Hierarchical Spatial Gossip for Multi-Resolution Representations in Sensor Networks

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In this paper we propose a lightweight algorithm for constructing multi-resolution data representations for sensor networks. At each sensor node u, we compute, $O(\log n)$ aggregates about exponentially enlarging neighborhoods centered at u. The ith aggregate is the aggregated data from nodes approximately within 2^i hops of u. We present a scheme, named the hierarchical spatial gossip algorithm, to extract and construct these aggregates, for *all* sensors simultaneously, with a total communication cost of $O(n \operatorname{polylog} n)$. The hierarchical gossip algorithm adopts atomic communication steps with each node choosing to exchange information with a node distance d away with probability $\sim 1/d^3$. The attractiveness of the algorithm attributes to its simplicity, low communication cost, distributed nature and robustness to node failures and link failures. We show in addition that computing multi-resolution aggregates precisely (i.e., each aggregate uses all and only the nodes within 2^i hops) requires a communication cost of $\Omega(n\sqrt{n})$, which does not scale well with network size. An approximate range in aggregate computation like that introduced by the gossip mechanism is therefore necessary in a scalable efficient algorithm. Besides the natural applications of multi-resolution data summaries in data validation and information mining, we also demonstrate the application of the pre-computed multi-resolution data summaries in answering range queries efficiently.

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1. INTRODUCTION

Distributed wireless sensor networks provide revolutionary ways to attain large scale, dense data collection and long-term environment monitoring. The immediate challenge is to develop efficient methods to extract, encode, and distribute information gathered by sensors, for improving the robustness and survivability of data, as well as increasing the flexibility and efficiency to answer user queries. In this paper we study the problem of constructing multi-resolution data representation in a sensor network. Our approach follows the principle of *fractional cascading* that states: "a sensor knows a fraction of the information from distant parts of the network, in an exponentially decaying fashion by distance" [Gao et al. 2004]. This multi-resolution, locality-preserving

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representation is motivated by observations that sensors are typically monitoring a physical phenomena, which exhibits high correlation in both the spatial and temporal domain. Naturally, information relevant to each node is decaying with the distance to this node.

In the basic setup of this paper we have n sensors deployed uniformly and densely inside a region monitoring a continuous data field. We compute, at each sensor node u, $O(\log n)$ aggregates about exponentially enlarging neighborhoods centered at u. The ith aggregate is the aggregated data among nodes approximately within 2^i hops of u. The specifics of aggregation techniques will be application dependent. For example, the aggregates can be the MAX/MIN or AVG, or more involved aggregates such as histogram [Shrivastava et al. 2004], parameter estimations [Xiao et al. 2005], or random linear projections used for compressed sensing and information recovery [Rabbat et al. 2006]. This multi-resolution scheme is inherently load-balanced. The storage requirement at each node is bounded by $O(\log n)$. We present a scheme to extract and construct these aggregates, for *all* sensors simultaneously, by a hierarchical spatial gossip algorithm. The total communication cost, measured by the total number of transmissions, is $O(n \operatorname{polylog} n)$, only a small polylogarithmic factor of the cost for flooding or information aggregation at a sink, yet we obtain multi-resolution aggregation for each and every sensor node in the network.

The multi-resolution data summaries provide a basis for information mining, data validation and efficient range queries. One of the major challenges in a sensor network is that nodes start with no idea of the big picture over the data field. Thus it is difficult for a node to assess whether its sensor reading is valid or not since detection of outlier or abnormality usually requires comparison with other sensor readings. In certain applications, the sensor field is deployed to detect events of interest to the owner. A sensor node often needs to decide, by itself, whether it holds interesting data or not. In some cases it is trivial, e.g., an unusually high reading by an acoustic sensor typically means activities in its vicinity. Sometimes this requires comparison with the average of sensor readings in an appropriate neighborhood. For example, the temperature threshold considered as 'high' in shaded area is different from that in open area. With the summarized data from each of its exponentially enlarging neighborhoods, a node has a basis against which its own reading can be compared, in order to spot local spikes which indicate data significance [Wang and Ramchandran 2006]. In addition, these partial aggregates can be used to support range queries injected from any node in the network. Queries for the aggregated value inside a geographical region can be answered by combining the pre-computed partial aggregates, without the necessity of examining each and every node in the geographical range. Thus both communication cost and query delay can be improved.

The major contribution of this paper are the development of a light-weight algorithm for constructing multi-resolution data representations for sensor networks, and the application of multi-resolution data for range queries. In the remaining of this section we will survey related work on data processing in a sensor network and gossip-based algorithms. We will then give a quick overview of our solution for constructing and using multi-resolution data representation.

1.1. Information Processing in Sensor Networks

Existing approaches for processing information in sensor networks can be classified into two main approaches: the standard sink model and distributed indexing and storage. In the standard sink model, data is delivered to the sink for out-of-network processing. Queries are disseminated from the sink to sensor nodes who will then report their readings. Data pruning and aggregation can be undertaken when data propagates up the tree to the sink (e.g., in TinyDB) [Madden et al. 2002]. The sink model assumes little or no in-network processing and most of the algorithmic challenges are handled outside the network.

The second approach uses in-network storage, builds distributed indices and stores partial aggregates to facilitate user queries. Examples of this category include DIMENSIONS [Ganesan et al. 2003; Ganesan et al. 2003; Ganesan et al. 2005], DIFS [Greenstein et al. 2003], DIM [Li et al. 2003], and fractional cascading [Gao et al. 2004]. As storage devices such as flash drives become cheaper and smaller, the approach of using collective distributed storage becomes increasingly feasible. A distributed indexing structure typically involves a hierarchy to bring together data across different attribute space or spatial separations (e.g., quad-tree or kd-tree). Partial aggregates are computed bottom up for each node in the hierarchy. Queries take a drill-down approach and traverse the hierarchy to visit nodes holding relevant data for detailed information. Important considerations for distributed indexing and storage include how the partial aggregates are computed and who holds the aggregated data/indices. A straight-forward way is to take a hashing scheme and make certain nodes be responsible to hold aggregated data/indices on the hierarchy (e.g., in DIMENSIONS and DIFS). Special care is typically taken for nodes holding data at high levels of the tree to alleviate communication and query bottleneck [Ganesan et al. 2005].

The approach of fractional cascading in [Gao et al. 2004] belongs to the second category and tries to avoid the bottleneck created by higher level nodes in the hierarchy. In [Gao et al. 2004], the sensor field is recursively partitioned by a standard quad-tree. Aggregates from each quad in the tree are computed and stored at all sensor nodes in the quad. Each node has the values of itself and aggregates of all the quads in which it resides. This improves data survivability and query efficiency as important information (e.g., the aggregates of larger regions) are naturally replicated more widely. Our multi-resolution representation can be considered as an alternative way to achieve fractional cascading. To see the difference of this paper with [Gao et al. 2004], instead of a fixed quad-tree partitioning, we keep the data summarization hierarchy of each node adaptive and centered on the node itself. Thus any two nodes will have slightly different world views at each scale, as their multi-resolution ranges differ, while two leaf nodes in a fixed quad-tree may share the same data of many high-level quads. Another novelty of this paper is to investigate gossip-based algorithm to disseminate information and construct the multi-resolution data representation. A survey of gossip algorithms and applications in sensor networks is covered in the next subsection.

1.2. Gossip Algorithms

Gossip is an attractive method for sensor networks, due to its distributed nature, robustness to network dynamics, and good load balancing. In a gossip algorithm each node picks, according to some underlying deterministic or randomized rule, another node and exchanges information with it [Hedetniemi et al. 1988; Shah 2008]. There are two important aspects in a gossip algorithm: the *gossip communication mechanism* that decides which node to communicate with; and the *gossip computation protocol* that decides what data to exchange.

In the literature two rules to select node to gossip with are common. In *uniform gossip*, each node chooses to communicate with a randomly chosen node at each step [Dimakis et al. 2006]. In *standard gossip* on a graph, a node picks, according to a probabilistic distribution, one of its immediate neighbors in the graph [Boyd et al. 2006; Xiao et al. 2005; 2006]. Of particular relevance to our work is the *spatial gossip* algorithm proposed by Kempe, Kleinberg and Demers [Kempe et al. 2001], where a node x selects a node y with probability proportional to $1/d^{\rho}$, where d is the distance between x and y and ρ is some constant parameter. The intuition of the spatial distribution complies with the principle of fractional cascading and our multi-resolution data representation. Data from a sensor node should, intuitively, be disseminated more to its nearby neighbors and less to far away neighbors.

