

A Game of Competitive Investment: Over-capacity and Under-learning

Jian Yang*, Yusen Xia[†] and Junmin Shi[‡]

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Abstract

We consider the situation where a number of firms decide their individual capacity investment levels. The total sum of these levels determines the total return, which the firms share in proportion to their contributions. Prior to their commitments, firms may spend efforts on learning a size indicator of the market. Using this model, we can explain the over-capacity phenomenon that appeared time and again in a slew of industries. The competitive learning aspect of the situation sheds light on the chronic neglect of due diligence when companies are supposed to conduct demand-forecast studies but do not do so.

Keywords: Investment Game; Over-capacity; Under-learning

*Department of Management Science and Information Systems, Rutgers Business School, Newark, NJ 07102, Email: jian.yang@rutgers.edu

[†]J. Mack Robinson College of Business, Georgia State University, Atlanta, GA 30303, Email: ysxia@gsu.edu

[‡]J. Mack Robinson College of Business, Georgia State University, Atlanta, GA 30303, Email: jmshi@gsu.edu.

1 Introduction

1.1 Motivation and Outline

In various product markets, especially those involving huge initial investments but comparatively little operational costs, one may observe that over-capacity, i.e., the presence of an industry-wide capacity that is more than desirable for the combined welfare of the producers, is a common phenomenon. In the DRAM industry, chip makers have been expanding their production capacities over recent years, resulting in continuous price drops that hurt the makers' profit in turn (Karkdos et al. 2008). In the LCD industry, LG Philips announced a \$335 million loss in 2006 due to a steep price decline resulting from an industry-wide capacity glut (Burns 2006). The auto industry is also plagued by excess capacity. In this industry, it has been estimated that the global over-capacity is around 20 percent (Dressler 2004).

Despite the prevalence and impact of over-capacity, there is but a dearth of research on the cause of this phenomenon. The few works dealing with this issue treat it as the result of established firms making credible threats to deter the entry of newcomers. However, it is hard to see that the self-harming strategy of over-capacity is used by established firms solely for the purpose of battling the remote chance of new entrants, since initial investments needed to enter the aforementioned industries all involve billions of dollars. It is against this backdrop that we propose a capacity-setting game, involving equally established firms. In this setting, over-capacity comes as a natural competitive outcome as firms jostle for market shares.

Key ingredients of our game-theoretic setting are: 1) the total revenue generated by all firms is increasing and concave in the total capacity built up by all firms, and 2) a firm's revenue share is proportional to its capacity share. As shall be clear later, these features are consistent with industries in which initial investments are comparatively costlier than day-to-day operations. Our setup naturally leads to the fact that a higher industry-wide capacity is needed in the competitive setting more than in the first-best setting, for a firm's marginal return on capacity investment to be matched by its marginal investment cost. This therefore leads to over-capacity.

Much is at stake when a firm commits to a multi-year project of building a multi-billion

dollar production facility from scratch, while future market outlook is still uncertain. Hence, forecasting of future demand is essential to a firm's survival and prosperity in the face of cutthroat competition. But learning under competition is a tricky business. With the ease at which data travel in this Internet age, it is impossible for a firm to keep what it has learned about the market in complete darkness. When multiple firms collect data about the same market, each participating firm may learn more than it would alone. Yet, as far as we know, there is no conclusive result on whether it positively or negatively affects each player when everybody gains more knowledge about market conditions. Therefore, we may contend that effects of information and learning in competitive environments are not yet well understood, even though benefits of better information have been well established in single-firm settings (e.g., Blackwell 1951, and Lehmann 1988).

From our capacity game, we take one step towards the better understanding of information and learning under competition. Specifically, we add one stage before the capacity-setting stage. In this stage, firms' efforts are focused on learning a size indicator of the common market. To make matters simple, we let random variables representing the size indicator and signals received by firms be bi-valued. Learning is reflected by relationships between these random variables and firms' efforts. Our learning framework reflects the externality in learning, so that the trust-worthiness of the signal received by a firm is determined by both the learning effort put in by the current firm and efforts put in by other firms. The framework also allows different firms to receive different signals. As the capacity game involving learning is difficult to analyze in its full generality, we let the investment return function take a special form. We then concentrate on the case where the information collected by every firm is public knowledge. For this case, we examine the under-learning effect, the phenomenon in which firms shirk from their learning responsibilities, hoping that others will do the dirty work for them.

The following is a summary of our main contributions:

1. we establish that the concavity of the investment return function and the proportionality of revenue allocation are primary culprits for over-capacity (cf. Theorem ??);
2. in a competitive setting, we introduce notions for both information structure and controlled learning; and,

3. for the case where firms cannot hide what they learn, we demonstrate the severity of the under-learning effect—the more numerous the firms, the less they will know about the market in which they all operate (cf. Theorem ??).

The above point 1 gives a plausible interpretation to the prevalent over-capacity phenomenon. Also, point 2 provides a functional alternative for modeling information and learning in competitive settings; and, point 3 offers firms forewarnings about the dire consequences of neglecting due diligence in their in-house market research before plunging into an uncertain market into which others are rushing as well (e.g., the sub-prime mortgage market from 2002 to 2007). Finally, we want to add that our under-learning results are consistent with firms' demand learning and capacity investment behaviors in several industries. For example, in the electronics industry, without sufficient and effectively learning demand information, firms invested too much in capacity, which led prices for DRAM chips to fall 70% in 2007 (Ihlwan 2007).

1.2 Literature Survey

In a duopolistic setting, Kreps and Scheinkman (1983) showed that a two-stage game involving capacity competition in the first stage and Bertrand-like price competition in the second stage results in a Cournot-like equilibrium. Davidson and Deneckere (1986) pointed out the earlier result's critical dependence on a particular demand rationing rule. Acemoglu, Bimpikis, and Ozdaglar (2009) considered a similar two-stage model in which consumers always exhaust a lower-priced firm's capacity before moving on to a higher-priced firm. They quantified inefficiencies of the game's equilibria and demonstrated that great differences exist between different equilibria. Anupindi and Jiang (2008) studied the role played by flexibility in a duopolistic game involving capacity decisions. The industrial organization literature has shown that excess capacity can be exploited by an established firm as a credible threat to deter entry; see, e.g., Dixit (1980) and Bulow, Geanakoplos, and Klemperer (1985).

The competitive-newsboy framework of Lippman and McCardle (1997) can certainly be used in a competitive-capacity study. However, this framework is more suitable for the situation where the set of firms under scrutiny constitutes only a small portion of the entire industry, to the effect that the total capacity built by these firms does not have any sway

over the sales price of the concerned product. Incidentally, as examined in Cachon (2003, Section 6.5.1), this framework also leads to an over-capacity phenomenon, due primarily to the fact that each firm ignores the demand-reducing effect on other firms when it builds excess capacity. The same reason is behind the over-capacity effect identified by Mahajan and van Ryzin (2001) in their dynamic consumer choice framework. Our study of over-capacity across entire industries necessitates a different setup.

Elastic demand was indeed considered in Deneckere, Marvel, and Peck (1997) and Cachon (2003, Section 6.5.2). However, their models allow for an unlimited supply of nonatomic firms. New firms will come into competition as long as the market price has not been driven to zero. Our setting is oligopolistic with a fixed number of firms. We shall demonstrate over-capacity at the individual-firm level.

To leave room for the later learning-stage addition, we do not explicitly model a pricing-and-rationing stage after the capacity-setting stage. Nevertheless, our setup takes into account the industry-wide capacity's dampening effect on firms' pricing powers. As mentioned, our fundamental assumptions on the capacity-setting stage are the concave total revenue function and proportional revenue allocation. One explanation for these assumptions is as follows: When production is relatively cheap, firms tend to fully utilize their capacities. This way, the total industry-wide capacity, together with the innate demand-price curve of the market, will determine the market-clearing price, and hence the total industry-wide revenue. Since all firms face the same price, each firm's revenue share is its share of the total capacity.

Under this setup, a firm has the incentive to keep on expanding its capacity until the marginal return on its own build-up can no longer offset the required investment. With revenue being shared proportionally, a firm's marginal return will be more diluted when there are more competitors in the market. On the other hand, it takes a higher capacity for a diluted marginal return to match its un-diluted counterpart. This is the main reason behind the over-capacity phenomenon.

The literature on learning in monopolistic settings is extensive. For instance, Burnetas and Gilbert (2001) showed that demand learning can help reduce procurement costs; under various circumstances, Lariviere and Porteus (1999), Bensoussan, Cakanyildirim, and Sethi (2007), and Chen and Plambeck (2008) demonstrated that Bayesian learning can help firms

cope with unobserved lost sales. A few works in economics dealt with “learning by doing” in competitive settings. Rob (1991) treated a multi-period rational expectations model in which firms base their decisions of entering and exiting a market on past information generated by incumbent firms’ actions. Aghion, Espinosa, and Jullien (1993) studied a multi-period pricing game involving firms that produce differentiated products, whereby all firms receive the same information on the market demand which is influenced by past actions of all firms. These works emphasized the “public good” aspect of information and their equilibria often exhibit the “free-riding” phenomenon.

Our setup for information and learning can demonstrate the above informational externality features as well. Furthermore, it has improved over existing frameworks in the aspects that learning has been made explicit and separately controllable. That is, “learning” is no longer entangled with “doing.” This way, investments in market studies and experiments can be modeled directly, and tradeoffs between information-acquisition costs and gains due to better information can be more clearly analyzed; in addition, our framework can be more readily transplanted to different settings.

Shin and Tunca (2009) showed that over-learning can occur when retailers ordering from one single supplier are themselves engaged in Cournot competition. However, they assumed that there is no externality in learning, and hence every retailer is singularly responsible for its own learning. When retailers’ learning efforts (but not the signals they acquired) become public, the same authors demonstrated that the over-learning effect will be further amplified. One of our main results is almost complementary. It says that, when all firms receive the same signal produced by their collective efforts, firms will tend to shirk from their forecasting responsibilities. Also, we assume that the efforts put into learning are public knowledge throughout.

