, contains all this and a rule for detaching tensed actual-ought statements from conditional-ought statements in certain circumstances. After showing how the paradox is resolved in 3-D we conclude with some suggestions concerning the application of the system to moral and legal reasoning.

First, let's review Chisholm's paradox.<sup>1</sup> Consider the following sentences:

- (1) It ought to be the case that Arabella buys a train ticket to visit her grandmother.
- (2) It ought to be that if Arabella buys the ticket she calls to tell her that she is coming.
- (3) If Arabella does not buy the ticket it ought to be that she not tell her that she is coming.
- (4) Arabella does not buy the ticket.

It appears that (i) the statements 1–4 are consistent. Furthermore it appears that (ii) no one of these statements logically implies any other one.

Paradox results when we try to represent 1–4 in standard deontic logic, SDL, while honoring conditions (i) and (ii).<sup>2</sup> SDL is the system that results from adding a monadic deontic operator  $O$ , it ought to be that, to propositional logic.<sup>3</sup> Axioms are tautologies,  $Op \supset \sim O \sim p$ ,  $O(p \supset q)$ ,  $Op \supset Oq$ , and  $O(T)$ ; rules of proof are modus ponens and substitution. Some deontic logicians have suggested further axioms, e.g.,  $Op \supset OOp$ ,  $OOp \supset Op$ ,  $O(Op \supset p)$ , but these embellishments will not concern us. It is clear that 1 and 4 should be translated by (1\*) $Ob$  and (4\*) $\sim b$ , respectively. But what of 2 and 3? The most natural suggestion is to translate 2 by (2\*)  $O(b \supset c)$  and 3 by (3\*)  $\sim b \supset O \sim c$ . This translation respects *ii*. But since 1\* and 2\* imply  $Oc$  and 3\* and 4\* imply  $O \sim c$  and their conjunction is inconsistent in SDL, the translation violates *i*. There seem to be only three other candidates for translating 2 and 3 into SDL. These are obtained by varying whether or not  $O$  is within or without the scope of  $\supset$ . Each of these possibilities violates condition *ii*. If 2 is translated by  $b \supset Oc$  then it is implied by 4 and if 3 is translated as  $O(\sim b \supset \sim c)$  then it is implied by 1. So it seems that there is no adequate way of paraphrasing 1–4 into SDL.

There are two further requirements on an adequate formalization of (1) and (4) which have been widely discussed. Some deontic logicians have argued that (1) and (2) (or rather analogous sentences) logically imply<sup>4</sup>

- (5) It ought to be that Arabella calls her grandmother to tell her she is coming.

Others have claimed that (3) and (4) logically imply<sup>5</sup>

- (6) It ought to be that Arabella does not call her grandmother to tell her she is coming.

We will refer to the requirements that a formalization of (1) and (4) validate the first and second inferences by (iii) and (iv), respectively. There is some plausibility to each of these requirements though it is clear that they cannot both be accommodated in SDL without violating (i). It's our view that an adequate resolution of the paradox should account for whatever plausibility (iii) and (iv) have.

Like all paradoxes, there is more than one way out of Chisholm's. Most deontic logicians follow Von Wright who finds the difficulty to reside in the representation of (2) and (3). Von Wright's suggestion is

that one or both of these should be represented not as obligations of conditionals or as conditionals of obligations but as genuinely conditional obligations. This involves the augmentation of SDL with a two-place deontic operator  $O(B/A)$ , read as, it ought to be that  $B$  given that  $A$ . The old monadic operator  $O(A)$  is defined as:  $OA \equiv O(A/T)$ , (where  $T$  abbreviates  $pv \sim p$ ).  $OA$  thus defined conforms to SDL. If one employs the dyadic operator, (2) and (3) can be translated by (2\*\*)  $O(c/b)$  and (3\*\*)  $O(\sim c/\sim b)$ , respectively. We will discuss logics for  $O(B/A)$  in the next two sections. For now, to see how the paradox is supposed to be resolved, it suffices to mention some formulas which are not valid in any of the logics suggested for  $O(B/A)$ . Neither  $O \sim A \supset O(B/A)$  nor  $\sim A \supset O(B/A)$  is valid, so condition (ii) is satisfied. Furthermore, the set comprised by 1\*, 2\*\*, 3\*\*, and 4\* is consistent, so (i) is satisfied.

As we previously mentioned, it is not possible to simultaneously satisfy *iii* and *iv* without violating *i*. Dyadic deontic logics divide over whether they contain the theorem

$$(7) \quad OA \cdot O(B/A) \supset OB;$$

or the theorem

$$(8) \quad A \cdot O(B/A) \supset OB.$$

Since these principles allow for the derivation of a monadic deontic statement from a conditional deontic statement and other statements, we will follow Patricia Greenspan and call them "detachment principles";<sup>7</sup> the first is deontic detachment, the second is factual detachment. No consistent extension of SDL can contain both. Of course whether or not the use of the dyadic  $O$  really resolves the paradox depends on the interpretation. In the next section we examine a logic which validates (7), and in the subsequent section we examine a logic which validates (8). We will see that they capture very different concepts of conditional obligation.

## 2.

David Lewis<sup>8</sup> has proposed and discussed several different systems of dyadic deontic logic all of which validate (7) and none of which validates (8). For simplicity we will use his system CUAL (a.k.a.

'VTAL') in our discussion, sometimes referring to this system as *Lewis*.

The language of CUAL is obtained by adding dyadic deontic operators  $O(-/-)$  and  $P(-/-)$  to a propositional language containing the unary connectives  $T$  and  $F$ . The deontic operators are interdefinable:

$$O(A/B) \equiv \sim P(\sim A/B)$$

and

$$P(A/B) \equiv \sim O(\sim A/B).$$

Possible-world semantics for CUAL is formulated in terms of model structures  $(W, \leq)$ , where  $W$  is a set of possible worlds and  $\leq$  is a simple ordering on the members of  $W$ . CUAL contains the 'limit assumption' which says that every subset of  $W$  includes a least set of worlds. While acceptance or rejection of the limit assumption has no affect on the set of valid formulas, it does play a role in the system we develop in section 4.

A Lewis model is a model structure  $(W, \leq)$  and a function  $[ ]$  which assigns to each atomic sentence  $A$  of the language a subset  $[A]$  of  $W$  and which obeys the usual conditions for truth-functional compounds as well as the following condition for the dyadic  $O$ :

$$w \in [O(B/A)] \text{ iff } \exists u(u \in [A \cdot B] \text{ and}$$

$$\forall v(\text{if } v \in [A \cdot \sim B] \text{ then not } (v \leq u))).$$

This says that  $O(A/B)$  is true at  $w$  in  $\langle W, \leq, [ ] \rangle$  just in case worlds at which  $A \cdot B$  are true are ranked before any world at which  $A \cdot \sim B$  are true.

Intuitively,  $\leq$  is thought of as a ranking of worlds induced by some system of values or ethics. The worlds ranked first are the deontically ideal worlds. In these worlds all obligations are met, no rights violated, etc. Of course these worlds may be far from ideal in ways that are ethically irrelevant. If  $u \leq v$  then  $u$  is at least as close to deontically ideal worlds as  $v$ , that is,  $u$  is at least as acceptable as  $v$  with respect to the system of values that determines  $\leq$ . To evaluate  $O(B/A)$  we look at the worlds which are closest to being ideal at which  $A$  is true. If  $B$  is true at all these worlds then so is  $O(B/A)$ . Notice that  $O(A/T)$  says that at all the most ideal worlds  $A$  is true.