On top of the gossip communication mechanism, a gossip computation protocol specifies what information to be exchanged. In probably the simplest setting, information spreading [Kempe et al. 2001], gossip is used to disseminate a piece of data from one node to the rest of the network. When two nodes communicate, the message is propagated. The protocol stops when all the nodes receive the message. More sophisticated information exchange protocols can be used to compute aggregations and global statistics among the gossip nodes. For the problem of distributed averaging [Boyd et al. 2006], each node takes the average of the values of itself and its gossip partner. The algorithm converges when all nodes hold values close to the true average. Gossip-type protocols have also been developed in various settings to compute, in a distributed way, consensus [Moallemi and Van Roy 2006; Boyd et al. 2006], various aggregates [Kempe et al. 2003; Mosk-Aoyama and Shah 2006], distributed linear parameter estimation [Xiao et al. 2005; 2006], spectral analysis [Kempe and McSherry 2004] or random linear projections of the data field for information compression and recovery [Rabbat et al. 2006]. For consensus computation, most of the algorithms mentioned above assume bidirectional communication links, recently Fagnani and Zampieri [Fagnani and Zampieri 2008] and Aysal *et al.* [Aysal et al. 2009] developed gossip-based algorithms using only directional communication.

1.3. The Challenge and Our Contribution

To construct the multi-resolution data representation, we first note that simple flooding and aggregation from each node will incur too high communication cost – $O(n^2)$ since each node incurs a cost of $O(n)$ to flood the network. In this paper we investigate gossip algorithms with almost linear communication cost.

In our setting the metric we care most is the total communication cost of the gossip algorithm, which depends on two factors: the cost of communication for each iteration step, and the number of iterations for it to converge. Many existing gossip protocols either assume that every two nodes can communicate with a unit cost (e.g., in peer-to-peer networks and distributed systems), or allow only immediate neighbors to gossip (e.g., in the standard gossip model). In our setting, we allow far away nodes to be chosen as gossip partners, and communication between them is performed by multi-hop routing. Thus the cost of each gossip step may involve any two nodes and have a higher cost if the nodes are far apart. This idea is also adopted in geographical gossip to reduce the communication cost of distributed averaging in a random geometric graph [Dimakis et al. 2006].

Under the objective of minimizing the total communication cost, the selection of gossip communication mechanism needs to balance two important factors. First, the fast convergence of a gossip protocol depends critically on the selection of gossip partners. Intuitively, fast convergence requires information to be well mixed — one of the best is to select a random node in the network as the gossip partner. On the other hand, if we choose a random node to gossip in each iteration, the cost of communication with multi-hop routing is proportional to the distance to a random node in the network, which is roughly $O(\sqrt{n})$ in a grid-like network with uniformly deployed sensors. To reduce the communication per each iteration, the best is to simply gossip with its immediate neighbors. But analysis of standard gossip on a random geometric graph or a 2-dimensional grid shows a slow convergence of roughly $O(n^2)$ gossip steps[1] [Xiao and Boyd 2004; Boyd et al. 2006], which is asymptotically the same order with that of naive flooding.

[1]Here we use 'gossip step' to refer to the atomic operation of one node gossiping with its partner.

The second challenge of the gossip algorithm in this paper, different from all the other gossip protocols, is on its multi-resolution nature. We would like information to be exchanged and mixed for fast convergence but also want to make sure that information does not travel too far and pollute the aggregates at other nodes. Thus the two conflicting considerations – fast convergence and restricted propagation range – need to be carefully balanced.

We propose to use a hierarchical spatial gossip algorithm that automatically takes care of all the issues above. Our hierarchical gossip algorithm proceeds in $O(\log n)$ phases. In phase i, we compute, for *all* sensor nodes, their respective aggregates in a roughly 2^i neighborhood. This is achieved by a spatial gossip algorithm in a restricted range, where each node x picks, from nodes within distance 2^i, a node y with probability proportional to $1/d^3$, where d is the distance between x and y, $1 \leq d \leq 2^i$ [2]. Each phase stops after $O(\text{poly}(i))$ iterations, $i \leq \log n$. At the end of phase i, we compute for each node u the aggregate of a subset of nodes $S_i(u)$ that contains all the nodes within distance 2^i from u with high probability, and does not contain any node more than distance $\text{poly}(i)2^i$ away. The total communication cost over all phases is bounded by $O(n \,\text{polylog}\, n)$. Notice that this achieves a substantial improvement in terms of communication cost to the naive flooding approach and is only at most a polylogarithmic factor away from an obvious lower bound of $\Omega(n \log n)$ for constructing the multi-resolution data representation[3].

What is critical to the success of our hierarchical gossip algorithm is that we use order and duplicate insensitive synopsis (ODI-synopsis) [Nath et al. 2004; Considine et al. 2004] to compute and represent the partial aggregates. The idea of an ODI-synopsis is that the same data can be aggregated multiple times but it is counted only once. Certain aggregates such as MAX/MIN are naturally ODI-synopses. ODI-synopsis for other aggregates such as COUNT and SUM/AVG are available, by implementation through MAX/MIN or boolean OR computations [Nath et al. 2004; Considine et al. 2004; Cohen 1997; Gao et al. 2007]. ODI-synopsis combined with gossip algorithm removes the above trouble caused by the same data disseminated and aggregated multiple times. In addition, ODI-synopsis is helpful for range queries as we do not need to worry about over-counting resulting from partial aggregates from overlapping regions.

One last note is that our gossip-based method is randomized. The multi-resolution aggregation covers roughly the 2^i neighborhood, for $i = 0, \cdots, \log n$. The question of computing an accurate set of multi-resolution aggregates, i.e., the aggregate of all the nodes precisely within 2^i hops, is considered in Section 4. We describe a deterministic algorithm to achieve this. This algorithm has a communication cost of $\Theta(n\sqrt{n})$. We show that this is in fact asymptotically optimal, and there is a lower bound of $\Omega(n\sqrt{n})$ for the message complexity. Accurate multi-resolution computation therefore does not scale well with network size. This makes it necessary to introduce approximate neighborhoods, as considered in our spatial gossip method.

[2]To implement the gossip step, we let each node choose a geographical location p^* with the above spatial distribution and use geographical routing towards p^*. The message will eventually arrive at the sensor node closest to p^*. Given a roughly uniform node distribution this will generate approximately the required spatial distribution on the sensor nodes. Notice that a node only needs the knowledge of the general span of the sensor field (e.g., a bounding box) and does not need the global topology of the network, nor the location of other sensors.

[3]For each sensor node simply reading in their $\log n$ data summaries it requires a communication cost of $\Omega(n \log n)$.

2. PRELIMINARIES

2.1. Network Setup

We consider a network of n sensor nodes in a square region. Each sensor node knows its own location and generates a reading which is the sample of an underlying data field at the location of this sensor. Since the sensors are discrete, a general point p^* in the sensor domain may not have a sensor located right at the spot. We assume that the value of the signal at p^* is taken to be the value at the closest sensor p. Thus our signal function f under consideration is assumed to be a piecewise constant function that takes the value at a sensor p for all the points p^* in the Voronoi cell[4] of p. This natural representation of the signal is particularly suitable for sensor networks. Geographic routing schemes for sensor networks have the property that a route targeted for p^* will terminate at the nearest sensor p, thus automatically comply to our assumed function.

The object of interest to us is the multi-resolution data summaries of the *signal*, through the data obtained from the sensors. In most typical sensor network applications, the sensor nodes are densely deployed in the region to ensure sufficient coverage and redundancy. In this case we compute the multi-resolution representations on the sensors, which is a good multi-resolution data summary of the signal. In the case when sensors are not a dense and uniform sample of the domain, we compute the multi-resolution data representations of the signal on a virtual grid. We overlay an abstract grid in the sensor field, and each grid point acts as a virtual sensor sampling our function f. The actions required of the virtual sensor are performed by the real sensor that is closest to that grid point. The resolution of the grid will be proportional to the aggregation resolution desired. This method is more suitable when the intent is to obtain the aggregate of the piecewise constant function f and it can adapt to sparse networks with holes. The two approaches are discussed in detail below.