The rest of the chapter is organized as follows. In Section ??, we set up the capacity investment game and provide basic analyses; in Section ??, we introduce notions of information structure and controlled learning; in Section ??, we analyze the under-learning effect; in Section ??, we shed light on a potential extension to the case where different firms may acquire different signals; finally, we conclude the chapter in Section ??.

2 The Over-capacity Effect

2.1 Setup

Our investment game involves n identical firms. These firms compete in utilizing costly capital to build capacities with hopes of generating future returns. Each firm's cost for capital follows a function $c : \mathcal{R}^+ \rightarrow \mathcal{R}^+$, where \mathcal{R}^+ stands for $[0, +\infty)$. The return to an individual firm is not solely determined by its own investment level. Rather, the total return to all firms is governed by a function $r : \mathcal{R}^+ \rightarrow \mathcal{R}^+$ of these firms' total investment level. The return to each firm i is proportional to its investment level $x_i \in \mathcal{R}^+$. Therefore, when the profile of other firms' investment levels is $x_{-i} = (x_j \mid j \neq i)$, firm i will receive a profit $f(x_i, x_{-i})$, where

$$f(x_i, x_{-i}) = \frac{x_i}{x_i + \sum_{j \neq i} x_j} \cdot r(x_i + \sum_{j \neq i} x_j) - c(x_i). \quad (1)$$

We now give one potential explanation to the above setup. Suppose the demand function of the concerned product is given by $d = D(p)$, whose inverse is $p = P(d)$. Also, there is one production run after the capacity build-up. Finally, suppose that, relative to $c'(0)$, the unit production cost c_P is negligible. This way, firms will tend to produce at full capacity. The total supply on the market will be the total capacity level $\sum_{i=1}^n x_i$, which will lead to a market-clearing price $P(\sum_{i=1}^n x_i)$. The total revenue made by all firms will therefore be

$$r(\sum_{i=1}^n x_i) = (\sum_{i=1}^n x_i) \cdot P(\sum_{i=1}^n x_i), \quad (2)$$

while firm i 's share of the total revenue will be $(x_i / (x_i + \sum_{j \neq i} x_j)) \cdot r(x_i + \sum_{j \neq i} x_j)$. When there are an infinite number of production stages after the settlement of capacity levels, where the unit production cost at every stage is c_P and the per-stage discount factor is δ , we will need $c_P / (1 - \delta) \ll c'(0)$ for the above explanation to work.

Due to the apparent symmetry in (??), we may define function g , so that

$$g(x, y) = \frac{x}{x + y} \cdot r(x + y) - c(x). \quad (3)$$

Note that

$$f(x_i, x_{-i}) = g(x_i, \sum_{j \neq i} x_j). \quad (4)$$

We suppose that the capital cost function c is smooth. In addition, we assume the following:

(c0) $c(0) = 0$, as is expected;

(c1) $c'(0^+) \geq 1$, which reflects the lost opportunity of invested capital;

(c2) $c''(x) \geq 0$ for any $x \in (0, +\infty)$, so that the marginal cost of capital is increasing.

We further suppose that the return function r is smooth. In addition, we assume the following:

(r0) $r(0) = 0$, as is expected;

(r1) $r'(x) > 0$ for any $x \in (0, +\infty)$, so that more investment leads to a higher return;

(r2) $r''(x) < 0$ for any $x \in (0, +\infty)$, so that the marginal rate of return to investment decreases with the investment level;

(r3) $\lim_{x \rightarrow +\infty} r'(x) = 0$, so that return to investment will diminish to zero when capital is injected in indefinitely.

Between functions c and r , we assume that:

(cr) $r'(0^+) > c'(0^+)$, so that investment will be lucrative when the industry-wide capacity is low enough.

One immediate consequence of (r0) and (r2) is that $r'(x) < (r(x) - r(0))/(x - 0) = r(x)/x$.

We put this in the following:

(r02) $r'(x) < r(x)/x$ for any $x \in (0, +\infty)$.

A further consequence of the above is that $(r(x)/x)' = (r'(x) - r(x)/x)/x < 0$. Therefore, we have:

(r02b) $r(x)/x$ is decreasing in x .

2.2 Competitive Analysis

From (??), we have

$$\frac{\partial g(x, y)}{\partial x} = \frac{x}{x+y} \cdot r'(x+y) + \frac{y}{(x+y)^2} \cdot r(x+y) - c'(x), \quad (5)$$

and

$$\frac{\partial^2 g(x, y)}{\partial x^2} = \frac{x}{x+y} \cdot r''(x+y) + \frac{2y}{(x+y)^2} \cdot [r'(x+y) - \frac{r(x+y)}{x+y}] - c''(x). \quad (6)$$

From (r2), (r02), (c2), and (??), we know that $\partial^2 g(x, y)/\partial x^2 < 0$, and hence $g(x, y)$ is strictly concave in x .

We set out to see if a pure symmetric equilibrium ($x_i = x_n^* \mid i = 1, 2, \dots, n$) exists. Here, the subscript “ n ” in “ x_n^* ” signifies the presence of n firms. From the strict concavity of $g(x, y)$ in x and (??), we see that x_n^* can be found by solving the following:

$$h_n(x_n^*) = \frac{\partial g(x, y)}{\partial x} \Big|_{x=x_n^*, y=(n-1)x_n^*} = \frac{1}{n} \cdot r'(nx_n^*) + \frac{n-1}{n} \cdot \frac{r(nx_n^*)}{nx_n^*} - c'(x_n^*) = 0, \quad (7)$$

whenever the equality is achievable. The desired x_n^* is not only in existence, but also unique.

Proposition 1 *There is a unique pure symmetric equilibrium investment level x_n^* .*

We have relegated all proofs to appendices that follow the main text. For instance, the proof of the above proposition appears in Appendix A. Now we study the total investment level $z_n^* = nx_n^*$. To this end, define function i_n so that $i_n(z) = h_n(z/n)$. By (??), we have

$$\begin{aligned} i_n(z) &= (1/n) \cdot r'(z) + ((n-1)/n) \cdot r(z)/z - c'(z/n) \\ &= r(z)/z + (1/n) \cdot [r'(z) - r(z)/z] - c'(z/n). \end{aligned} \quad (8)$$

Being a re-scaled version of h_n , the function i_n is positive at 0^+ , decreasing in z , and negative at large z values. Just because x_n^* is the unique root of h_n , we know that z_n^* is the unique root of i_n . More importantly, we know that the market-wise investment level z_n^* increases with the number of participants n .

Proposition 2 *The equilibrium total investment level z_n^* is increasing in n .*

2.3 Comparison with the First-best Solution

For the total payoff of the n firms to be maximized, a social planner will solve the following problem:

$$\begin{aligned} \max \quad & r(z) - n \cdot c(z/n) \\ \text{s.t.} \quad & z \in \mathcal{R}^+. \end{aligned} \quad (9)$$

Therefore, the first-best total investment level z_{1n}^* will be a solution to

$$i_{1n}(z) = r'(z) - c'\left(\frac{z}{n}\right) = 0. \quad (10)$$

Here, the subscript “1” signifies optimality under one decision maker, and the subscript “ n ” still signifies the presence of n firms. By (r2) and (c2), we know that $i_{1n}(z)$ is strictly

decreasing in z ; by (cr), we know that $i_{1n}(0^+) > 0$; and, by (r3), (c1), and (c2), we know that $i_{1n}(z) < 0$ will occur when z is large enough. Therefore, z_{1n}^* is in existence and unique. Moreover, we can predict the trend for z_{1n}^* when the number of participants n changes.

Proposition 3 *The first-best total investment level z_{1n}^* is increasing in n , while the first-best individual investment level $x_{1n}^* = z_{1n}^*/n$ is decreasing in n .*

We can show the important result that competition brings in over-capacity.

Theorem 1 *Compared to the first-best total investment level z_{1n}^* , the equilibrium level z_n^* is greater.*

Our earlier assumptions and the above result essentially reflect that over-capacity is a consequence of the concavity of the investment return function and the proportionality of revenue allocation. In general, the difference function $(i_n - i_{1n})$ as defined in (??) indicates the degree of over-capacity. We may see that it increases with n , and saturates at the function $(r(z)/z - r'(z))$ as n tends to $+\infty$.

As a specific case, suppose $r(x) = x^\gamma$ for some $\gamma \in (0, 1)$ and $c(x) = x$. Then, from (??) and (??), we may find that $z_n^* = ((n-1)/n + \gamma/n)^{1/(1-\gamma)}$ and $z_{1n}^* = \gamma^{1/(1-\gamma)}$. Hence, a more direct measure of over-capacity, $z_n^*/z_{1n}^* = ((n-1)/(n\gamma) + 1/n)^{1/(1-\gamma)}$, grows with n fairly quickly. When $\gamma = 1/2$, it follows that $z_n^*/z_{1n}^* = (2n-1)^2/n^2$, which converges to 4 as $n \rightarrow +\infty$.

3 Information Structure and Learning

We introduce a framework that allows uncertainty in the size of the market faced by all firms. Through individual efforts, firms may exert control on the precision levels of the common signal received by all of them.

3.1 A Learning Framework

Instead of the previous r , let now the return function R_ω be parameterized by some $\omega \in \{L, H\}$, where $H > L > 0$. Before obtaining any information, all firms believe that ω is the

realization of the random variable Ω , which satisfies

$$P[\Omega = L] = P[\Omega = H] = \frac{1}{2}. \quad (11)$$

To describe the information structure to be used, we introduce random variable Θ , which serves as a common signal that reflects the commonality of firms' knowledge about Ω . More specifically, we let Θ be a bi-valued random variable ranging in $\{L, H\}$. There is a constant $a \in [0, 1]$ such that

$$\begin{cases} P[\Theta = L | \Omega = L] = (1 + a)/2 = P[\Theta = H | \Omega = H], \\ P[\Theta = L | \Omega = H] = (1 - a)/2 = P[\Theta = H | \Omega = L]. \end{cases} \quad (12)$$

From the above, we may derive that

$$P[\Omega = L] = P[\Omega = H] = P[\Theta = L] = P[\Theta = H] = \frac{1}{2}. \quad (13)$$

Our framework can be extended beyond the current bi-valued case. We opt for the current case for the ease of later derivations involving controlled information acquisition.