A formula  $\phi$  is valid in Lewis iff it holds at all worlds of all models. Here is a sampling of valid formulas and nonvalid formulas.

Valid formulas:

- (7)  $OA \cdot O(B/A) \supset OB$ ;
- (9)  $O(A/A)$ ;
- (10)  $O(T/A)$ ;
- (11)  $O(A/C) \supset \sim O(\sim A/C)$ ;
- (12)  $O(A/C) \cdot O(B/C) \supset O((A \cdot B)/C)$ .

Nonvalid formulas:

- (8)  $A \cdot O(B/A) \supset OB$ ;
- (13)  $\sim A \supset O(B/A)$ ;
- (14)  $OA \supset O(A/B)$ ;
- (15)  $O \sim A \supset O(B/A)$ ;
- (16)  $O(B/A) \supset O(B/(A \cdot C))$ ;
- (17)  $O(B/A) \supset O(B/(A \vee C))$ ;
- (18)  $A \cdot C \cdot O(B/A) \supset O(B/A \cdot C)$ .

Some of these formulas are of special interest to us. The validity of (7) can be seen by noting that it says that if  $OA$  holds in the most ideal worlds and if in the most ideal worlds at which  $A$  holds  $B$  holds, then in the most ideal worlds  $B$  holds. In contrast, (8) is not valid since it is possible for  $A$  to be true and for the most ideal  $A$  worlds to be  $B$  worlds even though both  $\sim A$  and  $\sim B$  hold at some of the most ideal worlds.

The sentences of Chisholm's story can be translated into Lewis in the following way:

- (1\*)  $O(b)$ ;
- (2\*\*)  $O(c/b)$ ;
- (3\*\*)  $O(\sim c/\sim b)$ ;
- (4\*)  $\sim b$ .

It is easy to check that requirements (i) and (ii) are satisfied by this translation. Also since (7) is valid (iii) is met. Does this show that the paradox can be resolved in Lewis? We think not. Our reason is that (1\*) and (3\*\*) are not adequate representations of (1) and (3). (1) does not mean that in deontically perfect worlds, Arabella buys a ticket. Instead, it says that in this world Arabella has an obligation to buy a ticket. The actual world may be far from deontically perfect – presumably there have been many ethical lapses – so the fact that Arabella ideally ought to buy the ticket may be irrelevant to her

actual obligations. A similar remark can be made concerning (3). It apparently says that if Arabella fails to buy a ticket then she has an actual obligation not to call. (3\*) says that in the most ideal worlds at which  $\sim b$  holds  $\sim c$  holds. But the actual world may not be one of those most ideal worlds at which  $\sim b$  holds. Furthermore, it seems plausible that if (3) and (4) are true then Arabella has an actual, not merely a conditional, obligation not to call.

Within Lewis's framework, it appears that we cannot even express actual obligations. As we observed,  $O_p$  says that  $p$  holds in deontically ideal worlds. Even if OA, it might be that actually it ought to be that  $\sim A$ .  $O(A/B)$  doesn't seem to express an actual obligation either. It expresses the conditional obligation that  $A$  given  $B$ . Even if  $B$  is true it doesn't follow that there is an actual obligation that  $A$ , for  $O(\sim A/B \cdot C)$  and  $B \cdot C$  might be true as well. If  $O(A/B)$  is true then  $B$  provides a reason for there being an actual obligation that  $A$ , since in all the deontically ideal  $B$  worlds  $A$  is true. But it is a *defeasible* reason. This suggests that we interpret  $O(A/B)$  as saying that there is a prima facie obligation that  $A$  at least when  $B$  is true. In fact, Lewis appears to be a logic of prima facie ought statements. At best it is incomplete. In so far as Chisholm's story involves *actual* obligations it cannot even be expressed in Lewis.

## 3.

In this section we will examine some systems of deontic logic which contain the principle of factual detachment (8). Such systems have been devised by Peter Mott, Brian Chellas, and Azizah-al-Hibri.<sup>9</sup> The latter develops her system in the most detail, so we will discuss it and some of its extensions. Al-Hibri's system  $S$  is characterized axiomatically as follows:

- A1. Tautologies are axioms;
- A2.  $\sim O(F/A)$ ;
- A3.  $O(B/A) \supset (A \supset O(B/T))$ ;
- A4.  $O(B/A) \cdot O(B/C) \supset O(B/A \vee C)$ .

Rules for the system are modus ponens and

- R1 
$$\frac{\vdash(A \cdot B) \supset D}{\vdash(O(A/C) \cdot O(B/C)) \supset O(D/C)}$$
- R2 
$$\frac{\vdash A \supset C}{\vdash O(B/A) \supset O(B/C)}$$

Possible-world semantics for  $S$  are as follows:

An  $S$  model structure is a pair  $\langle W, f \rangle$  where  $W$  is a set of possible worlds and  $f$  is a function from  $W \times P(W)$  to  $P(W)$  which satisfies the following conditions:<sup>10</sup>

- (a) Not  $f(w, W) = \Lambda$ ;
- (b) If  $w \in X$  and  $f(w, X) = Y$  then  $f(w, W) \subseteq Y$ ;
- (c) If  $f(w, X) = Y$  and  $f(w, X') = Y'$  then  $f(w, X \vee X') \subseteq Y \vee Y'$ ;
- (d) If  $f(w, X) = Y$  and  $f(w, X) = Y'$  then  $f(w, X) \subseteq Y \cap Y'$ .

The function  $f$  is thought of by al-Hibri as assigning to a world  $w$  and a condition  $A$  a set of worlds which are the "best achievable from  $w$  with respect to condition  $A$ ."<sup>11</sup> We will postpone for the moment discussing what she might mean by this phrase. The definitions of an interpretation  $[ ]$  on a model structure  $\langle W, f \rangle$  and validity in  $S$  proceed in the usual way. The only clause of the definition of truth with respect to interpretation  $[ ]$  on structure  $\langle W, f \rangle$  which we should note is:

$$w \in [O(B/A)] \text{ iff } f(w, [A]) \subseteq [B].$$

It is instructive to compare  $S$  and Lewis with regard to theorems and nontheorems. With the exception of the factual-detachment principle A3 (which is the same as (8)) each axiom of  $S$  is valid in Lewis and the rules of  $S$  are also correct for Lewis. Also, (13)–(17) are not valid in either  $S$  or Lewis. However, (7), (9), and (10) are valid in Lewis but not in  $S$ . Of these (10) is not germane to our interests in this paper. Al-Hibri seems to have a special dislike for it which we do not share. In any case, it is a simple matter to alter her semantics to validate it or Lewis's to invalidate it.<sup>12</sup> (7) and (9), on the other hand, do mark important differences between her system and Lewis.