Clustering in dense networks. For a dense network we assume the sensor nodes are deployed with sufficient sensing coverage such that any unit disk centered inside the region contains at least 1 sensor node. Notice that the above assumption guarantees sufficient coverage but does not prevent regions with dense node distribution. We can further improve the uniformity of the sensor sampling by clustering [Gao et al. 2006]. We compute a set of clusterheads such that every two clusterheads are of distance at least 1 away and every node is within distance 1 of at least one clusterhead. The clustering can be easily implemented by a greedy and distributed algorithm[5].

The set of clusterheads has both upper and lower bounded density. Every two clusterheads are at least distance 1 apart, as specified by the algorithm. Further, inside any disc of radius 2, denoted by D_2, there are at least 1 clusterhead — this is because any clusterhead outside this disc cannot cover the unit-radius disk D_1 co-centric with D_2. Thus by the sampling assumption there is at least one node inside the unit disk D_1, whose clusterhead must be within D_2.

Virtual grid network. When the nodes are sparsely distributed, we overlay a virtual grid and compute the multi-resolution representation on the grid points. In this case n is the number of grid points rather than the actual sensors. A sensor node takes over the computation for the grid points in its Voronoi cell. There are distributed methods for computation of Voronoi diagrams [Bash and Desnoyers 2007], such that each sensor node is aware of its own Voronoi cell. Essentially each node is trying to figure out the Voronoi vertices of its Voronoi cell. The nodes u start with the Voronoi cell C by

[4]The Voronoi cell of a site p is the collection of points that have p as the closest one among all sensors.

[5]Each node checks its nearby nodes to see if there is a clusterhead within distance 1. Otherwise it will promote itself as a clusterhead. By local communication the nodes can select a subset of nodes as clusterheads as desired above.

considering only the immediate neighbors. C might be bigger than the real Voronoi cell. Thus we need to refine it. To do so, one can use geographical routing to route towards each vertex of the convex polygon C to see if its nearest node is u. If its nearest node is $v \neq u$, then one discovers how to refine C by incorporating the bisector of uv. This iterative procedure continues until all the vertices of the refined polygon have u as nearest node. In this case node u finds its own Voronoi cell. The computation of the Voronoi diagram is only a one time computation at initialization, and can be used by all aggregations afterward.

The gossip step is between two virtual nodes on the abstract level and is actually carried out by the corresponding sensor nodes in charge of them. In particular, at each round, a sensor node, on behalf of each of its grid point, chooses another random grid point and gossips with it. Note that to send a message to a virtual node, geographical routing will automatically deliver it to the sensor node closest to it (in charge of it). Thus a node does not need to know who is in charge of a virtual node. The algorithm can be carried out with only the information about the local Voronoi cell. In the following discussion, the correctness of the multi-resolution representation is not affected by the use of a virtual grid. The analysis for the gossip communication cost holds as well, as long as the assumption holds that the routing communication cost between two virtual grid vertices is $O(d)$ where d is their Euclidean distance. If the above assumption is violated, i.e., it might take a much longer detour to reach the sensor in charge of a virtual grid vertex, the communication cost will be increased by this factor.

In the following sections, we will describe the spatial gossip for a set of nodes such that (i) any two sensor nodes are of distance at least 1 apart; (ii) any disk of radius 2 contains at least one sensor node. These hold regardless of whether the grid or the dense clustered network is in use.

2.2. Communication model

In analyzing communications costs of our scheme, we assume that two sensor nodes can communicate with each other directly if they lie within a small distance of each other. However, we do not enforce that the connectivity corresponds to a unit disk graph or any specific model. For the analysis we assume that the deployment permits the existence of a multi-hop routing algorithm that can carry a message from node x to node y using at most $O(d_{x,y})$ hops, where $d_{x,y}$ is the Euclidean distance between the two nodes. For sensors uniformly deployed, simple geographical forwarding would suffice to find a path with length proportional to the Euclidean distance between them. In all cases, similar results will hold if the cost is a polynomial in $d_{x,y}$ instead. If no such bound is available, then the communication costs may be higher. We remark that the correctness of the algorithms and the accuracies of the computed results are not affected by higher communication costs.

2.3. Order and duplicate insensitive aggregates

All the aggregates in our scheme are order and duplicate insensitive synopsis. In particular, given a set of values S, an ODI-synopsis is an aggregate computed for values in S that remains the same no matter how many times one duplicates some values in S or what order the aggregation was performed. For example, MAX/MIN are naturally ODI-synopsis. ODI-synopsis for a large variety of other aggregates such as COUNT (counting distinct items), SUM, AVG, most popular items, second moment, uniform sample, and set memberships (Bloom filter) are available [Nath et al. 2004; Considine et al. 2004; Cohen 1997; Gao et al. 2007] by essentially implementing them by MAX/MIN or Boolean operations. To give a quick idea of ODI-synopsis, we take COUNT as an example. The idea is to use probabilistic counting [Flajolet and Martin 1985]. Given

n distinct values, we use a hash function to hash each value to a k-bit long $0/1$ vector with one bit set as 1 and all other bits 0. In fact, the probability that the i-th bit is 1 is $1/2^i$, for $1 \leq i \leq k-1$. The probability that the last bit is 1 is $1/2^{k-1}$. To count the number of distinct values, we simply take the boolean OR of all the vectors. Probabilistic analysis will show that this vector has all 1's at early bits and all 0's at the last bits. The first zero is at the $\log(0.7753n)$-th bit with high probability. Therefore the position of the first zero can be used to deduce the value n in a probabilistic manner. For sensor network aggregation, we simply generate the bit vectors at each sensor and the aggregation function corresponds to taking the boolean OR of these vectors. Since boolean OR is order and duplicate insensitive the aggregate is ODI as well. In this paper we use MIN as the example, but the algorithm works with any ODI-synopsis.

The main benefit of using ODI-synopsis is that we do not need to explicitly keep track of which value has been included in the aggregates — this allows great flexibility in the use of routing schemes. If the value happens to travel to a node through multiple paths, the aggregated value is not affected by the double counting. Without using ODI-synopsis, we must enforce the value arriving at a node to travel along a tree, or be labeled and tracked independently of others. With ODI-synopsis, we can allow a general communication pattern. Since gossip algorithm is a randomized scheme and it is hard to control carefully what information flows where, the usage of ODI-synopsis allows us to separate the gossip communication protocol and the gossip computational protocol, and focus on the optimal design of the former one.

3. SPATIAL GOSSIP

In this section we describe the hierarchical spatial gossip algorithm to compute multi-resolution data summaries for every sensor node.

3.1. Hierarchical Spatial Gossip

We use a gossip mechanism where each node selects from a restricted neighborhood a node to gossip with and sends a message to it. The algorithm proceeds in *phases*. The phase i calculates for each node the aggregate of values inside a roughly 2^i neighborhood centered at itself. The phases are completely independent so that phase $i+1$ starts fresh. Since we have a network of n nodes, with a lower bound on density, $O(\log n)$ phases are sufficient for the phase with the largest neighborhood to cover the entire network.

For phase i, we adopt a restricted spatial gossip algorithm. We implement the selection of gossip partner with geographical routing. At each round, a node x chooses a *location* y^* in the sensor field with probability:

$$p_i(x, y^*) = \begin{cases} \frac{1}{\pi} \cdot \frac{1}{(|xy^*|+1)^3}, & |xy^*| \leq 2^i; \\ 0, & |xy^*| > 2^i. \end{cases}$$

where $|xy^*|$ is the Euclidean distance between nodes x and y^*. Notice that y^* is not necessarily the location of a sensor. x will send the information towards y^* using geographical routing and eventually reach the node y whose location is closest to y^*. Then y is x's gossip partner and takes the information delivered by x. Notice that a node only needs the knowledge of the general span of the sensor field (e.g., a bounding box) and does not need the global topology of the network, nor the location of other sensors.