Now, we suppose that the information structure itself is subject to firms' controls. That is, we allow the random variable Θ to be dependent on firms' efforts. There are stages 0 and 1. In stage 0, firms may invest to learn the market; in stage 1, they may participate in the investment game described in Section ???. Before stage 0, all firms believe that the return function follows $R_\Omega(x)$. Given stage-0 firm-effort vector $x_0 = (x_{0i} | i = 1, 2, \dots, n)$, we suppose that Θ is in the form of $\tilde{\Theta}(x_0)$. Thus, $\tilde{\Theta}(x_0)$ reflects the learning effect. The true value of Ω will be revealed only after stage 1.

We suppose that the a used in (??) is replaced by some function $\tilde{a}(x_0)$. With effort-dependent substitutions, we can achieve counterparts of the earlier (??). For the function $\tilde{a}(\cdot)$, we suppose that there is a positive constant α , so that

$$\tilde{a}(x_0) = \frac{\alpha \sum_{j=1}^n x_{0j}}{1 + \alpha \sum_{j=1}^n x_{0j}}. \quad (14)$$

This function form reflects that more can be learned through the exertion of greater efforts, and that the marginal return in learning decreases with effort levels. We have simplified the matter by letting $\tilde{a}(x_0)$ depend on $\sum_{j=1}^n x_{0j}$ only. Due to this, we later write $\tilde{a}(\sum_{j=1}^n x_{0j})$ in the place of $\tilde{a}(x_{01}, \dots, x_{0n})$.

Also in the above, α indicates the effectiveness of learning. At the extreme of $\alpha = 0$, the intermediate signal $\tilde{\Theta}(x_0)$ will be useless noise regardless of the amount of effort spent; at the other extreme of $\alpha = +\infty$, $\tilde{\Theta}(x_0)$ will be Ω itself under the convention that $+\infty \cdot 0 = +\infty$.

3.2 Particulars of the Investment Game

Let us specify the particular form of the parameterized return function $R_\omega(\cdot)$:

$$R_\omega(x) = \omega \cdot r\left(\frac{x}{\omega}\right), \quad \forall x \in \mathcal{R}^+. \quad (15)$$

Note that

$$\frac{dR_\omega(y)}{dy} \Big|_{y=\omega x} = \omega \cdot \frac{1}{\omega} \cdot r'(z) \Big|_{z=y/\omega=\omega x/\omega=x} = r'(x). \quad (16)$$

That is, the marginal return of an ωx -level investment under the $R_\omega(\cdot)$ -return regime is the same as the marginal return of an x -level investment under the $r(\cdot)$ -return regime. Thus, ω in $R_\omega(\cdot)$ can be thought of as a market-size indicator. Now, we make the simplifying assumption that the total cost to firm i is $x_{0i} + x_{1i}$ when it has spent learning effort x_{0i} and capacity investment x_{1i} in the two stages. Now (cr) in Section ?? means that $r'(0^+) > 1$.

We may use $x_i = (x_{0i}, x_{1i}(L), x_{1i}(H))$ to describe firm i 's strategy. In it, x_{0i} is the firm's stage-0 learning effort, while $x_{1i}(\theta)$ is its stage-1 investment level when it has learned θ as the realization of $\tilde{\Theta}(x_0)$. Let us use $f(x_i, x_{-i})$ to describe the average payoff to firm i , when it adopts policy $x_i = (x_{0i}, x_{1i}(L), x_{1i}(H))$ while others have adopted policy profile $x_{-i} = ((x_{0j}, x_{1j}(L), x_{1j}(H)) \mid j \neq i)$. We have

$$f(x_i, x_{-i}) = \sum_{\theta \in \{L, H\}} (1/2) \cdot \sum_{\omega \in \{L, H\}} q(\omega \mid \theta; x_{0i}, x_{0,-i}) \times [(x_{1i}(\theta)/(x_{1i}(\theta) + \sum_{j \neq i} x_{1j}(\theta))) \cdot \omega \cdot r((x_{1i}(\theta) + \sum_{j \neq i} x_{1j}(\theta))/\omega) - x_{0i} - x_{1i}(\theta)], \quad (17)$$

where, according to the effort-dependent version of (??),

$$q(\omega \mid \theta; x_{0i}, x_{0,-i}) = \begin{cases} (1 + 2\alpha x_{0i} + 2\alpha \sum_{j \neq i} x_{0j}) / (2 \cdot (1 + \alpha x_{0i} + \alpha \sum_{j \neq i} x_{0j})), & \text{when } \theta = \omega, \\ 1 / (2 \cdot (1 + \alpha x_{0i} + \alpha \sum_{j \neq i} x_{0j})), & \text{when } \theta \neq \omega. \end{cases} \quad (18)$$

Inside (??), the $1/2$ is the chance for the common observation Θ to be either L or H , and $q(\omega \mid \theta; x_{0i}, x_{0,-i})$ is the conditional probability $\mathbf{P}[\Omega = \omega \mid \Theta = \theta]$ under the same

learning-effort vector. Note that firm i 's decision is only dependent on θ , whereas its payoff is dependent on the actual Ω -realization ω . As efforts are measured in costs, the cost term $(x_{0i} + x_{1i}(\theta))$, involving unit coefficients, in no way indicates that information acquisition and capacity expansion costs are comparable.

4 The Under-learning Effect

By (??), we can condense other firms' action profile x_{-i} into $y = (y_0, y_1(L), y_1(H))$ with $y = \sum_{j \neq i} x_j$, meaning, component-wise, that $y_0 = \sum_{j \neq i} x_{0j}$, $y_1(L) = \sum_{j \neq i} x_{1j}(L)$, and $y_1(H) = \sum_{j \neq i} x_{1j}(H)$.

4.1 Stage-1 Competitive Analysis

Define function $g(x, y)$, so that $g(x_i, \sum_{j \neq i} x_j) = f(x_i, x_{-i})$ as defined in (??). For ease of presentation, we shall use x_0, x_L, x_H to represent $x_0, x_1(L), x_1(H)$, respectively, and do the same for the y 's. Hence, x_0 stands for the current firm's stage-0 learning effort and y_0 stands for other firms' total stage-0 learning effort; for $\theta = L, H$, x_θ stands for the current firm's stage-1 investment level and y_θ stands for other firms' total stage-1 investment level.

Now (??) can be simplified into

$$\begin{aligned}
g(x, y) &= g(x_0, x_L, x_H, y_0, y_L, y_H) = -x_0 - x_L/2 - x_H/2 \\
&+ [(1 + 2\alpha x_0 + 2\alpha y_0)/(4 + 4\alpha x_0 + 4\alpha y_0)] \times \\
&\quad \times [(x_L/(x_L + y_L)) \cdot L \cdot r((x_L + y_L)/L) + (x_H/(x_H + y_H)) \cdot H \cdot r((x_H + y_H)/H)] \quad (19) \\
&+ [1/(4 + 4\alpha x_0 + 4\alpha y_0)] \times \\
&\quad \times [(x_L/(x_L + y_L)) \cdot H \cdot r((x_L + y_L)/H) + (x_H/(x_H + y_H)) \cdot L \cdot r((x_H + y_H)/L)].
\end{aligned}$$

From this, we can derive that

$$\frac{\partial g(x, y)}{\partial x_L} = g'_L(x_0 + y_0, x_L, y_L), \quad (20)$$

where

$$\begin{aligned}
g'_L(z_0, x_L, y_L) &= -1/2 + [(1 + 2\alpha z_0)/(4 + 4\alpha z_0)] \times \\
&\quad \times [(x_L/(x_L + y_L)) \cdot r'((x_L + y_L)/L) + (y_L/(x_L + y_L)^2) \cdot L \cdot r((x_L + y_L)/L)] \\
&\quad + [1/(4 + 4\alpha z_0)] \times \\
&\quad \times [(x_L/(x_L + y_L)) \cdot r'((x_L + y_L)/H) + (y_L/(x_L + y_L)^2) \cdot H \cdot r((x_L + y_L)/H)].
\end{aligned} \tag{21}$$

Symmetrically, we can find the expression for $\partial g(x, y)/\partial x_H = g'_H(x_0 + y_0, x_H, y_H)$. These lead to the following important property.

Lemma 1 *We have $\partial g'_L(z_0, x_L, y_L)/\partial x_L < 0$ and $\partial g'_H(z_0, x_H, y_H)/\partial x_H < 0$.*

We seek a pure symmetric equilibrium ($x_i = x_n^* \mid i = 1, 2, \dots, n$), where x_n^* is made up of three components: x_{n0}^* , x_{nL}^* , and x_{nH}^* . For the time being, we concentrate on firms' stage-1 subgame perfect equilibrium actions when all firms' stage-0 actions are known. Let $\tilde{x}_{nL}^*(z_0)$ be each firm's pure symmetric equilibrium stage-1 action when it is known that the total learning effort in stage 0 is z_0 and the firm itself has observed the L signal. We may similarly define $\tilde{x}_{nH}^*(z_0)$. Once the stage-0 equilibrium x_{n0}^* has been determined, we can let $x_{nL}^* = \tilde{x}_{nL}^*(nx_{n0}^*)$ and $x_{nH}^* = \tilde{x}_{nH}^*(nx_{n0}^*)$.