To understand this difference we must discuss the notion of obligation that al-Hibri is attempting to formalize. She insists that the notion is one of *actual* obligation as opposed to *prima facie* obligation. The pertinent difference between the two is that *prima facie* obligations are defeasible while actual obligations are indefeasible. So if it actually ought to be the case that  $B$  given  $A$  and  $A$  holds, then it actually ought to be that  $B$ . In contrast, if there is a *prima facie* obligation that  $B$  given  $A$  and  $A$  holds, it cannot be concluded that it ought to be that  $B$  since other conditions  $C$  may hold which defeat the obligation. Since factual detachment holds in  $S$  but not in Lewis it

is reasonable to interpret the former as a logic of actual obligation and the latter as a logic of prima facie obligation. Further notice that in Lewis  $O(B/T)$  means that  $B$  is true in all the ethically ideal worlds. While  $OB$  may be an ideal it need not be a guide to action since we might have  $O(\sim B/A)$  and  $A$  true. In contrast,  $O(B/T)$  in  $S$  means that actually it ought to be that  $B$ . This is an ought that can direct action since it cannot be defeated.

If these interpretive remarks are correct then  $S$  and Lewis are not in competition since they formalize different concepts of obligation. It would remain to be seen how, or if, the two systems could be combined in a comprehensive deontic logic.

However, there are a number of difficulties with al-Hibri's system. The meaning of  $O(B/A)$  in  $S$  is not entirely clear. Based on her informal discussion it seems that  $O(B/A)$  is supposed to formalize " $B$  is obligatory given  $A$  all things considered."<sup>13</sup> But this would lead one to expect (18),  $A \cdot C \cdot O(B/A) \supset O(B/A \cdot C)$ , to be valid in  $S$  since if  $B$  is obligatory given  $A$ , all facts considered, and both  $A$  and  $C$  are true then the fact that  $A$  and  $C$  are true must have been considered. However (18) is not valid in  $S$ .<sup>14</sup> In any case, the phrase "all things considered" is far from clear. Perhaps the semantics will shed some light on the meaning of  $O(B/A)$ . The function  $f(w, A)$  is supposed to select the set of worlds which are "the best achievable with respect to condition  $A$ ." But what does this mean? A partial answer to this question may result from embedding  $S$  in a somewhat richer system  $K$ .  $K$  is a system which contains SDL without  $O(T)$  and a counterfactual conditional  $\rightarrow$ , whose logic is that constructed by Lewis in *Counterfactuals*. If  $O(B/A)$  is defined as  $A \rightarrow OB$  then each of the axioms and rules of  $S$  hold in  $K$ . So  $S$  is a subsystem of  $K$ . None of (7), (9), (10), (13)–(17) holds. This suggests that one should interpret al-Hibri's  $O(B/A)$  as saying that if  $A$  were the case then it actually ought to be that  $B$ .  $f(w, A)$  is the set of worlds which are determined in the following way: first look at the worlds which are most similar to  $w$  (according to the counterfactual similarity relation) at which  $A$  holds, then find the worlds which are best with respect to each of these worlds.  $f(w, A)$  is the union of these sets.<sup>15</sup>  $K$  is more expressive than  $S$ . For example,  $O(A \rightarrow B)$  and  $A \rightarrow \sim O \sim B$  are not expressible in  $S$ . The latter is especially interesting since it seems to be the appropriate formalization of "it is permissible that  $B$  given  $A$ ." Also (18) is valid in  $K$  though not in  $S$ . As we noted it appears that on

al-Hibri's informal interpretation of  $O(B/A)$  one would expect (18) to be valid. If this is correct then al-Hibri's system is not only unnecessarily obscure but is also incomplete.  $K$  seems to capture her intuitions better than  $S$ .

There is a much more serious objection to al-Hibri's account. It does not satisfactorily deal with Chisholm's paradox. Let's take another look at the paradox. At some time  $t$  Arabella actually ought to buy a ticket to visit her grandmother. She also has an actual obligation to call if she buys the ticket. These are formalized in  $S$  by

(19)  $O(b/T)$  and

(20)  $O(c/b)$ .

In  $S$ , (19) and (20) do not imply

(21)  $O(c/T)$ ,

since deontic detachment does not hold. A good thing this is, since  $O(\sim c/T)$  is derivable from  $\sim b$  and  $O(\sim c/\sim b)$ , the formalizations of 3 and 4, respectively. But is it correct to say that Arabella never has an actual obligation to call her grandmother to tell her that she is coming? It seems to us that if (1) and (2) are true then it may very well be that Arabella actually ought to call. Of course al-Hibri cannot agree with this since she is committed to the truth of  $O(\sim c/T)$ . It seems to us that the natural thing to say is that at one time Arabella had an obligation to visit and to call, but once she was determined not to visit she has an obligation not to call. Obligations change with time. At one time we were obligated to finish this paper by August 1, 1981. But when the editor extended the deadline we no longer had this obligation. The trouble with al-Hibri's account is that it has no way of registering the obvious fact that actual obligations vary in time. If we do take this into account then, as we will see, it is possible to detach both  $Oc$  and  $O\sim c$ . There is no contradiction since the time indices with respect to which the two formulas are evaluated are different.

Let's stop to summarize our survey to this point. Chisholm's paradox spurred logicians on to develop systems of dyadic deontic logic. Two kinds of systems have been developed represented here by Lewis and al-Hibri. They differ primarily in that Lewis contains a principle of deontic detachment while al-Hibri contains a principle of factual detachment. This suggested that the two systems are concerned with different notions of ought: Lewis with what *prima facie*

ought to be and al-Hibri with what actually ought to be. We argued that Lewis is incomplete since it does not contain the resources to express actual obligations and no way of inferring actual obligations from conditional ones. Since Chisholm's paradox seems to involve actual obligations, *Lewis* cannot resolve the paradox. In contrast al-Hibri's system does concern actual obligation. But, aside from the obscurity of her semantical explanations, the system *S* is unable to handle Chisholm's story since it ignores the fact that actual obligations change from time to time.

## 4.

Richard Thomason and Richard Grandy and Marcia Barron have suggested<sup>16</sup> that the key to resolving Chisholm's paradox is taking time and tense into account. They further suggest that once we do this we will have no need for conditional deontic operators. We partially agree with the first and strongly disagree with the second of these sentiments. Our reasons will emerge in what follows.

There are a number of ways of combining obligation and tense. To simplify the presentation we choose to construct a system *OT* in which reference to time is explicit and in which formulas are evaluated with respect to possible histories but not with respect to times. This means that our system will not contain the usual tense operators, although these can be added without difficulty. For the sake of simplicity we will also assume that times are discretely ordered and that there is a first time.

### *The Language of OT*

*OT* contains propositional variables  $p, q, r$ , etc., the usual truth-functional connectives, time terms  $t_1, t_2$ , etc., a two place predicate  $\geq$ , a deontic operator  $O$ , a necessity operator  $\square$  and a counterfactual operator  $\rightarrow$ . The wffs of *OT* are characterized as follows: (a) propositional variables are wffs; (b) if  $A, B$  are wffs and  $t$  is a time term then the following are wffs:  $\sim A, A \cdot B, A \vee B, O_t A, \square_t A, A \rightarrow B, t_1 \geq t_2$ .  $O_t A$  means that at  $t$  it actually ought to be the case that  $A$ .  $\square_t A$  means that at  $t$  it is settled that  $A$ .<sup>17</sup>  $t_1 \geq t_2$  means that  $t_1$  is simultaneous with or later than  $t_2$ .