With the above gossip algorithm and the uniformity of sensors, the probability that a node x chooses a sensor node located at y (also denoted by y by slightly abusing the notation) is also proportional to $1/|xy|^3$. The proof of the following Lemma appears in the Appendix.

LEMMA 3.1. *At phase i, let the probability that a node x gossips with a node y be $q_i(x, y)$. Then if $2 \leq |xy| \leq 2^i + 2$,*

$$q_i(x, y) \leq \frac{4}{(|xy| - 1)^3};$$

and if $|xy| \leq 2^i - 2$,

$$q_i(x, y) \geq \frac{1}{16(|xy| + 3/2)^3};$$

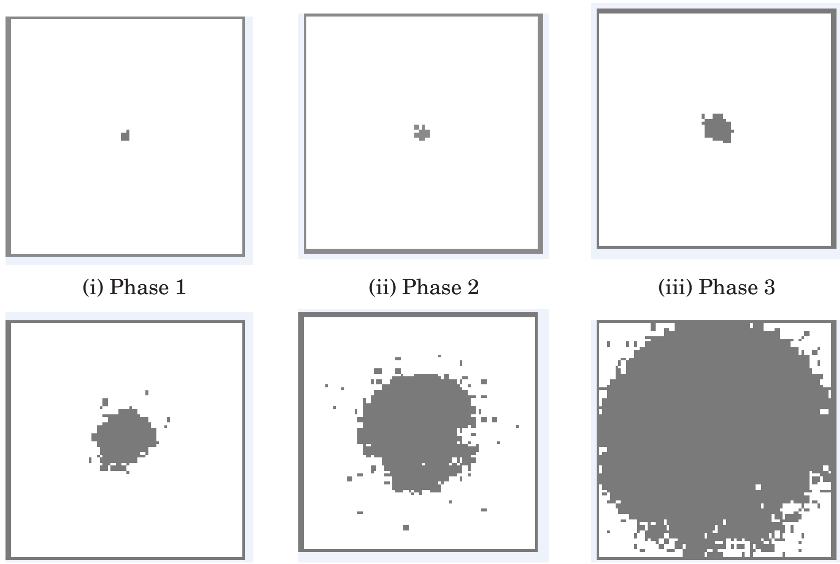
if $|xy| \geq 2^i + 2$, $q_i(x, y) = 0$.

We assume that all the nodes gossip in a synchronous way. At each clock tick, every node selects and shoots its information to its respective gossip partner. We consider each clock tick as a *round*. Once a node x chooses another node, say y, with distance at most 2^i from it, x sends its current synopsis to y. y will incorporate the information it receives from x and maintain the aggregation of synopsis of its old value with the synopsis from x. Note that this is asymmetric as only node y updates its synopsis and node x keeps its current synopsis value. The asymmetry is an attractive feature as reliable round-trip multi-hop routing adds communication overhead and implementation difficulty. Denote by $s_{i,j}(x)$ the synopsis at any node x after round j of phase i. The original value at x is thus given by $s_{0,0}(x)$. After round j, each node updates its synopsis to be the aggregation of its synopsis at round $j - 1$ and all the values it received in this round. The value computed at node x at completion of phase i is denoted by $s_i(x)$.

The use of ODI-synopsis is key to the success of the spatial gossip algorithm for constructing multi-resolution data representation. The insight is that aggregation by ODI-synopsis tremendously simplifies gossip computation protocols. Each node u keeps only a value $s(v)$ which is the ODI-synopsis of the set of values it has received so far and does not keep the set of values in its original form. When one node u chooses to gossip with v, u sends to v its aggregate $s(u)$ and v computes and keeps the ODI-aggregation of the synopsis of both u and v. $s(v) \leftarrow s(u) \oplus s(v)$, where $\oplus$ represents the aggregation function of the ODI-synopsis. This not only reduces the cost of transmission as only one aggregated value is delivered each step, but also guarantees that over-counting is eliminated although the same value may potentially be received multiple times. In short, with ODI-synopsis the model of gossip computation is the same as alarm spreading — each node starts with its own value and in each gossip step one node will send all the values it has received so far — but with reduced communication cost since only the aggregate (not the whole set of values) is delivered. When the algorithm stops, a node keeps the aggregate of all the values it has received.

To make the analysis easier, we also denote by $S_{i,j}(x)$ the set of nodes whose values x should have received if we deliver all the original values instead of a synopsis in the gossip algorithm. In other words, $s_{i,j}(x)$ is the aggregation of the values in the set $S_{i,j}(x)$. The set corresponding to the value $s_i(x)$ at node x at the completion of phase i is denoted by $S_i(x)$. Figure 1 shows the idea for a node in the center of the network.

To summarize, there are at most $O(\log n)$ phases in the hierarchical spatial gossip algorithm. In phase i, every node executes $O(i^{3.4})$ synchronous *rounds*. Each round consists of a single gossip operation performed by every node, and each phase consists of sufficient number of rounds so that nodes x and y that lie within a distance 2^i of each-other obtain each-other's values with high probability. Thus, at the end of phase i, any node has considerable information about values within a distance 2^i from it. Thus the synopsis aggregate at each node has incorporated sufficiently many nodes within its 2^i neighborhood.



(i) Phase 1 (ii) Phase 2 (iii) Phase 3

(iv) Phase 4 (v) Phase 5 (vi) Phase 6

Fig. 1. Propagation of one piece of data of the node x located in the center of the field. The shaded region in each case shows the set $S_i(x)$ for each phase i.

The gossip algorithm for each phase is very similar to the spatial gossip protocol proposed by Kempe *et al.* [Kempe et al. 2001], except that we restrict the maximum range of gossip partners. This modification is to reduce the level of pollution such that a node does not receive information from nodes too far away, as will be made clear later.

3.2. Multi-resolution Representations

In this section, we analyze the multi-resolution information computed by the algorithm described above. We show that if we stop the algorithm at phase i after $j^* = O(i^{3.4})$ rounds, the synopsis kept at node x, i.e., the aggregated value of a set of nodes $S_{i,j^*}(x)$, captures the information is a roughly 2^i neighborhood around x. Without loss of generally we denote by $S_i(x)$ and $s_i(x)$ the respective values when $j = j^*$.

Specifically, we show upper and lower bounds for the set of nodes in $S_i(x)$. Theorem 3.4 says that $S_i(x)$ includes with high probability each node within distance 2^i from x. Theorem 3.5 says that $S_i(x)$ does not include nodes $O(2^i i^{3.4})$ away for sure and with high probability does not include nodes with distance $O(2^i i^{2.4})$ or more away.

Before we prove our main theorems, we first observe that when we run more iterations of the gossip algorithm, the amount of information each node gets is monotonically non-decreasing within a phase. Recall that $S_{i,j}(x)$ is the set of nodes whose values have reached x, after j rounds at phase i. Thus,

OBSERVATION 3.2. $S_{i,j}(x) \subseteq S_{i,j+1}(x)$*, for any* i, j, x.

3.2.1. Lower bound. We first show a lemma that bounds the rate of information propagation by the restricted spatial gossip algorithm. Intuitively the lemma says that after $O(\text{polylog}\, d)$ rounds information at one node reaches a node at distance d with high probability.

LEMMA 3.3.
In phase i, if the distance between nodes x and y is $d \leq 2^i$ then $S_{i,j}(x) \subseteq S_{i,j+\alpha}(y)$ within $\alpha = O(\log^{3.4} d)$ rounds of iterations with probability at least $1 - O(\frac{1}{d})$.

The proof is an adaptation of the proof in [Kempe et al. 2001] that bounds the information spread rate in spatial gossip, with necessary modification that additionally takes care of the restricted range. While the essential proof is the same, the adjustment to get the specific result is not entirely trivial. We therefore include the complete proof in the appendix. This lemma shows that information propagates pretty fast in the network. Thus we can stop the algorithm in $O(\text{poly}(i))$ rounds for phase i, $i \leq \log n$, in order to collect information from almost all nodes inside the desired range 2^i.