By Lemma ??, we know that, under a given total stage-0 learning effort z_0 , the pair $(\tilde{x}_{nL}^*(z_0), \tilde{x}_{nH}^*(z_0))$ can be found by solving for

$$g'_L(z_0, \tilde{x}_{nL}^*(z_0), (n-1)\tilde{x}_{nL}^*(z_0)) = 0, \quad g'_H(z_0, \tilde{x}_{nH}^*(z_0), (n-1)\tilde{x}_{nH}^*(z_0)) = 0, \tag{22}$$

whenever the above equalities are achievable. According to (??), the above requirement on $\tilde{x}_{nL}^*(z_0)$ is that it be a root for function $h_{nL}(z_0, \cdot)$ defined by

$$h_{nL}(z_0, x_L) = -\frac{1}{2} + \frac{1 + 2\alpha z_0}{4 + 4\alpha z_0} \cdot j_n\left(\frac{nx_L}{L}\right) + \frac{1}{4 + 4\alpha z_0} \cdot j_n\left(\frac{nx_L}{H}\right), \tag{23}$$

where

$$j_n(x) = \frac{1}{n} \cdot r'(x) + \frac{n-1}{n} \cdot \frac{r(x)}{x}. \tag{24}$$

By (r2) and (r02b), we know that $j_n(x)$ is strictly decreasing in x . Symmetrically, we may resort to function $h_{nH}(z_0, \cdot)$ to find $\tilde{x}_{nH}^*(z_0)$, where

$$h_{nH}(z_0, x_H) = -\frac{1}{2} + \frac{1 + 2\alpha z_0}{4 + 4\alpha z_0} \cdot j_n\left(\frac{nx_H}{H}\right) + \frac{1}{4 + 4\alpha z_0} \cdot j_n\left(\frac{nx_H}{L}\right). \tag{25}$$

We can establish the existence and uniqueness of a stage-1 subgame perfect equilibrium.

Proposition 4 *On $(0, +\infty)$, both $h_{nL}(z_0, \cdot)$ and $h_{nH}(z_0, \cdot)$ are strictly decreasing functions starting with strictly positive values and ending with strictly negative values. Consequently, when firms contribute a total learning effort z_0 , there will be a unique low-signal investment level $\tilde{x}_{nL}^*(z_0)$ and a unique high-signal investment level $\tilde{x}_{nH}^*(z_0)$ all will be willing to adopt.*

From the following, we see that when better informed, firms will place more trust in the common signal they receive, and more boldly place differentiated bets on the market.

Proposition 5 *First, we have $\tilde{x}_{nL}^*(0) = \tilde{x}_{nH}^*(0)$. Then, as z_0 increases, $\tilde{x}_{nL}^*(z_0)$ will decrease, while $\tilde{x}_{nH}^*(z_0)$ will increase.*

Using the same logic as above, we can show that increases in learning effectiveness α will result in the widening of the gap between $\tilde{x}_{nL}^*(z_0)$ and $\tilde{x}_{nH}^*(z_0)$.

Proposition 6 *First, we have $\tilde{x}_{nL}^*(z_0) = \tilde{x}_{nH}^*(z_0)$ when $\alpha = 0$. Then, as α increases, $\tilde{x}_{nL}^*(z_0)$ will decrease, while $\tilde{x}_{nH}^*(z_0)$ will increase.*

4.2 Stage-0 Competitive Analysis

We now come back to stage 0 to study its equilibrium decision. Define function $\tilde{g}_n(x_0, y_0)$, so that

$$\tilde{g}_n(x_0, y_0) = g(x_0, \tilde{x}_{nL}^*(x_0 + y_0), \tilde{x}_{nH}^*(x_0 + y_0), y_0, (n-1)\tilde{x}_{nL}^*(x_0 + y_0), (n-1)\tilde{x}_{nH}^*(x_0 + y_0)), \quad (26)$$

where g is given in (??). The newly defined function is the payoff to a firm which spends an x_0 effort in stage 0, when the other $n - 1$ firms spend a total of y_0 effort in this stage and all firms adopt their subgame perfect equilibrium responses in stage 1. By (??), we have

$$\begin{aligned} \tilde{g}_n(x_0, y_0) = & -x_0 - \tilde{x}_{nL}^*(x_0 + y_0)/2 - \tilde{x}_{nH}^*(x_0 + y_0)/2 \\ & + [(1 + 2\alpha x_0 + 2\alpha y_0)/(4 + 4\alpha x_0 + 4\alpha y_0)] \times [(L/n) \cdot r(n\tilde{x}_{nL}^*(x_0 + y_0)/L) \\ & + (H/n) \cdot r(n\tilde{x}_{nH}^*(x_0 + y_0)/H)] + [1/(4 + 4\alpha x_0 + 4\alpha y_0)] \times \\ & \times [(H/n) \cdot r(n\tilde{x}_{nL}^*(x_0 + y_0)/H) + (L/n) \cdot r(n\tilde{x}_{nH}^*(x_0 + y_0)/L)]. \end{aligned} \quad (27)$$

Since the above \tilde{g}_n is difficult to analyze, we shall start to make the simplifying assumption:

$$r(x) = \sqrt{x}. \quad (28)$$

Now, from (??), we will have

$$j_n(x) = \frac{2n-1}{2n\sqrt{x}}. \quad (29)$$

Hence, from (??) and (??), we have

$$\begin{cases} h_{nL}(z_0, x_L) &= -1/2 + ((2n-1)/(8n+8n\alpha z_0)) \times \\ &\quad \times [(1+2\alpha z_0) \cdot \sqrt{L/(nx_L)} + \sqrt{H/(nx_L)}], \\ h_{nH}(z_0, x_H) &= -1/2 + ((2n-1)/(8n+8n\alpha z_0)) \times \\ &\quad \times [(1+2\alpha z_0) \cdot \sqrt{H/(nx_H)} + \sqrt{L/(nx_H)}], \end{cases} \quad (30)$$

from which we get

$$\begin{cases} \tilde{x}_{nL}^*(z_0) &= (2n-1)^2 \times (\sqrt{L} + \sqrt{H} + 2\sqrt{L} \cdot \alpha z_0)^2 / (16n^3 \cdot (1 + \alpha z_0)^2), \\ \tilde{x}_{nH}^*(z_0) &= (2n-1)^2 \times (\sqrt{H} + \sqrt{L} + 2\sqrt{H} \cdot \alpha z_0)^2 / (16n^3 \cdot (1 + \alpha z_0)^2). \end{cases} \quad (31)$$

Plugging (??) and (??) into (??), we obtain a closed-form expression for \tilde{g}_n :

$$\begin{aligned} \tilde{g}_n(x_0, y_0) &= -x_0 + [(2n-1)/(16n^3 \cdot (1 + \alpha x_0 + \alpha y_0)^2)] \times \\ &\quad \times \{2\sqrt{HL} \cdot (1 + 2\alpha x_0 + 2\alpha y_0) + (L + H) \cdot [1 + 2\alpha(x_0 + y_0) \cdot (1 + \alpha x_0 + \alpha y_0)]\}. \end{aligned} \quad (32)$$

After some algebra, it can be found that

$$\frac{\partial \tilde{g}_n(x_0, y_0)}{\partial x_0} = Q(A_n, \alpha(x_0 + y_0)), \quad (33)$$

where

$$Q(a, w) = \frac{aw}{(1+w)^3} - 1, \quad (34)$$

and

$$A_n = \frac{(2n-1)\alpha(\sqrt{H} - \sqrt{L})^2}{8n^3}. \quad (35)$$

At various a levels, the function $Q(a, \cdot)$ possesses useful properties.

Lemma 2 *When $a \in [0, 27/4)$, we have $Q(a, w) < 0$ for $w \in \mathcal{R}^+$; when $a \in [27/4, +\infty)$, the function $Q(a, \cdot)$ has two positive real roots $w^0(a) \leq 1/2$ and $w^*(a) \geq 1/2$, such that the function is below 0 when $w \in [0, w^0(a))$, above 0 when $w \in [w^0(a), w^*(a)]$, and below 0 again when $w \in (w^*(a), +\infty)$.*

Suppose parameters α , L , and H are such that the A_n as defined by (??) is above $27/4$. Then, we may define $B_n = w^*(A_n)$, where $w^*(a)$ is the larger one of the two roots of function $Q(a, \cdot)$ as defined in Lemma ??. Also, define C_n so that

$$C_n = n\alpha \cdot \left[\tilde{g}_n\left(\frac{B_n}{n\alpha}, \frac{(n-1)B_n}{n\alpha}\right) - \tilde{g}_n\left(0, \frac{(n-1)B_n}{n\alpha}\right) \right], \quad (36)$$

which, after some algebra while using (??) and the fact that $Q(A_n, B_n) = Q(A_n, w^*(A_n)) = 0$, can be found to be the same as

$$C_n = \frac{(2n-2) \cdot B_n^3 + nB_n \cdot (B_n - 1)}{2 \cdot (nB_n - B_n + n)^2}. \quad (37)$$

We can express the largest stage-0 equilibrium x_{n0}^* in terms of the above constants.