*Semantics for OT*

An OT model structure is a 5-tuple  $\langle T, W, H, F, \$ \rangle$ , where  $T$  is the set of natural numbers (the set of times),  $W$  is a set of momentary world stages,  $H$  is a subset of the set of functions from  $T$  into  $W$  (these functions are possible histories),  $F$  is a function which assigns to a pair  $h \in H$  and  $t \in T$  a subset of  $H$ , and  $\$$  is a function which assigns to each  $h$  a counterfactual similarity ordering on  $H$ .

$W$  is intended to be the set of possible instantaneous states. A history is then an assignment to each time of a world state.  $F$  is intended to assign to a history and a time a set of deontic alternatives to  $h$  at  $t$ .

Since we are interested in actual obligation, it seems reasonable to place certain restrictions on which histories can count as deontic alternatives to  $h$  at  $t$ . If in  $h$  at  $t$  it is no longer possible to make it the case that  $A$ , then  $A$  cannot be an actual obligation in  $h$  at  $t$ . We can represent this semantically in this way. First define  $h \approx_t h'$  as  $h$  and  $h'$  are identical for all moments at or before  $t$  and we restrict the members of  $F(h, t)$  to a subset of histories  $h'$  such that  $h \approx_t h'$ .

An interpretation [ ] on an OT model structure is defined as follows: [ ] assigns to each propositional variable a subset of  $H$ , the time term  $t_k$  is assigned the number  $k$ . Recursion clauses for the truth functional and counterfactual connectives are the usual ones. The interesting clauses are these:

$$\begin{aligned} h \in [t_m \geq t_n] &\text{ iff } m \geq n; \\ h \in [O_t A] &\text{ iff } F(h, t) \in [A]; \\ h \in [\Box_t A] &\text{ iff } \forall h'(h' \approx_t h \supset h' \in [A]). \end{aligned}$$

Here are some interesting valid and invalid formulas of OT.

Formulas valid in OT:

- (19)  $(\Box_t A \cdot u \geq t) \supset \Box_u A;$
- (20)  $(\Box_t A \cdot u \geq t) \supset \Box_t \Box_u A;$
- (21)  $\Box_t A \supset O_t A;$
- (22)  $O_t A \subset \sim \Box_t \sim A;$
- (23)  $(O_t A \cdot u \geq t) \supset O_t O_u A.$

Formulas not valid in OT:

- (24)  $(\Box_t A \cdot t \geq u) \supset \Box_u A;$
- (25)  $A \supset O_t A;$

- (26)  $O_t A \supset A$ ;  
 (27)  $(O_t A \cdot u \geq t) \supset O_u A$ ;  
 (28)  $(O_t A \cdot t \geq u) \supset O_u A$ .

At first (21) may seem counterintuitive since it says that if the truth of  $A$  is settled at  $t$  then at  $t$  it ought to be that  $A$ . A disturbing example<sup>18</sup> is that since it is now settled that Hitler murdered millions of Jews in the 1940s, it now ought to be that Hitler murdered millions of Jews in the 1940s. This certainly sounds odd, but since (28) is not valid it does not follow that in the 1930s it ought to be that Hitler murders millions of Jews in the 1940s. Nor does it mean that we cannot now blame Hitler. The validity of (21) results from our decision to count as permitted at  $t$  only those states of affairs which are possible to achieve at  $t$ . This decision does seem appropriate for considering ones actual obligations in what Thomason calls "the context of deliberation."<sup>19</sup> As he points out, we can introduce a different notion of obligation  $O^n$  appropriate for "contexts of judgment" as follows:  $O_t^n A$  is true at  $h$  iff  $F(h, t - n) \subseteq [A]$ . In other words,  $O_t^n A$  holds just in case at  $n$  moments before  $t$  it ought to be that  $A$ . Of course,  $\Box_t A \supseteq O_t^n A$  is not valid. So in the context of judgment we can say that  $O_t^n$ (Hitler did not murder millions of Jews in the 1940s) for  $n$  that goes back before the 1940s. (22) is a plausible "ought implies can" principle which holds for the same sort of reasons that (21) holds. Note that  $O_t^n A \supset \sim \Box_t \sim A$  is not valid. The invalidity of (27) and (28) reflects the idea that obligations vary with time.

Let us see how Chisholm's paradox fares in OT. We will let  $t$  represent the time immediately before the moment at which it is not possible for Arabella to buy the ticket and  $u$  to represent the time immediately after that. Natural candidates for representing (1) to (4) are:

- (29)  $O_t b$ ;  
 (30)  $O_t(b \supset c)$ ;  
 (31)  $\sim b \supset O_u \sim c$ ;  
 (32)  $\sim b$ .

Are these paraphrases adequate? No doubt they are an improvement over previously discussed translations. (29) and (30) imply  $O_t c$ , and (31) and (32) imply  $O_u \sim c$ ; but there is no contradiction: just a case of changing obligations. Still there are some reasons to doubt that OT

does justice to all the elements involved in Chisholm's paradox. It is a bit awkward that (2) and (3) receive such different paraphrases in OT. It also seems inappropriate for (3) to be paraphrased by a formula which is implied by  $b$  and for (2) to be paraphrased by a formula which is implied by  $O_t \sim b$ . The situation can be partially remedied by exchanging the horseshoe for a stronger conditional, perhaps  $\rightarrow$ . Even with this remedy an important question remains. OT reflects the idea that obligations come into being and pass away. But from the semantical perspective of OT it must appear completely mysterious that at one time it ought to be that  $A$  while at another time it ought to be that  $\sim A$ . The question is how are the actual obligations at  $t$  determined? We formulate a logic capable of answering this question in the next section.

## 5.

What determines what actually ought to be the case in  $h$  at  $t$ ? Part of the answer is the facts at  $h$ , at least those that are ethically relevant. The other part of the answer is the system of values. Values and facts are like vectors whose resultant is the actual obligations that hold at  $t$ . But how are we to represent these values? Our suggestion is that Lewis's deontic system is tailor-made for representing a system of values. The ranking of worlds as better or worse embodies a value system.

We saw that a difficulty with *Lewis* is that there is no way to express actual obligations in it and of course no way to derive what actually ought to be from Lewis conditionals. Of course, we don't always want to derive an actual ought statement from a conditional. Suppose, for example, that  $O(A/B)$ . We do not want this to imply that it actually ought to be the case that  $A$  even if  $B$  happens to be true since there may be a true statement  $C$  such that  $\sim O(A/B \cdot C)$ . Sometimes though we may possess all the facts that are relevant to determining whether  $A$  is actually obligatory or actually forbidden or actually permissible. We will call such facts "ethically sufficient for  $A$ ."<sup>20</sup> Then if  $B$  is ethically sufficient for  $A$  and  $O(B/A)$  and  $A$  are true, it seems reasonable to conclude that it actually ought to be that  $A$ . In this way values represented in Lewis's system together with facts determine what actually ought to be when those facts are ethically sufficient.