THEOREM 3.4. *With probability at least $1 - O(1/2^i)$, the set $S_{i,w}(x)$ includes node y with $|xy| \leq 2^i$ in phase i consisting of $w = O(i^{3.4})$ rounds.*

PROOF. Obviously $x \in S_{i,0}(x)$. We apply Lemma 3.3 with $d = 2^i$ to obtain the theorem. This implies that in round i, any node collects information from each node in its 2^i neighborhood with probability at least $1 - O(1/2^i)$. □

3.2.2. Upper bound. The subsection above shows that in phase i, any node receives the information within a distance 2^i with good probability if we run the algorithm for $O(i^{3.4})$ rounds. Now we show an upper bound that a node does not get information from nodes too far away. Thus the 'pollution' from far away nodes is under control.

THEOREM 3.5. *After k rounds of phase i,*

(1) *$S_{i,k}(x)$ does not include nodes with distance $d > k2^i$ away from x, for sure.*
(2) *$S_{i,k}(x)$ does not include nodes with distance $d > \frac{3k2^i}{i+1}$ away from x with probability at least $1 - o(1/2^k)$, when i is greater than a sufficiently large constant.*

PROOF. To make the analysis easier, we assume that we actually propagate, by the hierarchical spatial gossip algorithm, the list of values together with their source nodes. Initially each node has only its own value. Then they propagate to other nodes. We examine, for the value of a node $u \in S_{i,k}(x)$, the path it may take to get from u to x, denoted by $P = \{u, u_1, \cdots, u_\ell = x\}$. $\ell \leq k$. In round j, u_j selects u_{j+1} as its gossip partner.

Claim 1. The value of u cannot travel further than $k2^i$ because in any iteration, a gossip step can go as far as 2^i at most and there are total k rounds.

Claim 2. Intuitively, for the value of u to reach a node x that is distance $d = \frac{(3+\varepsilon)k2^i}{i+1}$ away (ε is very small) via a path of length at most k, it must make enough number of long jumps. We argue that the probability for this to happen is small. The following analysis is to make this intuition rigorous.

First observe that the probability of a node u_j choosing a node u_{j+1} of distance $d' > 2^i/(i+1) - 2$ away is at most

$$\int_{2^i/(i+1)}^{2^i} \frac{2r}{(r+1)^3}\, dr \leq \frac{i+1}{2^{i-1}}.$$

Now consider a path P of at most k hops that starts from u and ends at x. Let k' be the minimum number of steps of length $2^i/(i+1)$ or more in P. Then the minimum value of k' satisfies the relation

$$(k - k')(\frac{2^i}{i+1} - 2) + k'(2^i + 2) \geq d = 2^i \frac{(3+\varepsilon)k}{i+1}.$$

When i is sufficiently large, $k' \geq (2+\varepsilon/2)\frac{k}{i}$. Therefore, for a k-hop path to reach node x, it needs to have at least k' long jumps, the probability of which is at most $\binom{k}{k'}\left(\frac{i+1}{2^{i-1}}\right)^{k'}$. Thus, the probability that a k-hop path P does not have k' or more links of length $2^i/(i+1)$ or more is at least $\left(1-\binom{k}{k'}\left(\frac{i+1}{2^{i-1}}\right)^{k'}\right)$.

In each round, a node that has a data sends a copy of it to another node. Thus, every existing copy gets replicated at a new node. At the end of k rounds, the total number of copies in the network is at most 2^k. We bound the probability that none of these 2^k paths reach x. This is at least

$$\begin{aligned} & \left(1-\binom{k}{k'}\left(\tfrac{i+1}{2^{i-1}}\right)^{k'}\right)^{2^k} \\ \approx\ & 1-\binom{k}{k'}\left(\tfrac{i+1}{2^{i-1}}\right)^{k'}2^k \quad \geq\ 1-2^{2k}\left(\tfrac{i+1}{2^{i-1}}\right)^{k'} \\ \geq\ & 1-\left(\tfrac{(2(i+1))^{2/i}}{2^{\varepsilon/2}}\right)^{k} \quad \geq\ 1-1/2^k. \end{aligned}$$

The last step is true when i is greater than a sufficiently large constant. □

For a phase i, with $k = i^{3.4}$ rounds, the probability that the value at a node does not spread beyond a distance $2^i\frac{3k}{i+1}$ is at least $1-o(1/2^{i^{3.4}})$. Thus with high probability $S_i(x)$ does not include nodes with distance $O(2^i i^{2.4})$ away.

3.3. Communication cost

In this section we show that the communication cost of constructing the multi-resolution data representation is almost linear.

LEMMA 3.6. *The expected communication cost incurred by any node in a single round of phase i is $O(i)$.*

PROOF. The expected distance to the gossip partner chosen by a node x is at most

$$2+\int_0^{2^i} 2r\frac{r}{(r+1)^3}dr \simeq O(i).$$

Since the cost of routing to a node distance d away is $O(d)$, the communication cost by any node in a round of phase i is $O(i)$. □

THEOREM 3.7. *The algorithm creates multi-resolution data as described above at every node using $O(\log^{4.4} n)$ rounds and total communication cost $O(n\log^{5.4} n)$. The storage requirement at each sensor node is $O(\log n)$.*

PROOF. In an n node network, with a constant lower bound on density, the maximum distance between any two nodes is $O(n)$. Thus, the number of phases required by the algorithm is $O(\log n)$. Each phase i consists of $O(i^{3.4})$ rounds. Thus, the number of rounds is $\sum_{i=1}^{\log n} O(i^{3.4}) = O(\log^{4.4} n)$. In phase i, at each round, a node uses a single message with an expected communication cost of $O(i)$. Thus, the communication cost per node for the algorithm is: $\sum_{i=1}^{\log n} O(i\cdot i^{3.4}) = O(\log^{5.4} n)$. The total communication cost is thus $O(n\log^{5.4} n)$. Notice that during the spatial gossip algorithm for phase i, each node at any time only keeps one value. The total storage requirement for each node is $O(\log n)$. □

3.4. Spatial gossip with metric ℓ_p

For ease of explanation, we have described the concepts in terms of Euclidean distances, but the ideas extend to other ℓ_p distance measures. In ℓ_2 measure, all points within a distance of d from point x form a circular disk of diameter d centered at x. Thus, the results of theorems 3.4 and 3.5 correspond to properties of data stored about disks of certain radii centered at each node in the network. For the type of rectangular range queries discussed in detail in section 5, it would be convenient to use ℓ_∞ as the metric. In ℓ_∞, *disks* correspond to axis aligned squares that can be used to cover the query region nicely.

4. ACCURATE MULTI-RESOLUTION DATA

The gossip based algorithm is randomized, and therefore has some inaccuracy associated with the aggregates it computes. In this section, we discuss a deterministic algorithm to compute multi-resolution aggregates and show a communication lower bound of $\Omega(n\sqrt{n})$ messages on computing multi-resolution data. These results show that approximation is necessary in order to achieve near linear communication cost.

For the ease of description we use the ℓ_∞ metric, and assume that the n nodes are placed on a unit grid in a square. A disk in this metric looks like a square. Suppose the aggregate minimum is being computed. The algorithm works as follows:

At step i, every node p finds the aggregate of the ℓ_∞ disk of radius 2^i centered at itself. This is done as follows: p collects the aggregates of step $i-1$ from each node q at distance 2^{i-1} from p, and computes the minimum to find the aggregate minimum of its 2^i neighborhood. Each node q needs to send its $(i-1)^{th}$ average to nodes at a distance 2^{i-1} from it. This is done by traversing the boundary of the disk of radius 2^{i-1}, at a cost of $O(2^{i-1})$.

The total cost per node is therefore $\sum_{i=0}^{\log\sqrt{n}} O(2^{i-1}) = O(\sqrt{n})$.

The following example shows that this is in fact a lower bound on the asymptotic complexity of computing multi-resolution data.

Suppose that the left topmost corner of the grid has position $(0,0)$. The node at row i and column j has position (i,j) and value $v_{ij} = i\sqrt{n}+j$. More importantly, this is also the rank of the value. Now consider the quadrant with $i,j \in [\sqrt{n}/2, \sqrt{n}]$ and in particular the node at $(\sqrt{n}/2, \sqrt{n}/2)$. The minimum of its $\sqrt{n}/2$ neighborhood is given by v_{00}. The corresponding aggregate of any node in the quadrant at $(\sqrt{n}/2+i, \sqrt{n}/2+j)$ is given by $v_{i,j}$. Therefore, each such value has to be transmitted a distance $\Omega(\sqrt{n})$. Since at least a constant fraction of the values have to be transmitted this distance, the lower bound on the message cost is $\Omega(n\sqrt{n})$.