Proposition 7 x_{n0}^* is always in existence; in addition, it is true that

$$x_{n0}^* = \begin{cases} 0, & \text{when } A_n < 27/4, \text{ or } A_n \geq 27/4 \text{ and } C_n < 0, \\ B_n/(n\alpha), & \text{when } A_n \geq 27/4 \text{ and } C_n \geq 0. \end{cases}$$

4.3 First-best Analysis

Suppose all firms adopt the same policy $x = (x_0, x_L, x_H)$. Then, according to (??), each of the firms will earn the following:

$$\begin{aligned} g_1(x_0, x_L, x_H) &= g(x_0, x_L, x_H, (n-1)x_0, (n-1)x_L, (n-1)x_H) = -x_0 - x_L/2 \\ &\quad - x_H/2 + [(1 + 2n\alpha x_0)/(4 + 4n\alpha x_0)] \times [(L/n) \cdot \sqrt{nx_L/L} + (H/n) \cdot \sqrt{nx_H/H}] \\ &\quad + [1/(4 + 4n\alpha x_0)] \times [(H/n) \cdot \sqrt{nx_L/H} + (L/n) \cdot \sqrt{nx_H/L}]. \end{aligned} \quad (38)$$

Hence, we have

$$\frac{\partial g_1(x_0, x_L, x_H)}{\partial x_L} = g'_{1L}(nx_0, x_L), \quad (39)$$

where

$$g'_{1L}(z_0, x_L) = -\frac{1}{2} + \frac{(1 + 2\alpha z_0) \cdot \sqrt{L} + \sqrt{H}}{(8 + 8\alpha z_0) \cdot \sqrt{nx_L}}. \quad (40)$$

It is easy to check that $g'_{1L}(z_0, \cdot)$ is strictly decreasing in x_L , $\lim_{x_L \rightarrow 0^+} g'_{1L}(z_0, x_0) > 0$, and $\lim_{x_L \rightarrow +\infty} g'_{1L}(z_0, x_L) < 0$. We can find similar properties for $\partial g_1(x_0, x_L, x_H)/\partial x_H = g'_{1H}(nx_0, x_H)$. Therefore, given total learning effort z_0 , the stage-1 first-best decisions $\tilde{x}_{1nL}^*(z_0)$

and $\tilde{x}_{1nH}^*(z_0)$ can be found by solving for $g'_{1L}(z_0, \tilde{x}_{1nL}^*(z_0)) = 0$ and $g'_{1H}(z_0, \tilde{x}_{1nH}^*(z_0)) = 0$, respectively. Hence, in view of (??), we have

$$\begin{cases} \tilde{x}_{1nL}^*(z_0) &= (\sqrt{L} + \sqrt{H} + 2\sqrt{L} \cdot \alpha z_0)^2 / (16n \cdot (1 + \alpha z_0)^2), \\ \tilde{x}_{1nH}^*(z_0) &= (\sqrt{H} + \sqrt{L} + 2\sqrt{H} \cdot \alpha z_0)^2 / (16n \cdot (1 + \alpha z_0)^2). \end{cases} \quad (41)$$

Comparing (??) and (??), we may see that over-capacity is present under the same level of learning. Also, the ratio of over-capacity approaches 4 from below when the number of firms tends to $+\infty$.

Theorem 2 *We have*

$$\frac{\tilde{x}_{nL}^*(z_0)}{\tilde{x}_{1nL}^*(z_0)} = \frac{\tilde{x}_{nH}^*(z_0)}{\tilde{x}_{1nH}^*(z_0)} = \frac{(2n-1)^2}{n^2}.$$

Plugging (??) into (??), we may obtain the payoff to an individual firm when every firm pitches in an x_0 learning effort and adopts the corresponding optimal stage-1 decision:

$$\begin{aligned} \tilde{g}_{1n}(x_0) &= g_1(x_0, \tilde{x}_{1nL}^*(nx_0), \tilde{x}_{1nH}^*(nx_0)) = -x_0 \\ &+ [2\sqrt{HL} \cdot (1 + 2n\alpha x_0) + (L + H) \cdot (1 + 2n\alpha x_0 + 2n^2\alpha^2 x_0^2)] / (16n \cdot (1 + \alpha nx_0)^2). \end{aligned} \quad (42)$$

Taking derivative, we find that

$$\frac{d\tilde{g}_{1n}(x_0)}{dx_0} = Q(A_1, n\alpha x_0), \quad (43)$$

where Q is defined in (??), whereas A_1 is defined in (??) but with $n = 1$. Suppose α , L , and H make $A_1 \geq 27/4$. Then, we may define $B_1 = w^*(A_1)$, as well as, C_1 so that

$$C_1 = n\alpha \cdot [\tilde{g}_{1n}(\frac{B_1}{n\alpha}) - \tilde{g}_{1n}(0)], \quad (44)$$

which by (??), is the same as C_1 being defined through (??) with $n = 1$. We can express the largest first-best individual-firm learning effort x_{1n0}^* in terms of these constants.

Proposition 8 *It is true that*

$$x_{1n0}^* = \begin{cases} 0, & \text{when } A_1 < 27/4, \text{ or } A_1 \geq 27/4 \text{ and } C_1 < 0, \\ B_1/(n\alpha), & \text{when } A_1 \geq 27/4 \text{ and } C_1 \geq 0. \end{cases}$$

4.4 Comparison between Sections ?? and ??

For $n = 2, 3, \dots$, let $z_{n0}^* = nx_{n0}^*$ be the total learning effort in equilibrium. Also, let $z_{10}^* = nx_{1n0}^*$ be the first-best total learning effort, which, according to Proposition ??, is independent of the number of firms n . Now we treat $n = 1, 2, \dots$ indiscriminately, with the understanding that $n = 1$ signifies the first-best case involving an arbitrary number of firms, while $n = 2, 3, \dots$ connotes the competitive case involving n firms.

From Propositions ?? and ??, it is clear that z_{n0}^* depends on L and H through the Ω -uncertainty indicator $\gamma = \sqrt{H} - \sqrt{L}$ only. We can further show that the total effort z_{n0}^* is decreasing in the number of decision makers n and will be encouraged by an increased level of return uncertainty γ ; in addition, the total learning effect αz_{n0}^* increases with the effectiveness of learning α .

Theorem 3 *For $n = 1, 2, \dots$, A_n , B_n , and C_n are all decreasing in n , increasing in γ , and increasing in α . Consequently, z_{n0}^* is decreasing in n and increasing in γ ; in addition, αz_{n0}^* is increasing in α .*

The above decrease of z_{n0}^* in n underscores the under-learning effect, that more intense competition leads to lower total investment in learning. With this, the individual learning effort x_{n0}^* will drop with n even faster. The above effect should be expected, as communal learning in some sense encourages “free riding.” To help better understand the trends for A_n and C_n , we draw Figures 1 and 2. In Figure 1, we draw, in the (α, γ) -plane, iso-value curves $A_n = 27/4$ at different n values and gradients $(\partial A_n / \partial \alpha, \partial A_n / \partial \gamma)$ at different points of the curves. In Figure 2, we repeat the same for C_n .

Let us consider the total stage-1 capacity investment levels. By (??) and (??), we can combine the competitive and first-best cases to obtain the following: for $n = 1, 2, \dots$,

$$\tilde{z}_{nL}^*(0) = \tilde{z}_{nH}^*(0) = \frac{(2n-1)^2 \cdot (\sqrt{L} + \sqrt{H})^2}{16n^2}, \quad (45)$$

and

$$\begin{cases} \tilde{z}_{nL}^*(B_n/\alpha) &= (2n-1)^2 \cdot (\sqrt{L} + \sqrt{H} + 2\sqrt{L} \cdot B_n)^2 / (16n^2 \cdot (1+B_n)^2), \\ \tilde{z}_{nH}^*(B_n/\alpha) &= (2n-1)^2 \cdot (\sqrt{H} + \sqrt{L} + 2\sqrt{H} \cdot B_n)^2 / (16n^2 \cdot (1+B_n)^2). \end{cases} \quad (46)$$

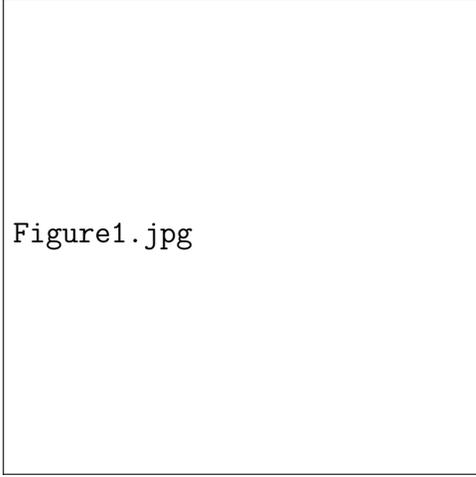


Figure 1: Iso-valued curves $A_n = 27/4$

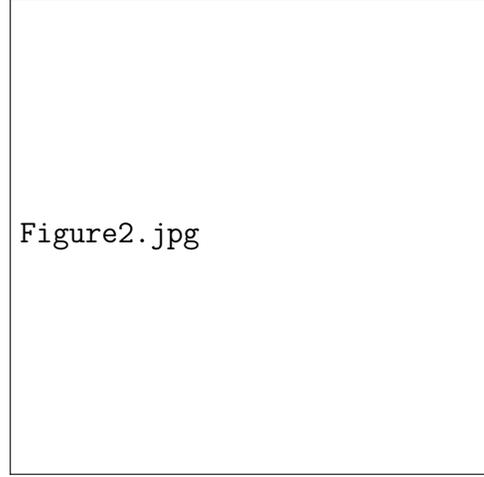


Figure 2: Iso-valued curves $C_n = 0$

We may define k_n^* for $n = 1, 2, \dots$ as a ratio:

$$k_n^* = \frac{\tilde{z}_{nH}^*(z_{n0}^*)}{\tilde{z}_{nL}^*(z_{n0}^*)}. \quad (47)$$

It measures the confidence a firm has in its acquired information about the market outlook.

By Propositions ?? and ??, (??), (??), and (??), we have, for $n = 1, 2, \dots$,

$$k_n^* = \begin{cases} 1, & \text{when } A_n < 27/4, \text{ or } A_n \geq 27/4 \text{ and } C_n < 0, \\ K(B_n), & \text{when } A_n \geq 27/4 \text{ and } C_n \geq 0, \end{cases} \quad (48)$$

where

$$K(b) = \frac{(\sqrt{L} + \sqrt{H} + 2\sqrt{H} \cdot b)^2}{(\sqrt{H} + \sqrt{L} + 2\sqrt{L} \cdot b)^2}. \quad (49)$$

In the competitive setting, a firm's confidence in its learning effect decreases with the number of firms n ; a firm's confidence in this setting is always below that in the first-best case; also, confidence in both settings increases with the stage-0 learning effectiveness α .