To implement this idea we will combine *Lewis* with the system OT. The resulting system is called 3-D. We will add to the union of the languages of OT and *Lewis* two 2-place operators  $R(B, A)$  read as “ $A$  is ethically sufficient for  $B$ ” and  $R_t(B, A)$  as “ $A$  is ethically sufficient for  $B$  at time  $t$ .” The exact meanings of these will be explained shortly. A 3-D model structure is a tuple  $\langle W, T, H, F, \$, \leq \rangle$  where  $W, T, H, F, \$$  are as in the OT system and  $\leq$  is as in *Lewis* with the modification that the ranking  $\leq$  is on members of  $H$ . Our basic idea for connecting the ranking  $\leq$  with  $F$ , that is, connecting conditional obligations with actual obligations is this: At the first instant of time  $t_0$  we will assume that some of the histories which are ideal according to  $\leq$  can be achieved. But as time proceeds, events and the actions of men may render the ideal histories unattainable. Still at every moment the actual obligation is to bring about one among those best histories that remain. We can express this as a condition relating  $F$  and  $\leq$  as follows:

$$(PP) \quad h \in F(h^*, t) \text{ iff } (h_i h^* \text{ and } \forall_j (\text{if } j_i h^* \text{ then } h \leq j))$$

That is,  $F(h^*, t)$  is the set of histories which are the best possible at  $t$ . Our characterization of  $F$  presupposes that there is a set of best possible histories at  $h, t$  – and this is equivalent to the Limit Assumption mentioned in Section 2. That detachment of actual oughts from a system of values requires the Limit Assumption is an excellent reason for accepting the Limit Assumption on deontic orderings.<sup>21</sup> Suppose that  $A$  is a complete description of everything which is settled at time  $t$ . If  $O(B/A)$  holds and  $O(B/A)$  then clearly  $O_t B$  holds. (Same for  $P(B/A)$  etc.) This is guaranteed by (PP).

We have defined  $O_t$  so that what is settled at  $t$  determines what ought to be at  $t$ . We can introduce other  $O$  operators which are detachable from the value system on the basis of other considerations. For example,  $O_t^n \phi$  is true when  $\phi$  is true at all the best histories achievable from the actual history from  $t - n$ .  $O_{t-t} \phi$  is just the ideal obligation operator  $O(\phi/T)$ . The sequence of operators  $O_{t-t} \dots O_{t-n} \dots O_{t-1} O_t$  can be thought of as descending from the ideal as more and more of the unsavory facts about our world are taken into account. We could also introduce a context dependent  $O$  operator for which an analogue of (PP) holds that allows detachment with respect to certain facts determined by context.

It is important to emphasize the way (PP) (and its analogues for

other operators) determines the truth conditions of  $O$  statements. Given a value system (a Lewis ranking of worlds), the actual obligations which hold at time  $t$  in history  $h$  are determined by certain features of  $h$ . No new semantical apparatus needs to be introduced to provide truth conditions for  $O_t\phi$ .

Although (PP) provides the background which makes detachment of actual obligations from conditional obligations possible, we never know everything that is settled at  $t$ . Fortunately, in most value systems, most of what is settled is irrelevant to what ought to be the case at  $t$ . Presumably whether or not George Washington chopped down the cherry tree is irrelevant to what Arabella ought to do today. It is our view that the concept of *relevance* occupies a central place in deontic reasoning.

How can we represent appropriate notions of relevance? There are two options which we consider, one time independent, the other time indexed.

- R.  $h^* \in [R(B, A)]$  iff  $\cdot \forall h \forall h' \forall t$  (if  $h, h' \in [\Box_t A \cdot \Diamond_t B]$  then  $F(h, t)_{\bar{B}} F(h', t)$ );
- $R_m$ .  $h^* \in [R_m(B, A)]$  iff  $\forall h \forall h' \forall t$  (if  $h_{\bar{m}} h^*$  and  $h_{\bar{m}} h^*$  and  $h, h' \in [\Box_t A \cdot \Diamond_t B]$  then  $F(h, t)_{\bar{B}} F(h', t)$ ).

where

$$F(h, t)_{\bar{B}} F(h', t) \text{ iff } (((F(h, t) \subseteq [B] \text{ iff } F(h', t) \subseteq [B] \text{ and } (F(h, t) \subseteq [\sim B] \text{ iff } F(h', t) \subseteq [\sim B]))).$$

$R$  says that  $A$  is ethically sufficient to  $B$  just in case in any two histories  $h, h'$  for any time  $t$  at which  $A$  is settled and  $B$  is possible,  $B$  is obligatory at  $h$  iff  $B$  is obligatory at  $h'$  and  $B$  is forbidden at  $h$  iff  $B$  is forbidden at  $h'$ .  $R_m$  is similar to  $R$  except that the histories we look at to see whether or not  $R_m(B, A)$  are restricted to those which agree with the actual history until time  $m$ . In both  $R$  and  $R_m$  we require for  $R(B, A)$  only that  $F(h, t)_{\bar{B}} F(h', t)$  for certain times, those at which  $\Box_t A$  and  $\Diamond_t B$ . The reason for the first restriction is that if the truth of  $A$  is not settled until after the truth of  $B$  we do not want the fact that  $A$  will be true to determine the deontic status of  $B$ . (For example, suppose it is better than Arabella not buy the ticket and not call than that she buy the ticket and not call. Suppose as a matter of fact that she will not call but that this is not yet settled. We certainly do not want to conclude from this that she now ought not buy the ticket.)

The reason for the second restriction is that once the truth value of  $B$  is settled so is its deontic status.

There are some logical features of  $R(B, A)$  and  $R_m(B, A)$  that are worth noting. First, the truth value of  $R(B, A)$  depends on  $F(h, t)$  for certain values of  $h, t$  and, given (PP),  $F(h, t)$  depends on the value structure  $\leq$ . So, whether or not  $R(B, A)$  holds depends only on the nature of the system of values. In contrast  $R_m(B, A)$  also depends on the history of the world until  $m$ . If  $S_m$  describes everything settled at  $m$  then the following holds:

$$(33) \quad R_m(B, A) \text{ iff } R(B, A \cdot S_m)$$

and of course

$$(34) \quad R(B, A) \supset R_m(B, A)$$

is also valid.

Some other valid formulas concerning  $R$  are:

$$(35) \quad R_m(B, A) \supset R_m(\sim B, A);$$

$$(36) \quad R_m(B, A) \cdot (\Box_m A \supset \Box_m C) \\ \cdot (\Box_m \sim A \supset (\Box_m \sim C)) \supset R_m(B, A \cdot C);$$

$$(37) \quad R_m(B, A) \cdot m' \geq m \supset R_m(B, A).$$

Some invalid formulas concerning  $R$  are:

$$(38) \quad R_m(B, A) \supset R_m(B \sim A);$$

$$(39) \quad R_m(B, A) \supset R_m(B \cdot C, A);$$

$$(40) \quad R_m(B, A) \supset R_m(B \vee C, A).$$

Our reason for introducing  $R(B, A)$  and  $R_m(B, A)$  is to enable us to formulate detachment rules. It turns out that in 3-D we can have both factual and deontic detachment. The following are valid schema:

<p>I.</p> $\begin{array}{l} O(B/A) \\ R(B/A) \\ \Box_t A \\ \diamond_t B \\ \hline O_t B \end{array}$	<p>III.</p> $\begin{array}{l} O(B/A) \\ R(B/A) \\ O_t A \\ \diamond_t B \\ \hline O_t B \end{array}$
<p>II.</p> $\begin{array}{l} O(B/A) \\ R_t(B, A) \\ \Box_t A \\ \diamond_t B \\ \hline O_t B \end{array}$	<p>IV.</p> $\begin{array}{l} O(B/A) \\ R_t(B, A) \\ O_t A \\ \diamond_t B \\ \hline O_t B \end{array}$

I and II are factual detachment principles while III and IV are deontic detachment principles. Although  $R_t(B, A)$  may hold while  $R(B, A)$  fails, exactly the same actual obligations can be detached from I, III as from II, IV, at least as long as everything that is settled at  $t$  can be described by a single statement. This holds in virtue of (33), since whenever we can detach  $O_t B$  using II( or IV) we can detach  $O_t B$  using I (or III) with  $O(B/A)$ ,  $R(B, A)$  and  $\Box_t A$  replaced by  $O(B, A \cdot S_t)$ ,  $R(B, A \cdot S_t)$  and  $\Box_t(A \cdot S_t)$ . Still II and IV are much more useful in constructing arguments for actual obligations since one may have reason to believe  $R_t(B, A)$  without being able to describe everything that is settled at  $t$ .