5. RANGE QUERIES

The pre-computed data summaries by the hierarchical spatial gossip algorithm can be useful in answering user queries about aggregates in large regions of the network with reduced cost. For example, the aggregate for the entire network is available at any single node. Similarly, it is possible to obtain probabilistic information about a large region of radius 2^i by visiting a single node at its center. If the query requires better estimates of the aggregate, then it can be answered by making use of the different ODI synopses computed at different phases of the algorithm. Thus, the query response mechanism can adapt to the quality of estimate and restriction on pollution desired by the user.

In the rest of this section we discuss a case where the user wishes to obtain with high probability the correct ODI synopsis of a rectangular region, without any pollution. The query consists of an $a \times b$ axis aligned rectangular area A, and a small probability δ. The response to the query is the ODI synopsis s corresponding to a set S, such that, for any node x, if $x \in A$ then $x \in S$ with probability at least $1-\delta$, and if $x \notin A$ then

$x \notin S$. That is, no node outside the region A should be included in set S, and no node inside A should be excluded with a probability more than δ. Without loss of generality, we can assume that $a \leq b$.

By Theorem 3.4, after phase i, the ODI synopsis at any node x includes the value at any other node inside a disk of radius 2^i with a high probability. For distances measured in the L_∞ metric, this disk corresponds to a square of side $2 \cdot 2^i$. We refer to such a square as a square of radius 2^i (analogous to a disk of same radius), and use a set of such squares to *cover* the given query region.

We denote by $B_i(x)$ a square of radius 2^i centered at node x. For a node $y \in B_i(x)$, by Theorem 3.4, $y \notin S_i(x)$ with probability $O(1/2^i)$. Corresponding to any square $B_i(x)$, there is a square $G_i(x)$ of radius $\eta i^{3.4} 2^i$, for a proper constant η, such that for any node $y \notin G_i(x)$, $y \notin S_i(x)$, by Theorem 3.5. If the user query requires no pollution from outside the query region, the bigger square $G_i(x)$ must be completely inside the query range.

We refer to a square $B_i(x)$ as a *maximal piece* if $G_i(x) \subseteq A$ and $G_{i+1}(x) \nsubseteq A$, and i as the maximal level of node x. Let $B_p(x)$ be the largest maximal piece in A, then formally

$$p = \max_{x \in A} \{i : B_i(x) \text{ is a maximal piece}\}.$$

Then, we have

$$\eta p^{3.4} 2^p \leq \frac{a}{2} < \eta (p+1)^{3.4} 2^{(p+1)}.$$

This implies that $p = O(\log a)$. Now we can collect the partial aggregates from these maximal pieces to answer the query. This can be done in a manner similar to that in [Gao et al. 2004] by starting at the boundary and spiraling inward accumulating synopsis for maximal pieces that together cover the entire region. Additionally, we must ensure that the probability of any node being excluded in the synopsis is small.

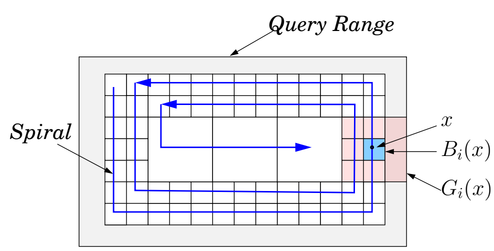


Fig. 2. The spiral used for response for a given query region. Nodes are visited individually in the shaded region at the perimeter. The figure also shows the maximal square $B_i(x)$ for a node x of maximal level i, and the corresponding pollution region $G_i(x)$.

We use a spiral path that guarantees the required probability for every node in the query region. By Theorem 3.4, if a node is covered by a maximal piece $B_i(\cdot)$, the probability of it being included in the corresponding synopsis set $S_i(\cdot)$ increases with the size of $B_i(\cdot)$. This implies that given a δ, nodes more than a certain distance (depending on delta) away from the boundary are covered by one or more maximal pieces that provide the required probability. Thus, our spiral starting at the boundary accumulates synopsis from all individual nodes up to this distance, and makes use of maximal pieces to obtain the synopsis for the rest of the region. Figure 2 shows a schematic representation of this idea. The following theorem gives the cost for such a computation.

LEMMA 5.1.

Given a query (A, δ) where A is an $a \times b$ rectangular axis aligned query region, the query can be answered at a cost of $O(\max(a,b)\log^{4.4}\min(a,b) + \max(a,b)(1/\delta)\log^{3.4}(1/\delta))$.

PROOF. For a node x, let d_x be the distance of node x from the perimeter of the region A. If i is the maximal level of x, then $\eta i^{3.4}2^i \leq d_x < \eta(i+1)^{3.4}2^{(i+1)}$. This implies that all nodes of maximal level i occur in an annular rectangular region of inner boundary $(b - 2\eta(i+1)^{3.4}2^{(i+1)}) \times (a - 2\eta(i+1)^{3.4}2^{(i+1)})$ and outer boundary $(b - 2\eta i^{3.4}2^i) \times (a - 2\eta i^{3.4}2^i)$. The thickness of this annular rectangle is $O(i^{3.4}2^i)$.

To obtain the result with parameter δ, we start at the perimeter of region A, and spiral inward accumulating the synopsis s. At a distance $\eta d \log^{3.4} d$ from the boundary, a maximal piece of level $\log d$ can be used, and the probability of a node at this level being missed by a maximal piece is $O(1/d)$. By the requirements of the query, it has to be ensured that $\delta = O(1/d)$. Thus, the spiral visits each individual node until the distance to the boundary reaches $O((1/\delta)\log^{3.4}(1/\delta))$. At every node x, the synopsis is updated as $s = s \oplus s_0(x)$. The cost of such a path is $O((a+b)\frac{1}{\delta}\log^{3.4}(1/\delta))$.

After this point, the synopsis are updated according to maximal levels. At a node x of maximal level i, we set $s = s \oplus s_i(x)$, which is equivalent to the operation $S = S \cup S_i(x)$. The lowest maximal level that we can use for the given query is $\gamma = O(\log(1/\delta))$. The cost incurred to process nodes at any maximal level $i \geq \gamma$ is $O((a - i^{3.4}2^i)i^{3.4} + (b - i^{3.4}2^i)i^{3.4}) = O(b \cdot i^{3.4})$.

The cost for the spiral covering all maximal levels i for $\gamma \leq i \leq p$ is given by

$$\sum_{i=\log(1/\delta)}^{p} O(b \cdot i^{3.4}) = O(bp^{4.4}) = O(b \log^{4.4} a).$$

Thus, the total communication cost for answering the query is $O(\max(a,b)\log^{4.4}\min(a,b) + \max(a,b)(1/\delta)\log^{3.4}(1/\delta))$. $\square$

Spatial gossip with no maximum range restriction. We note that the hierarchical spatial gossip for phase i makes only one change to the spatial gossip algorithm as in [Kempe et al. 2001]. Essentially a node chooses its gossip partner with a maximum distance range 2^i. This way we are able to restrict the amount of pollution from distant nodes. In the above range query, we make use of the fact that the data summaries do not include information beyond a certain distance threshold (claim 1 in Theorem 3.5), to answer queries with no false positive errors.

For applications in which small false positive errors are not a problem, we can propose to use the single-phase spatial gossip algorithm to construct the multi-resolution data representations. Essentially, we just run the standard spatial gossip algorithm where each node chooses another node with distance d away with probability roughly $1/d^3$. We run the algorithm for $O(\log^{3.4} n)$ rounds. During the algorithm, we keep the current aggregation value after round $O(i^{3.4})$, as the data summary of the 2^i-hop neighborhood. Notice that the probabilistic upper bound on pollution as the second claim in Theorem 3.5 still holds. Thus the ith data summary we compute has a large probability to include every value inside a 2^i-hop neighborhood and not include values outside $2^i i^{2.4}$ neighborhood. This alternative solution saves a factor of $O(\log n)$ in the total communication cost, at the cost of more pollution from far away nodes. For range query, a probabilistic solution with both small false positives and small false negatives can be obtained. In practice either variation can be adopted, dependent on application requirements. We evaluated and compared the gain of each variation in the simulation section.