Theorem 4 *For $n = 1, 2, \dots$, k_n^* is decreasing in n . Also, every k_n^* is increasing in α .*

5 A Potential Extension

In an extended learning model, we may still use the Ω defined through (??) as firms' common belief of the size indicator before information acquisition. The relationship between each

realized ω and the return function may still be described by (??). However, this time we allow different firms to be able to acquire different information. On top of the random variable Θ , serving as firms' common observation through the relation (??), we introduce random variables $\Delta_1, \Delta_2, \dots$, and Δ_n for the n firms. We let each Δ_i be a bi-valued random variable ranging in $\{L, H\}$. When conditioned on $\Theta = \text{some } \theta$, the random variables $(\Delta_i | \theta)$ are independent of each other and $(\Omega | \theta)$. Also, there is a constant $b_i \in [0, 1]$ such that

$$\begin{cases} \mathbf{P}[\Delta_i = L | \Theta = L] = (1 + b_i)/2 = \mathbf{P}[\Delta_i = H | \Theta = H], \\ \mathbf{P}[\Delta_i = L | \Theta = H] = (1 - b_i)/2 = \mathbf{P}[\Delta_i = H | \Theta = L]. \end{cases} \quad (50)$$

Now, Θ serves as something intermediate between Ω and the Δ_i 's. Firm i 's final observation is the realization $\delta = L, H$ of the random variable Δ_i . The firm is to use this to make statistical inferences. By using Bayes' formula as well as other elementary probabilistic tools, it is easy to characterize the various random variables $(\Theta | \delta)$, $(\Delta_j | \delta)$ for $j \neq i$, and $(\Omega | \delta)$.

We have

$$\begin{cases} \mathbf{P}[\Theta = L | \Delta_i = L] = (1 + b_i)/2 = \mathbf{P}[\Theta = H | \Delta_i = H], \\ \mathbf{P}[\Theta = L | \Delta_i = H] = (1 - b_i)/2 = \mathbf{P}[\Theta = H | \Delta_i = L]; \end{cases} \quad (51)$$

for $j \neq i$,

$$\begin{cases} \mathbf{P}[\Delta_j = L | \Delta_i = L] = (1 + b_i b_j)/2 = \mathbf{P}[\Delta_j = H | \Delta_i = H], \\ \mathbf{P}[\Delta_j = L | \Delta_i = H] = (1 - b_i b_j)/2 = \mathbf{P}[\Delta_j = H | \Delta_i = L]; \end{cases} \quad (52)$$

and,

$$\begin{cases} \mathbf{P}[\Omega = L | \Delta_i = L] = (1 + ab_i)/2 = \mathbf{P}[\Omega = H | \Delta_i = H], \\ \mathbf{P}[\Omega = L | \Delta_i = H] = (1 - ab_i)/2 = \mathbf{P}[\Omega = H | \Delta_i = L]. \end{cases} \quad (53)$$

Like in Section ??, we may suppose that there are stages 0 and 1. In stage 0, firms may invest to learn the market; in stage 1, they may participate in the investment game described in Section ?. Before stage 0, all firms believe that the return function follows $R_\Omega(x)$. Given stage-0 firm-effort vector $x_0 = (x_{0i} | i = 1, 2, \dots, n)$, we suppose that Θ is in the form of $\tilde{\Theta}(x_0)$ and each Δ_i is in the form of $\tilde{\Delta}(x_{0i}, x_{0,-i})$. Thus, $\tilde{\Theta}(x_0)$ reflects the communal learning effect and the $\tilde{\Delta}(x_{0i}, x_{0,-i})$'s reflect firms' individual take-aways. The true value of Ω will be revealed only after stage 1.

We suppose that the a used in (??) is replaced by some function $\tilde{a}(x_0)$ and the b_i used in (??) is replaced by some function $\tilde{b}(x_{0i}, x_{0,-i})$. With effort-dependent substitutions, we

can achieve counterparts of the earlier (??), (??), and (??). For instance, for $\tilde{a}(\cdot)$, we may suppose that there is a positive constant α to satisfy (??); for function $\tilde{b}(\cdot)$, we may suppose the existence of some positive constant to satisfy

$$\tilde{b}(x_{0i}, x_{0,-i}) = \frac{\beta x_{0i}}{1 + \beta x_{0i}}, \quad \forall i = 1, 2, \dots, n. \quad (54)$$

These function forms reflect that more can be learned through the exertion of greater efforts, and that the marginal return in learning decreases with effort levels. We have simplified the matter by letting $\tilde{b}(x_{0i}, x_{0,-i})$ depend on x_{0i} only.

In the above, α again indicates the effectiveness of communal learning, while β indicates the strength of individual take-aways. At the extreme of $\beta = 0$, the final signal $\tilde{\Delta}(x_{0i}, x_{0,-i})$ will always be useless noise; at the other extreme of $\beta = +\infty$, $\tilde{\Delta}(x_{0i}, x_{0,-i})$ will be $\tilde{\Theta}(x_0)$ itself under the convention that $+\infty \cdot 0 = +\infty$. This last case is what we have just studied in Section ??.

We may use $x_i = (x_{0i}, x_{1i}(L), x_{1i}(H))$ to describe firm i 's strategy. In it, x_{0i} is the firm's stage-0 learning effort, while $x_{1i}(\delta)$ is its stage-1 investment level when it has learned δ as the realization of $\tilde{\Delta}(x_{0i}, x_{0,-i})$. Let us use $f(x_i, x_{-i})$ to describe the average payoff to firm i , when it adopts policy $x_i = (x_{0i}, x_{1i}(L), x_{1i}(H))$ while others have adopted policy profile $x_{-j} = ((x_{0j}, x_{1j}(L), x_{1j}(H)) \mid j \neq i)$. We have

$$\begin{aligned} f(x_i, x_{-i}) = & \sum_{\delta_i \in \{L, H\}} (1/2) \cdot \sum_{\delta_1 \in \{L, H\}} \cdots \sum_{\delta_{i-1} \in \{L, H\}} \cdot \sum_{\delta_{i+1} \in \{L, H\}} \cdots \sum_{\delta_n \in \{L, H\}} \\ & \{ \prod_{j \neq i} p(\delta_j \mid \delta_i; x_{0i}, x_{0j}) \times \sum_{\omega \in \{L, H\}} q(\omega \mid \delta_i; x_{0i}, x_{0,-i}) \times [(x_{1i}(\delta_i) / (x_{1i}(\delta_i) \\ & + \sum_{j \neq i} x_{1j}(\delta_j))) \cdot \omega \cdot r((x_{1i}(\delta_i) + \sum_{j \neq i} x_{1j}(\delta_j)) / \omega) - x_{0i} - x_{1i}(\delta_i)] \}, \end{aligned} \quad (55)$$

where, according to effort-dependent versions of (??) and (??),

$$p(\delta_j \mid \delta_i; x_{0i}, x_{0j}) = \begin{cases} (1 + \beta x_{0i} + \beta x_{0j} + 2\beta^2 x_{0i} x_{0j}) / (2 \cdot (1 + \beta x_{0i}) \cdot (1 + \beta x_{0j})), \\ \quad \text{when } \delta_i = \delta_j, \\ (1 + \beta x_{0i} + \beta x_{0j}) / (2 \cdot (1 + \beta x_{0i}) \cdot (1 + \beta x_{0j})), \\ \quad \text{when } \delta_i \neq \delta_j, \end{cases} \quad (56)$$

and according to effort-dependent versions of (??), (??), and (??),

$$q(\omega \mid \delta_i; x_{0i}, x_{0,-i}) = \begin{cases} (1 + (\alpha + \beta)x_{0i} + \alpha \sum_{j \neq i} x_{0j} + 2\alpha\beta x_{0i} \cdot (x_{0i} + \sum_{j \neq i} x_{0j})) / \\ \quad / (2 \cdot (1 + \alpha x_{0i} + \alpha \sum_{j \neq i} x_{0j}) \cdot (1 + \beta x_{0i})), & \text{when } \delta_i = \omega, \\ (1 + (\alpha + \beta)x_{0i} + \alpha \sum_{j \neq i} x_{0j}) / \\ \quad / (2 \cdot (1 + \alpha x_{0i} + \alpha \sum_{j \neq i} x_{0j}) \cdot (1 + \beta x_{0i})), & \text{when } \delta_i \neq \omega. \end{cases} \quad (57)$$

Inside (??), the $1/2$ is the chance for firm i 's observation Δ_i to be either L or H , $p(\delta_j \mid \delta_i; x_{0i}, x_{0j})$ is the conditional probability $\mathbf{P}[\Delta_j = \delta_j \mid \Delta_i = \delta_i]$ when firms' learning-effort vector is $x_0 = (x_{0i} \mid i = 1, 2, \dots, n)$, and $q(\omega \mid \delta_i; x_{0i}, x_{0,-i})$ is the conditional probability $\mathbf{P}[\Omega = \omega \mid \Delta_i = \delta_i]$ under the same learning-effort vector. Note that firm i 's decision is only dependent on δ_i , whereas its payoff is dependent on the actual Ω -realization ω .

When $\beta < +\infty$, different firms receive different signals, and (??) is much more difficult to analyze than its counterpart (??) at $\beta = +\infty$. Outcomes will diverge depending on whether or not firms share these signals. It turns out that the concerned problem can become entangled even when we assume that there are $n = 2$ firms and that the return function $r(x) = \sqrt{x}$. Our preliminary analysis is numerical in nature, from which we may draw some conclusion on conditions that favor information sharing among firms. But more extensive analysis is still needed.

6 Concluding Remarks

We formulated a capacity investment game in which identical firms contest for market shares and watch out for their investment returns at the same time. Over-capacity appears as a natural outcome of this game. By introducing uncertainty to the market size and adding a stage 0 of competitive information gathering to the game, we enabled the investigation of information and learning in a competitive setting. Our theoretical analysis confirmed the severe under-learning effect when incentives exist for "free riding".