To better understand how the detachment schemas work, examine the representation of a 3-D model in Figure 1. Circled numbers at the ends of branches provide the place in the ranking of the history represented by the branch. Note that in this model the following holds:  $O(\sim s)$ ,  $O(b/s)$ ,  $O(c/b)$ ,  $O(\sim c/\sim b)$ ,  $R(\sim c, \sim b)$ ,  $R(c, b)$ . At  $t_0$  the best attainable histories are  $(\sim s \cdot b \cdot c)$  and  $(\sim s \cdot \sim b \cdot \sim c)$ . Suppose that actually  $s$  is true so  $\Box_{t_1} s$ . Since  $R(s, b) \cdot \Diamond_{t_1} b$  and  $O(b/s)$  we can use I to conclude  $O_{t_1} b$ . Of course this is what we want since once  $s$  is true the best attainable history is  $s \cdot b \cdot c$ . Notice that although  $\sim R(c, s)$ , since whether or not  $b$  makes a difference to the deontic status of  $c$ ,  $R(c, b)$  does hold. Since  $O_{t_1} b$ , we can use III to conclude

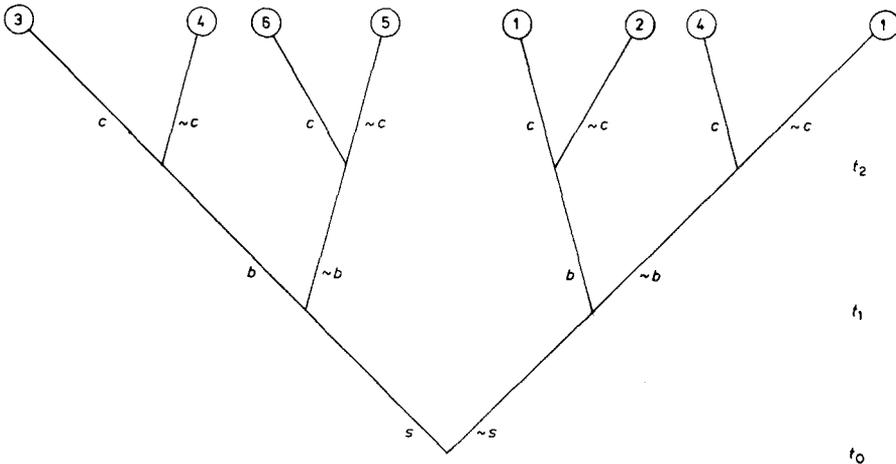


Fig. 1.

$O_{t_1}c$  from  $\diamond_{t_1}c$ ,  $O_{t_1}b$ ,  $O(c/b)$  and  $R(c, b)$ . Of course we cannot at  $t_1$  conclude  $O_{t_2}c$  since it is still possible for  $\sim b$  to turn out to be true and in that case  $O_{t_2}\sim c$  would hold. Suppose that  $s$ ,  $b$ , and  $c$  stand respectively for 'Arabella's grandmother is sick at  $t_1$ ,' 'Arabella buys a train ticket at  $t_2$ ' and 'Arabella calls at  $t_3$ .' We will suppose also that the 'tree' in section 5 represents a part of the system of values relative to which we are evaluating Arabella's situation. Let's also suppose that  $s$  is true and the time is  $t_1$ . At this point we can assert  $O_{t_1}b$ . Suppose we represent statements (2) and (3) of Chisholm's story as  $O(c/b)$  and  $O(\sim c/\sim b)$ . Our claim is that this representation interpreted within the 3-D framework resolves the paradox.

We previously observed that commentators on Chisholm's paradox have disagreed concerning whether or not (1) and (2) imply (5) and whether or not (3) and (4) imply (6). Our representation of these sentences in 3-D clarifies the matter.  $O_{t_1}b$  and  $O(c/b)$  do not by themselves imply  $O_{t_1}c$ . In the context of the story it may be reasonable to assume  $R(c, b)$  and  $\diamond_{t_1}c$ . This justifies the conclusion  $O_{t_1}c$ . However, it does not justify  $O_{t_2}c$ . Our view is that the hesitancy to draw the conclusion  $O_{t_1}c$  has two sources. First is the failure to keep track of time and to confuse  $O_{t_1}c$  with  $O_{t_2}c$ . And secondly, there is the uncertainty of whether or not  $b$  is ethically sufficient for  $c$ . Chisholm's story leaves ethical sufficiency relations undetermined. Similar remarks apply to the inference of (6) from (3) and (4). Since Arabella does not buy the ticket at  $t_1$  we have  $\square_{t_2}\sim b$ . And  $\square_{t_2}\sim b \cdot O(\sim c/\sim b) \cdot \diamond_{t_2}\sim c \cdot R(\sim c, \sim b)$  implies  $O_{t_2}\sim c$ . Again the conclusion of  $O_{t_2}\sim c$  is justified on the assumption of an appropriate sufficiency relation. Hesitancy to draw this conclusion is due, we think, to the fact that Chisholm's story does not make clear whether or not  $R(\sim c, \sim b)$  holds. Our representation of Chisholm's paradox in 3-D satisfies condition (i) and (ii), allows for both deontic and factual detachment, and explains why there is hesitancy in detaching (5) and (6). In these respects it is superior to all other accounts of the paradox.

## 6.

If the only application of 3-D were to Chisholm's paradox then our paper would certainly be a case of attempting to kill a flea with a cannon. But we claim that the arguments representable in 3-D but not

in simpler systems are central to moral and indeed all practical reasoning. As an example consider the following legal scenario. In the famous Riggs case, the question before the court was whether Riggs should inherit the money willed to him by his grandfather, Riggs having murdered his grandfather. As is typical in the law, there were reasons in favor of any particular solution and reasons against it. The fact that the will was valid is a reason for allowing Riggs the inheritance. Does it follow that he actually ought to receive it? No, for other facts may be legally relevant. In fact the court decided that despite the fact that the will was valid, Riggs actually ought not receive the inheritance since no one should be permitted to profit from his own wrongdoing. It is precisely this kind of practical reasoning that can be represented in 3-D.

There are two interesting philosophical applications of 3-D which are worth mentioning. The first concerns Searle's famous attempt to derive an ought statement from 'is' statements.<sup>22</sup> According to Searle, it is 'tautological' that promises ought to be kept. Given the 'is' statement that Arabella promises to visit her grandmother, Searle claims that it can be concluded that it ought to be that Arabella makes the visit. This is supposed to be a case of an 'is' statement logically implying an ought statement.