Error introduced by ODI synopsis. The analysis above considers the probabilistic error introduced by the gossip. ODI-synopses for aggregates such as SUM, AVG are probabilistic with small probabilities of error. Thus, the overall system error may incorporate this factor, which will depend on the actual ODI synopsis used.

6. SIMULATIONS

In this section, we show simulation results that confirm our expectations of the hierarchical spatial gossip. The gossip algorithm was found to be efficient in communication. We monitored the spread of information, and found that representation is sharper than the worst case theoretical bounds. That is, when we perform gossip to compute aggregates in 2^i disks, the data typically does not spread too far beyond the disk, therefore does not create too many outliers that would pollute aggregates at distant nodes. The range query procedure from the previous section is also seen to be efficient. Finally, we found that the method adapts well to unreliable links and message losses.

We focus on evaluating the performance of our approaches at the algorithm level, and ignore specifics of lower level protocols and hardware. We use geographic routing in the simulations. Each packet transmitted only contains necessary location information and a piece of aggregate data of the source node. All simulations are on a unit-disk graph model. For the simplicity of explanation, we denote the set of nodes within 2^i distance from node x as $D_i(x)$. The aggregate of $D_i(x)$ is referred as the aggregate of resolution level i at node x. We compute the aggregate MIN as an example in the following simulations, other ODI-synopsis can be evaluated in the same way. All simulation results are averaged on 10 runs.

6.1. Total Communication Cost

We simulated a grid network where the sensor nodes have a fixed transmission range 2. Nodes can communicate directly if they are within the transmission range of each other. Keeping the density of the network constant, we vary the number of nodes from 256 to 4900, and vary the size of the sensor field from 32×32 to 140×140.

Each phase i of the gossip was terminated at the average number of rounds when at least $(1-0.5/i)$ fraction of nodes received the minimum for i^{th} resolution. This condition was found to provide a reasonable balance between fast information propagation and low pollution rates.

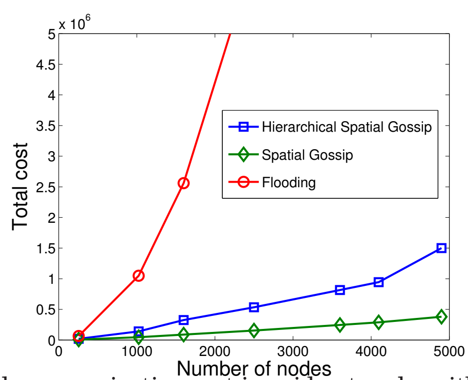


Fig. 3. Total communication cost in grid networks with various size.

Figure 3 shows the total communication cost in grid networks with various sizes. We compare hierarchical spatial gossip with the flat single phase spatial gossip and simple flooding. Flooding incurs dramatically higher cost as expected. In a network with n nodes, it requires $O(n)$ transmissions for propagating one piece of data, and $O(n^2)$ transmissions in total. The hierarchical spatial gossip costs slightly more than the spatial gossip, but is still almost linear in the network size.

6.2. Effectiveness of Multi-Resolution Representation

In this subsection, we test the sharpness of the multiresolution representation. Each phase i is expected to compute the aggregate in a region of radius 2^i surrounding a node. We wish to verify that the gossip algorithm does not deviate far from this goal.

We take one piece of data s of the node located in the center of the network as a representative, and evaluate the entire process of its propagation. All other data is propagated in the same way. Intuitively, an ideal multi-resolution representation should compute aggregates at level i of almost all nodes belonging to D_i, and little or no pollution beyond D_i.

The example of one execution in Figure 1 shows different phases of the propagation of s in the hierarchical spatial gossip. We can see that the information s is propagated within a restricted range in each phase and pollutes very few nodes beyond a certain distance. In the following we evaluate this property using more quantitative measures.

Flooding can be very precise in this regard. It can compute the accurate multi-resolution data summaries by labeling each flood message with the location of its starting point. This is expensive, but each node then receives data from all nodes, and can simply maintain the i^{th} aggregate as aggregate of those originating within its 2^i disk. Flooding therefore creates perfect multi-resolution data at a high cost. Thus we only compare the effectiveness of multi-resolution representation of our approach with the single phase spatial gossip here.

We compare the standard spatial gossip with hierarchical spatial gossip when they reach roughly the same state. For example, if round 15 of spatial gossip is the first round at which at least a fraction of $(1-0.5/3)$ nodes correctly compute the aggregates of resolution level 3, then the state of the 15^{th} round is comparable to phase 3 in hierarchical spatial gossip. The following simulations are conducted in a 128×128 grid network with 4096 sensor nodes.

Coverage. We define the *percentage of coverage* at distance d as the percentage of the number of nodes at distance d from the origin of s that receive s. In Figure 4, we show the percentage of coverage in an intermediate phase (phase 4) for both standard spatial gossip and hierarchical spatial gossip. The result confirms that there is a disk such that nodes within it receive the value with high probability. And the probability of a node outside this disk receiving the values falls sharply with the distance from the origin.

In the hierarchical spatial gossip, all nodes within a disk with radius 8 from the center receive s. The percentage of coverage decreases quickly as the distance increases, and goes below 10% beyond distance 30. The propagation quickly stops at distance 44.6. In standard spatial gossip, all nodes within a disk with radius 6 from the center receive s, but it pollutes the information at distant nodes up to a distance of 78, almost to the boundary of the network.

Pollution. The small coverage in hierarchical spatial gossip implies low pollution rates. This is visible in figure 4. The single-phase spatial gossip always selects nodes from the entire network, thus it cannot guarantee a comparable restriction on pollution. We characterize and compare the pollution caused by the two approaches using two more criteria - maximum distance and relative pollution. We define the *maximum distance* of phase i as the distance between the center and the furthest node receiving s in phase i. The *relative pollution* of phase i is defined as the ratio of the number

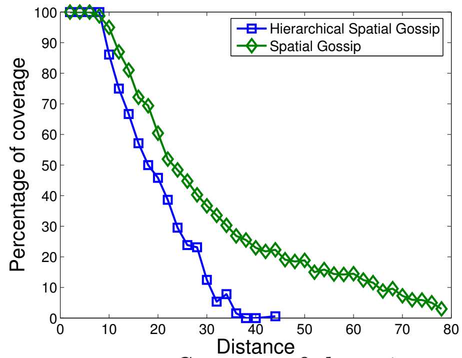


Fig. 4. Coverage of phase 4.

of nodes receiving s beyond $D_i(center)$ and the number of nodes receiving s within $D_i(center)$.

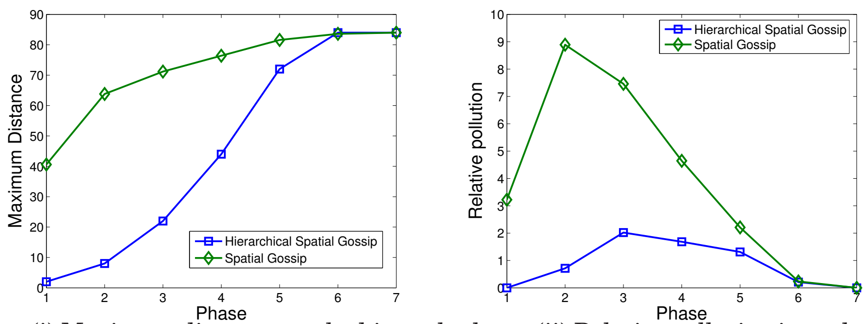


Fig. 5. (i) Maximum distance reached in each phase. (ii) Relative pollution in each phase.

Figure 5(i) shows the maximum distance reached at the end of each phase in both approaches. Since we simulate in a 128×128 grid network, the farthest point from the center is at a distance of about 90 units. In the hierarchical spatial gossip, the maximum distance increases relatively slowly with phases, while in the single-phase spatial gossip, the data often reaches distant nodes within the first few rounds. From Figure 5(ii), we can see that there is a big gap between the single-phase spatial gossip and the hierarchical spatial gossip in terms of relative pollution. The peaks are 9 and 2 respectively. Since we compare the states of the standard spatial gossip at the point of reaching the same state in the hierarchical spatial gossip, the number of nodes getting s within D_i is roughly the same in both approaches. However, to build up the same level resolution, the single-phase spatial gossip would pollute data at about 4 times as many nodes beyond that level than the hierarchical spatial gossip.