Extending on the learning framework in Section ??, we may further model, as we did in Section ??, the case where different firms receive different signals. Outcomes will diverge depending on whether or not firms share these signals. We may then study whether or not information sharing brings in additional benefits. So far we have found that analysis would

become numerically entangled even for two firms. But we hope that, in future research, some minor adjustments to the current framework could pave the way for more major insights.

Appendices

A. Proof of Proposition ??: It all hinges on h_n . Note that

$$h'_n(x) = r''(nx) + \frac{n-1}{nx} \cdot [r'(nx) - \frac{r(nx)}{nx}] - c''(x), \quad (58)$$

which, by (r2), (r02), and (c2), is strictly negative. By l'Hôpital's rule,

$$h_n(0^+) = r'(0^+) - c'(0^+), \quad (59)$$

which is strictly positive by (cr). Also, by (r3), (c1), and (c2), we know that $h_n(x) < 0$ will occur when x is large enough. Therefore, x_n^* is in existence and unique. ■

B. Proof of Proposition ??: By (r02), (c2), and (??), we know that $i_n(z)$ is increasing in n at any fixed z . Thus, z_n^* is increasing in n , as it is the unique root of the decreasing function $i_n(z)$. ■

C. Proof of Proposition ??: By (c2) and (??), we know that $h_{1n}(z)$ is increasing in n at any fixed z . Thus, z_{1n}^* is increasing in n , as it the unique root of the decreasing function $h_{1n}(z)$. On the other hand, $x_{1n}^* = z_{1n}^*/n$ is the unique root for the decreasing function $r'(nx) - c'(x)$. By (r2), we know that the function is decreasing in n . Thus, we know that x_{1n}^* is decreasing in n . ■

D. Proof of Theorem ??: We clearly have $z_1^* = z_{11}^*$. Let us focus on the case where $n \geq 2$. Comparing (??) with (??), we have

$$i_n(z) - i_{1n}(z) = \frac{n-1}{n} \cdot [\frac{r(z)}{z} - r'(z)], \quad (60)$$

which is strictly positive by (r02). With z_n^* and z_{1n}^* being roots, respectively, of the functions i_n and i_{1n} , we therefore have $z_n^* > z_{1n}^*$. ■

E. Proof of Lemma ??: By (??), we have

$$\begin{aligned}
\partial g'_L(z_0, x_L, y_L)/\partial x_L &= [(1 + 2\alpha z_0)/(4 + 4\alpha z_0)] \times \\
&\times [(x_L/(L \cdot (x_L + y_L))) \cdot r''((x_L + y_L)/L) + (2y_L/(x_L + y_L)^2) \cdot r'((x_L + y_L)/L) \\
&\quad - (2y_L/(x_L + y_L)^3) \cdot L \cdot r((x_L + y_L)/L)] \\
&+ [1/(4 + 4\alpha z_0)] \times [(x_L/(H \cdot (x_L + y_L))) \cdot r''((x_L + y_L)/H) \\
&\quad + (2y_L/(x_L + y_L)^2) \cdot r'((x_L + y_L)/H) - (2y_L/(x_L + y_L)^3) \cdot H \cdot r((x_L + y_L)/H)].
\end{aligned} \tag{61}$$

That is, we have

$$\frac{\partial g'_L(z_0, x_L, y_L)}{\partial x_L} = S_1 \cdot (Q_1 + S_2 T_1) + S_3 \cdot (Q_2 + S_2 T_2), \tag{62}$$

where

$$\begin{cases} Q_1 &= (x_L/(L \cdot (x_L + y_L))) \times r''((x_L + y_L)/L), \\ Q_2 &= (x_L/(H \cdot (x_L + y_L))) \times r''((x_L + y_L)/H), \end{cases} \tag{63}$$

$$\begin{cases} S_1 = (1 + 2\alpha z_0)/(4 + 4\alpha z_0), & S_2 = 2y_L/(x_L + y_L)^2, \\ S_3 = 1/(4 + 4\alpha z_0), \end{cases} \tag{64}$$

and

$$\begin{cases} T_1 &= r'((x_L + y_L)/L) - r((x_L + y_L)/L)/((x_L + y_L)/L), \\ T_2 &= r'((x_L + y_L)/H) - r((x_L + y_L)/H)/((x_L + y_L)/H). \end{cases} \tag{65}$$

By (r2), we have $Q_1 < 0$ and $Q_2 < 0$; S_1 , S_2 , and S_3 are all apparently strictly positive; also, by (r02), we have $T_1 < 0$ and $T_2 < 0$. Therefore, we have $\partial g'_L(z_0, x_L, y_L)/\partial x_L < 0$. Symmetrically, we can show that $\partial g'_H(z_0, x_H, y_H)/\partial x_H < 0$. ■

F. Proof of Proposition ??: Since $j_n(x)$ is strictly decreasing in x , we know from (??) that $h_{nL}(z_0, x_L)$ is strictly decreasing in x_L . By l'Hôpital's rule, we know that $j_n(0^+) = r'(0^+)$, and hence

$$h_{nL}(z_0, 0^+) = \frac{r'(0^+) - 1}{2}, \tag{66}$$

which is strictly positive by (cr). Also, by (r3), we know that $h_{nL}(z_0, x_L) < 0$ will occur when x_L is large enough. Therefore, the root $\tilde{x}_{nL}^*(z_0)$ is in existence and unique. The result on $\tilde{x}_{nH}^*(z_0)$ can be achieved symmetrically. ■

G. Proof of Proposition ??: From (??) and (??), we see that

$$h_{nL}(0, x) = h_{nH}(0, x) = -\frac{1}{2} + \frac{1}{4} \cdot [j_n(\frac{nx}{L}) + j_n(\frac{nx}{H})]. \quad (67)$$

Therefore, the unique roots $x_{nL}^*(0)$ and $x_{nH}^*(0)$ of the two identical functions above should be equal to each other. Rewriting (??) and (??), we note that

$$h_{nL}(z_0, x_L) = -\frac{1}{2} + \frac{1}{2} \cdot j_n(\frac{nx_L}{L}) + \frac{1}{4 + 4\alpha z_0} \cdot [j_n(\frac{nx_L}{H}) - j_n(\frac{nx_L}{L})], \quad (68)$$

and

$$h_{nH}(z_0, x_H) = -\frac{1}{2} + \frac{1}{2} \cdot j_n(\frac{nx_H}{H}) + \frac{1}{4 + 4\alpha z_0} \cdot [j_n(\frac{nx_H}{L}) - j_n(\frac{nx_H}{H})]. \quad (69)$$

Since $j_n(x)$ is strictly decreasing in x and $H > L > 0$, we know that $j_n(nx_L/H) > j_n(nx_L/L)$ and $j_n(nx_H/L) < j_n(nx_H/H)$. Hence, $h_{nL}(z_0, x_L)$ is decreasing in z_0 and $h_{nH}(z_0, x_H)$ is increasing in z_0 . Being roots of decreasing functions $h_{nL}(z_0, \cdot)$ and $h_{nH}(z_0, \cdot)$, we therefore know that $\tilde{x}_{nL}^*(z_0)$ is decreasing in z_0 and that $\tilde{x}_{nH}^*(z_0)$ is increasing in z_0 . ■

H. Proof of Lemma ??: First, we always have $Q(a, 0) = -1$. Now we analyze $Q(a, \cdot)$ for $a \geq 0$. Note that

$$\frac{\partial Q(a, w)}{\partial w} = \frac{a \cdot (1 - 2w)}{(1 + w)^4}, \quad (70)$$

and

$$\frac{\partial^2 Q(a, w)}{\partial w^2} = \frac{6a \cdot (w - 1)}{(1 + w)^5}. \quad (71)$$

It is easy to see that $Q(a, \cdot)$ has a local maximum at $1/2$. We have

$$Q(a, \frac{1}{2}) = \frac{4a}{27} - 1. \quad (72)$$

Therefore, when $a \in [0, 27/4)$, the function $Q(a, \cdot)$ is never above 0 for $w \in \mathcal{R}^+$; when $a \in [27/4, +\infty)$, however, $Q(a, \cdot)$ is above 0 in an interval containing $1/2$, whose left- and right-end points are the two desired roots $w^0(a)$ and $w^*(a)$. ■

I. Proof of Proposition ??: We know the following from Lemma ??: When $A_n \in [0, 27/4)$, the function $Q(A_n, \cdot)$ is never above 0 for $w \in \mathcal{R}^+$, and hence $\tilde{g}_n(x_0, y_0)$ is never increasing in x_0 ; when $A_n \in [27/4, +\infty)$, however, $Q(A_n, \cdot)$ is above 0 in the interval $[w^0(A_n), w^*(A_n)]$ containing $1/2$.

Now, we know that $\tilde{g}_n(x_0, y_0)$ is decreasing in x_0 when $\alpha(x_0 + y_0) \in [0, w^0(A_n))$, increasing in x_0 when $\alpha(x_0 + y_0) \in [w^0(A_n), w^*(A_n))$, and decreasing in x_0 again when $\alpha(x_0 + y_0) \in [w^*(A_n), +\infty)$. When $A_n \geq 27/4$, we may define D_n so that

$$D_n = \alpha \cdot [\tilde{g}_n(\frac{B_n}{\alpha}, 0) - \tilde{g}_n(0, 0)]. \quad (73)$$

We can show that $(D_n \geq 0) \implies (C_n \geq 0)$, and hence $(C_n < 0) \implies (D_n < 0)$. By (??) and (??), we have

$$D_n = \frac{A_n B_n^2}{2 \cdot (1 + B_n)^2} - B_n, \quad (74)$$

which, since $Q(A_n, B_n) = Q(A_n, w^*(A_n)) = 0$, leads to

$$D_n = \frac{B_n \cdot (B_n - 1)}{2}. \quad (75)$$

By (??) and the fact that $B_n \geq 1/2$, we have $(D_n \geq 0) \implies (B_n \geq 1)$. But by (??), $B_n \geq 1$ leads to $C_n \geq 0$.