Jaakko Hintikka has discussed Searle's argument at length<sup>23</sup> and has, in our view, correctly pointed out that at best "If Arabella promises to visit them she does visit" is analytic only if the obligation is taken to be a prima facie obligation. He goes on to argue that we can conclude that Arabella has an actual obligation to visit only if we assume that there are no other considerations which overrule this prima facie obligation. In other words, that the fact that Arabella has promised is all that is relevant to her putative actual obligation to visit. Hintikka then correctly remarks that the statement that there are no other relevant considerations to itself a normative statement. So Searle has not succeeded in deriving an ought statement from only 'is' statements.

We basically agree with Hintikka's analysis of Searle's argument. However, Hintikka's discussion is flawed by the fact that he represents prima facie and actual obligation statements in SDL.<sup>24</sup> He claims that the prima facie obligation that  $q$  given  $p$  is represented by  $O(p \supset q)$  while the actual obligation that  $q$  given  $p$  is represented by  $p \supset Oq$ . There are many reasons why this is inadequate. For one, in

SDL  $O(p \supset q)$  implies  $O(p \cdot v \cdot \supset q)$ . This would seem to say that if there is a prima facie obligation that  $q$  given  $p$ , then there is a prima facie obligation that  $q$  given  $p$  and  $v$ . But surely this is not right. Given that Smith impregnates Arabella he has a prima facie obligation to marry her. But it doesn't follow that given that he impregnates her and that she is already married he has a prima facie obligation to marry her.

It is our contention that prima facie and actual obligations and the interplay between the two can all be represented in 3-D. There is a prima facie obligation that  $v$  just in case there is a settled  $s$  for which  $O(v/s)$ . In Searle's example  $p$  is the condition which gives rise to the prima facie obligation to visit. There is an actual obligation at  $t$  to visit which is represented of course by  $O_t v$ . Our suggestion is that Searle's argument is an instance of II.

(40)  $O(v/p)$

(41)  $\Box_t p$

(42)  $\Diamond_t v$

(43)  $\frac{R_t(v, p)}{O_t v}$

(44)

We will, with Hintikka, grant Searle his claim that (40) is 'tautological'. (41) and (42) are presumably 'is' statements. But the argument with premises (40), (41), (42) and conclusion (44) is not valid. The additional premise (43) is required for a valid argument. (43) is certainly a value statement. Can it plausibly be maintained that it is tautological? It hardly seems so since its truth value depends on the truth values of all the statements of the form  $O(v/p \cdot q)$ . Only if all these statements are tautological would the argument above be a derivation of an 'actual ought' statements from 'is' statements.

In the same paper, Hintikka also remarks that, in his view, we are likely to be more certain of our ideal or prima facie obligation than of our actual obligations.<sup>25</sup> Other philosophers, notably Prichard,<sup>26</sup> and Aristotle<sup>27</sup> apparently take the opposite view holding that we are more certain of our actual obligations and that, at best, we can infer conditional obligations by considering what our actual obligations would be under certain circumstances. We do not want to endorse either of these views but we do think that the issues can be illuminatingly discussed from the perspective of 3-D.<sup>28</sup>

Recall that in 3-D a value system is represented by  $\leq$ , a ranking of

possible histories in terms of how well they manifest some system of ethical values.  $\leq$  of course determines all the conditional or prima facie obligations. Given  $\leq$  and the facts of  $h^*$  to  $t$ , we can determine the actual obligations at  $h^*$  at  $t$ . This may make it appear that Hintikka's view that prima facie obligations are primary is more plausible. But even if prima facie obligations are ontologically primary they need not be epistemologically primary. It may be that often we have firm intuitions about our actual obligations at a certain time without knowing how they derive from the ethical system and relevant facts. This may be like 'knowing' that a certain expression is grammatical without knowing how to derive it from a grammar. Now the interesting thing is this: while we can derive actual oughts from conditional oughts and the facts, we cannot in general go the other way round. Even if we knew what ought to be at every moment of the actual history and the facts of the actual history, we could not in general recover a unique  $\leq$  from which these are derivable. Typically there will be many such  $\leq$ . Actual obligations underdetermine prima facie obligations. However, if we knew what actual obligations hold at all counterfactual histories at various times, we can recover a unique  $\leq$ . Formally, the situation is that we can define  $F(h, t)$  in terms of  $\leq$ , but we cannot define  $\leq$  in terms of  $F(h^*, t)$ , where  $h^*$  is the actual history. However, we can define  $\leq$  in terms of  $F(h, t)$ .

Let's suppose for the moment that there is some objective value system  $\leq$  which underlies our moral judgments. It is likely that no one completely knows  $\leq$ . More realistically each of us 'knows' some prima facie and some actual obligation statements. We can sometimes test one kind of knowledge against another and in that way modify or develop our beliefs about both prima facie and actual obligation. As we confront new ethical situation, opportunities for this testing and development increase. By considering what our actual obligations would be in counterfactual situations, it can be further increased. The picture is one of a kind of "reflective equilibrium" between prima facie and actual obligations in which our 'knowledge' of the underlying system of values is expanded.

It would probably be more realistic to consider the system of values  $\leq$  not as being objectively given but as being in the process of construction. There may be certain restrictions on the construction process, for example, that if there are ethical differences in two cases that there must be some relevant factual difference. If we thought of

our ethical system in this way then a model for it would be a sort of partial 3-D model, perhaps with restrictions on how the model can be extended. Such models might prove every interesting for investigating certain issues in philosophy of law, especially the controversy surrounding Dworkin's view that "there is always a right answer."<sup>28</sup> But we have already introduced enough complications into O's life, so we forbear, for the moment, from continuing these speculations.

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#### NOTES

<sup>1</sup> Or a perfectly analogous one. Cf. Roderick M. Chisholm, 'Contrary-to-Duty Imperatives and Deontic Logic' *Analysis* 24 (1963): 33-36.

<sup>2</sup> For accounts of SDL., cf. Dagfinn Føllesdal and Risto Hilpinen, 'Deontic Logic: An Introduction,' in Hilpinen (ed.), *Deontic Logic: Introductory and Systematic Readings*, D. Reidel, Dordrecht, 1971, pp. 13-15; and Bengt Hansson, 'An Analysis of Some Deontic Logics,' in Hilpinen (ed.), *op. cit.*, pp. 127-28.

<sup>3</sup> While the point is controversial, in this paper we assume that the 'ought to do' can be defined in terms of the 'ought to be': *S* ought to do *A* iff it ought to be that *S* does *A*. Cf. Roderick M. Chisholm, 'Practical Reason and the Logic of Requirement,' in Joseph Raz (ed.), *Practical reasoning*, Oxford, 1978, p. 127. (Reprinted from S. Korner (ed.), *Practical Reason* Blackwell, Oxford, 1974.

<sup>4</sup> Judith Wagner Decew, 'Conditional Obligation and Counterfactuals,' *Journal of Philosophical Logic* 10 (1981), 55-72. Also cf. the systems developed by David Lewis, *Counterfactuals*, Blackwell, Oxford, 1973; David Lewis, 'Semantical Analysis for Dyadic Deontic Logic,' in S. Stenlund (ed.), *Logical Theory and Semantic Analysis*, D. Reidel, Dordrecht, 1974, pp. 1-14; Bengt Hansson, 'An Analysis of Some Deontic Logics,' *op. cit.*, pp. 121-147; Bas van Fraassen, 'The Logic of Conditional Obligation,' *Journal of Philosophical Logic* 1 (1972), 417-483.