When $A_n < 27/4$, as $\tilde{g}_n(\cdot, 0)$ is decreasing, $\tilde{g}_n(0, 0)$ is greater than $\tilde{g}_n(x_0, 0)$ for any $x_0 \in \mathcal{R}^+$. Also, $\tilde{g}_n(x_0, (n-1)x_0)$ is smaller than $\tilde{g}_n(0, (n-1)x_0)$ for any $x_0 \in (0, +\infty)$, since $\tilde{g}_n(\cdot, (n-1)x_0)$ is decreasing too. Hence, we have $x_{n0}^* = 0$.

When $A_n \geq 27/4$ and C_n as defined in (??) is positive, it follows that $\tilde{g}_n(B_n/(n\alpha), (n-1)B_n/(n\alpha))$ is greater than $\tilde{g}_n(0, (n-1)B_n/(n\alpha))$, the only other local maximum of $\tilde{g}_n(\cdot, (n-1)B_n/(n\alpha))$; thus, $\tilde{g}_n(B_n/(n\alpha), (n-1)B_n/(n\alpha))$ is greater than $\tilde{g}_n(x_0, (n-1)B_n/(n\alpha))$ for any $x_0 \in \mathcal{R}^+$. Also, $\tilde{g}_n(x_0, (n-1)x_0)$ is smaller than $\tilde{g}_n(B_n/(n\alpha), (n-1)x_0)$ for any $x_0 \in (B_n/(n\alpha), +\infty)$, since $\tilde{g}_n(\cdot, (n-1)x_0)$ is decreasing when the argument is above $B_n/(n\alpha)$. Hence, we have $x_{n0}^* = B_n/(n\alpha)$.

When $A_n \geq 27/4$ but $C_n < 0$, we have $D_n < 0$ according to the above. Hence, it follows that $\tilde{g}_n(0, 0)$ is greater than $\tilde{g}_n(B_n/\alpha, 0)$, the only other local maximum of $\tilde{g}_n(\cdot, 0)$; thus, $\tilde{g}_n(0, 0)$ is greater than $\tilde{g}_n(x_0, 0)$ for any $x_0 \in \mathcal{R}^+$. Also, $\tilde{g}_n(x_0, (n-1)x_0)$ is smaller than $\tilde{g}_n((B_n/\alpha - (n-1)x_0) \vee 0, (n-1)x_0)$ for any $x_0 \in (0, +\infty)$. Hence, we have $x_{n0}^* = 0$. ■

J. Proof of Proposition ??: We know the following from Lemma ??: When $A_1 \in [0, 27/4)$, the function $Q(A_1, \cdot)$ is never above 0 for $w \in \mathcal{R}^+$, and hence $\tilde{g}_{1n}(x_0)$ is never increasing in x_0 ; when $A_1 \in [27/4, +\infty)$, however, $Q(A_1, \cdot)$ is above 0 in the interval $[w^0(A_1), w^*(A_1)]$

containing $1/2$. In the latter case, $\tilde{g}_{1n}(\cdot)$ has two local maximums, 0 and $B_1/(n\alpha)$. Whether $x_{1n0}^* = 0$ or $x_{1n0}^* = B_1/(n\alpha)$ depends solely on whether or not $\tilde{g}_{1n}(0) - \tilde{g}_{1n}(B_1/(n\alpha)) > 0$, which, according to (??), is the same as $C_1 < 0$. \blacksquare

K. Proof of Theorem ??: From (??), it is clear that A_n is decreasing in n , and increasing in γ and α . For $a \in (27/4, +\infty)$, we may, take derivative of a on the equation $Q(a, w^*(a)) = 0$ while in consultation with (??), to obtain

$$\frac{dw^*(a)}{da} = \frac{w^*(a)}{3(1+w^*(a))^2 - a} = \frac{(w^*(a))^2}{2(w^*(a))^3 + 3(w^*(a))^2 - 1}, \quad (76)$$

which is positive since $w^*(a) \geq 1/2$. Hence, just because A_n is so, $B_n = w^*(A_n)$ is decreasing in n , and increasing in γ and α .

By (??), we have

$$C_n = G(n, B_n), \quad (77)$$

where

$$G(k, b) = \frac{(2k-2) \cdot b^3 + kb \cdot (b-1)}{2 \cdot (kb - b + k)^2}. \quad (78)$$

Taking derivatives, we have

$$\frac{\partial G(k, b)}{\partial k} = \frac{b^2 \cdot [1 - (k-1) \cdot (2b^2 + 3b)] + kb}{D(k, b)}, \quad (79)$$

and

$$\frac{\partial G(k, b)}{\partial b} = \frac{N(k, b)}{D(k, b)}, \quad (80)$$

where

$$N(k, b) = 2(k-1)^2 \cdot b^3 + 6k(k-1) \cdot b^2 + k(3k-1) \cdot b - k^2, \quad (81)$$

and

$$D(k, b) = 2 \cdot (kb - b + k)^3. \quad (82)$$

From (??), we have, when treating A_n as a function of n , γ , and α ,

$$\frac{\partial A_n}{\partial n} = -\frac{(4n-3)\alpha\gamma^2}{8n^4} = -\frac{(4n-3) \cdot A_n}{2n^2 - n}. \quad (83)$$

In view of $Q(A_n, B_n) = 0$, this leads to

$$\frac{\partial A_n}{\partial n} = -\frac{(4n-3) \cdot (1+B_n)^3}{(2n^2 - n) \cdot B_n}. \quad (84)$$

Combining the above while treating C_n as a function of n , γ , and α , we obtain

$$\begin{aligned} & \partial C_n / \partial n \\ &= \partial G(k, b) / \partial k \big|_{k=n, b=B_n} + \partial G(k, b) / \partial b \big|_{k=n, b=B_n} \times dw^*(a) / da \big|_{w^*(a)=B_n} \times \partial A_n / \partial n \quad (85) \\ &= -(n-1) \cdot B_n \cdot (1+B_n) \cdot J(n, B_n) / [(2n^2-n) \cdot (nB_n - B_n + n)^3 \cdot (2B_n - 1)], \end{aligned}$$

where

$$J(k, b) = 3b^3 - b(1 + 9b(1 + b)) \cdot k + (1 + b)(8b^2 + 4b - 1) \cdot k^2. \quad (86)$$

Note that $J(\cdot, b)$ is a quadratic function with the minimum achieved at $k^0(b) = 1 - (7b^3 + 15b^2 + 5b - 2) / (2 \cdot (8b^3 + 12b^2 + 3b - 1))$, which is below 1 for $b \geq 1/2$. Thus, for $k \geq 1$ and $b \geq 1/2$, we have $J(k, b) \geq J(1, b) = 2b^3 + 3b^2 + 2b - 1 \geq 0$. Thus, we know that $\partial C_n / \partial n \leq 0$, and hence C_n is decreasing in n .

For $k \geq 1$ and $b \geq 1/2$, $D(k, b)$ as defined in (??) is clearly positive. Also, from (??), we have

$$N(k, b) \geq \frac{1}{4} \cdot (k-1)^2 + \frac{3}{2} \cdot k(k-1) + \frac{1}{2} \cdot k(3k-1) - k^2 = \frac{(9k-1)(k-1)}{4} \geq 0. \quad (87)$$

By (??), we may achieve from this the positivity of $\partial G(k, b) / \partial b$. Hence, just because B_n is so, C_n is increasing in γ and α .

The remaining results are simple consequences of the above trends of A_n , B_n , and C_n , as well as Propositions ?? and ??.

We now know that the strictly positive z_{n0}^* -value of B_n/α is decreasing in n . Suppose n has been increased. Then, it will be less likely for $A_n \geq 27/4$ and $C_n \geq 0$ to occur, and hence less likely for z_{n0}^* to assume the strictly positive value. Therefore, z_{n0}^* is decreasing in n .

We also know that the strictly positive z_{n0}^* -value of B_n/α is increasing in γ . Suppose γ has been increased. Then, it will be more likely for $A_n \geq 27/4$ and $C_n \geq 0$ to occur, and hence more likely for z_{n0}^* to assume the strictly positive value. Therefore, z_{n0}^* is increasing in γ .

Moreover, we know that the strictly positive αz_{n0}^* -value of B_n is increasing in α . Suppose α has been increased. Then, it will be more likely for $A_n \geq 27/4$ and $C_n \geq 0$ to occur, and hence more likely for αz_{n0}^* to assume the strictly positive value. Therefore, αz_{n0}^* is increasing

in α . ■

L. Proof of Theorem ??: From Theorem ??, we know that A_n and C_n are all decreasing in n . Hence, from (??), we have

$$k_n^*/k_{n+1}^* = \begin{cases} 1, & \text{when } A_n < 27/4, \text{ or } A_n \geq 27/4 \text{ and } C_n < 0, \\ K(B_n)/K(B_{n+1}), & \text{when } A_{n+1} \geq 27/4 \text{ and } C_{n+1} \geq 0, \\ K(B_n), & \text{in all other cases.} \end{cases} \quad (88)$$

From (??), it is easy to check that, as b increases from $1/2$ to $+\infty$, $K(b)$ increases from $(2\sqrt{H} + \sqrt{L})^2/(\sqrt{H} + 2\sqrt{L})^2$ to H/L . By Theorem ?? again, we know that B_n is decreasing in n . Combining these together, we will have $k_n^*/k_{n+1}^* \geq 1$ in all situations.

The increase of k_n^* in α is a simple consequence of (??), the above fact about the $K(b)$ function, and the increase of A_n , B_n , and C_n in α as stipulated in Theorem ??. ■

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