<sup>5</sup> Peter L. Mott, 'On Chisholm's Paradox,' *Journal of Philosophical Logic* 2 (1973), pp. 197-211; Brian F. Chellas, 'Conditional Logic,' in S. Stenlund (ed.), *op. cit.*; pp. 23-33; Azizah al-Hibri, *Deontic Logic: A Comprehensive Appraisal and a New Proposal*, University Press of America, Washington, D. C., 1978.

<sup>6</sup> Georg Henrik von Wright, 'A New System of Deontic Logic,' in Hilpinen (ed.), *op. cit.*, pp. 109, 115.

<sup>7</sup> P. S. Greenspan, 'Conditional Ought and Hypothetical Imperatives,' *Journal of Philosophy* 72 (1975), 259-276.

<sup>8</sup> D. Lewis, 'Semantical Analyses for Dyadic Deontic Logic,' *op. cit.*; and *Counterfactuals*, pp. 130-132.

<sup>9</sup> Peter L. Mott, 'On Chisholm's Paradox,' *Journal of Philosophical Logic* 2 (1973), pp. 197-211; Brian F. Chellas, 'Conditional Logic,' in S. Stenlund (ed.), *op. cit.*, pp. 23-33; Azizah al-Hibri, *Deontic Logic: A Comprehensive Appraisal and a New Proposal*, University Press of America, Washington, D. C., 1978.

<sup>10</sup> These semantics for  $S$  are a bit different from al-Hibri's. She formulates semantics in terms of a relation  $R$  between worlds, propositions, and propositions which satisfy analogues of (a)–(d). In any case the logics characterized are identical.

<sup>11</sup> al-Hibri, p. 159.

<sup>12</sup> To validate it in  $S$  add a condition guaranteeing that  $f(w, x) \neq \lambda$ ; to invalidate it in CUAL drop the  $U$  condition ("universality") which guarantees that all worlds are ranked by  $\leq$ .

<sup>13</sup> al-Hibri, pp. 49, 53, 64, 67, 82, 83.

<sup>14</sup> (18) does not hold at  $w$  if  $A \cdot C$  is true at  $w$  and if  $f(w, A) \supseteq [B]$  but  $f(w, A \cdot C) \not\supseteq [B]$ . This is possible without violation of the conditions on  $f$ .

<sup>15</sup> We are not certain that al-Hibri would find this interpretation of her system satisfactory. However we think it is illuminating to compare  $S$  and  $K$  when  $S$  is interpreted in  $K$ . Moreover, al-Hibri does regard Mott's solution to Chisholm's paradox as "basically acceptable" (p. 97) – and Mott translates (3) as does  $K$  – even though she rejects Mott's system because it validates OT; but we constructed  $K$  so that it does not contain OT.

<sup>16</sup> Richard Thomason, 'Deontic Logic as Founded on Tense Logic,' and 'Deontic Logic and the Rule of Freedom in Moral Deliberation'; Richard Grandy and Marcia Baron 'The Story of O Simplified.' None of these papers are published yet.

<sup>17</sup> Richmond H. Thomason, 'Indeterminist Time and Truth Value Gaps,' *Theoria* 36 (1970), 264–81; Bas C. van Fraassen, 'A Temporal Framework for Conditionals and Chance,' in William L. Harper, Robert Stalnaker, and Glenn Pearce, (eds.), *Ifs* D. Reidel, Dordrecht, 1981, pp. 326–327.

<sup>18</sup> Suggested by Paul Ziff.

<sup>19</sup> Thomason, 'Deontic Logic as Founded on Tense Logic,' in manuscript, p. 10.

<sup>20</sup> The term "ethical sufficiency" was suggested to us by Patricia Greenspan.

<sup>21</sup> If the limit assumption failed we there could be a sequence  $O(B/A_1)$ ,  $O(\sim B/A_1 \cdot A_2)$ ,  $I(B/A_1 \cdot A_2 \cdot A_3) \dots$  with  $A_1, A_2 \dots$  all settled. In this case no actual obligation would follow from the conditional ought statements and the facts.

<sup>22</sup> John R. Searle, 'How to Derive Ought from Is,' *Philosophical Review* 73 (1964), 43–58. In more recent writings, Searle seems to accept the actual/prima facie ought distinction, saying that only prima facie obligations can be derived from fact statements. Cf. *Speech Acts*, C.U.P., Cambridge, 1969, p. 181.

<sup>23</sup> Jaakko Hintikka, 'Some Main Problems of Deontic Logic,' in Hilpinen (ed.), *op. cit.*, pp. 87–101.

<sup>24</sup> Hintikka's system also contain  $O(OA \supset A)$ .

<sup>25</sup> Hintikka, *op. cit.*, p. 93.

<sup>26</sup> Prichard may seem to side with Aristotle when he ends 'Does Moral Philosophy Rest on a Mistake?' with this statement: "... if we doubt whether there is really an obligation to originate  $A$  in a situation  $B$ , the remedy lies not in any process of general thinking, but in getting face to face with a particular instance of the situation  $B$ , and then directly apprehending the obligation to originate  $A$  in that situation." (From Gorovitz (ed.), *Utilitarianism* (Bobbs-Merrill, 1971), p. 72.)

However, Prichard does not distinguish between immediate apprehension of actual obligations and that of moral principles, and indeed it seems that for him direct apprehension of the former involves that of the latter: "The plausibility of the view that obligations are not self-evident but need proof lies in the fact that an act which is referred to as an obligation may be incompletely stated..." (Gorovitz (ed.), p. 66).

Presumably Prichard regards complete "statement" of an obligation to do *A* as including specification of all relevant features of the situation, and this is grounds for saying he is neutral on the priority question.

<sup>27</sup> According to W. D. Ross, Aristotle held that "ethics reasons not from but to first principles; it starts not with what is intelligible in itself but with what is familiar to us, i.e. with the bare facts, and works back from them to the underlying reasons; and to give the necessary knowledge of the facts a good upbringing is necessary. Mathematics deals with a subject-matter the first principles of which are acquired by an easy abstraction from sense-data; the substance of mathematics is the deduction of conclusions from these first principles. The first principles of ethics are too deeply immersed in the detail of conduct to be thus easily picked out, and the substance of ethics consists in picking them out." (W. D. Ross, *Aristotle*, 5th ed. (Methuen, 1949), p. 189.)

But one should not conclude from this that Aristotle would have no use for the apparatus of 3-D. As Hardie notes about the above passage from Ross, "Aristotle does indeed say that the student must start in this way, working *towards* first principles. But he does not deny that politics, as a special science, derives consequences from its own first principles" (W. F. R. Hardie, *Aristotle's Ethical Theory* (Oxford, 1968), p. 36).

<sup>28</sup> Cf. Ronald Dworkin, 'Can Rights Be Controversial?' in *Taking Rights Seriously* (Harvard University Press, 1977), pp. 279–290; and 'No Right Answer?' in P. M. S. Hacker and J. Raz (eds.), *Law, Morality and Society* (Oxford, 1977), pp. 59–84.