6.3. Range Query Costs

We evaluated the range queries executed by the spiraling method described in section 5 on our computed aggregates. We evaluated the query costs of different sized query regions and also looked at the accuracy of the aggregate computed from the spiral. The data used to verify the query response was a continuous signal sampled at the sensor locations. The simulation was carried out in our largest network of 4900 nodes.

Figure 6 shows the communication costs incurred for the spiraling range query in square ranges of different sizes. The gossip phases for this simulation used the ℓ_∞

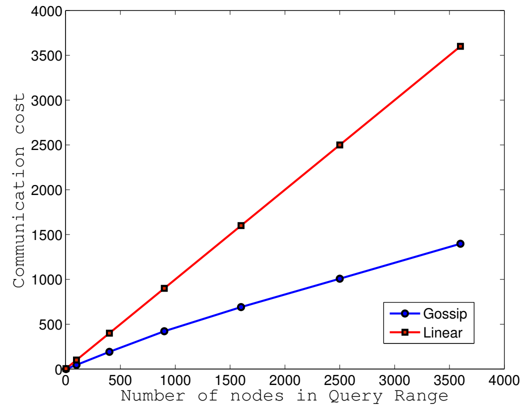


Fig. 6. Costs of querying squares of different sizes. Diameters are measured in ℓ_∞ metric. Axis aligned squares are disks in this metric.

metric, as described in section 5. The communication cost is slow growing. For comparison, we also show the linear plot denoting the number of nodes in corresponding squares. This will be the cost of performing the aggregation without any preprocssing, for example, with an aggregation tree.

Since the gossip preprocessing is probabilistic, there is a possibility of the aggregates being computed incorrectly. However, in simulations incorrect evaluations were very rare, less than 1% of queries. And in these cases, the computed minimum was found to be very close, within 5% of the true minimum.

6.4. Resilience to Link Failures

Wireless links may be unreliable. Messages can be lost without warning, introducing difficulties for an aggregation algorithm. One of our motivations for using a randomized gossip algorithm is its resilience to message loss. In case of a loss, it needs no special action, in fact, nodes can proceed oblivious to individual losses. However, frequent message loss does slow down the spread of information. Therefore in a lossy environment the spatial gossip takes longer to distribute information.

In our model transmission failures do not have a significant effect on the coverage/pollution as a transmission failure basically causes the gossip attempt to abort and none of the data is exchanged. The quality of the solution is not affected by the transmission failures. Therefore in the following we only investigate the increase in communication.

Figure 7 shows that the gossip algorithm scales well with link failures. The simulation was done on a network of 4900 nodes. We assigned a fixed probability p that any message transmission between adjacent nodes succeeds, and evaluated the communication costs for different p, to get the same information spread as before, that is until $(1 - 0.5/i)$ fraction of nodes in the 2^i disk is reached. The curve in red shows the cost with no transmission failures. The gossip efforts that are wasted due to link failures increase the overall communication costs, but do not help the spread of information. We checked how much the wasted efforts added to the communication costs. It is seen that the cost of the gossip for different phases do not increase much as the quality of the links drop.

The failure model above is particularly harsh. For example, if each link works with a 90% probability, then a 7 hop communication will fail in more than 50% of cases. Spatial gossip relies on such long communication. However, as long as one such long communication attempt succeeds information spreads quickly. In reality, wireless communication links do not always have the same failure rates, some are generally more

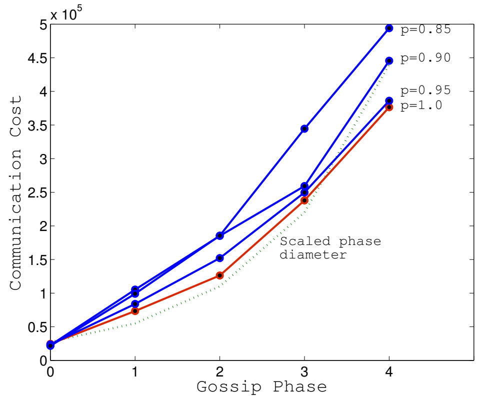


Fig. 7. Communication costs of the hierarchical spatial gossip algorithm when links are unreliable. p represents the probability that a message transmission to an adjacent node will succeed. The dotted line shows the curve 2^i (the diameter of phase i) scaled by a factor of 25000 – the approximate cost of phase 0.

dependable, but certain links and regions are likely to be lossy. In such models we expect spatial gossip to perform even better. Gossip does not rely on reliability of any particular link, and therefore is unaffected by a small number of bad links failing with high probability.

Figure 7 can be used to see how the gossip cost scales with phase (x-axis). We put as comparison a dotted curve for the phase diameter (i.e., 2^i), scaled by multiplying it with the cost of phase 0. We can observe that the costs at different phases scale roughly by the range diameter and the multiplicative factor remains almost the same in simulations.

7. CONCLUSION

In this paper, we propose an efficient algorithm with a total communication cost of $O(n \operatorname{polylog} n)$ to extract and construct sharp multi-resolution data representations for sensor networks. We believe that the multi-resolution data summary is a fundamental data storage paradigm to equip each node with compact sketches of the global picture of the data field. As the future work we will explore more applications of multi-resolution data summaries for advanced data processing and validation, as well as efficient query evaluations.

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Appendix

PROOF LEMMA 3.1. We compute the Voronoi diagram of all the sensor nodes (a partitioning of the region into cells such that all the points inside one cell are closest to the same sensor node) and only inspect the part inside the bounding square. In order for x to choose node y as its gossip partner, x must have chosen a location y^* that falls inside the Voronoi cell of sensor node y. Denote by $V(y)$ the Voronoi cell of y, then we have $q_i(x,y) = \int_{y^* \in V(y)} p_i(x, y^*)$.

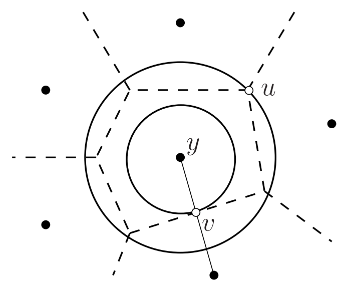

Fig. 8. The Voronoi cell of a sensor node y is enclosed inside a disk of radius 2 and contains a disk of radius $1/2$.

We now upper and lower bound the Voronoi region for y. $V(y)$ is a convex region. The point on the boundary of $V(y)$ furthest from y is realized at a Voronoi vertex (u in Figure 8), which has three sensors (including y) as its closest nodes. Thus the disk centered at u with radius $|yu|$ has no other sensor nodes inside. Since any disk of radius 2 has at least one sensor node inside, $|yu| < 2$. Thus $V(y)$ is enclosed by a disk centered at y with radius 2, denoted by $D_2(y)$. On the other hand, the point on the boundary of $V(y)$ closest to y, say v, is realized as the mid-point connecting y and one of its Delaunay neighbors (the sensors whose Voronoi cells are adjacent to that of y's). Thus $|yv| \geq 1/2$. Consider that y can be placed at the corner of the sensor bounding square. $V(y)$ includes at least 1/4 of a disk of radius $1/2$ centered at y.

With the upper and lower bound of $V(y)$, we will bound the probability $q_i(x,y)$. Take the point in $V(y)$ closest to x, denoted by w. $|xw| \geq |xy| - 2$. Therefore $q_i(x,y) = \int_{y^* \in V(y)} p_i(x,y^*) \leq p_i(x,w) \cdot \pi 2^2 = 4/(|xw|+1)^3 \leq \frac{4}{(|xy|-1)^3}$. Similarly, we have $q_i(x,y) \geq \frac{1}{16(|xy|+3/2)^3}$.

The above bound is valid when $V(y)$ is completely within distance 2^i from x, which is true if $|xy| \leq 2^i - 2$. If $|xy| \geq 2^i + 2$, then all points in $V(y)$ are of distance 2^i away. Thus y will never be chosen as x's partner. $q_i(x,y) = 0$. □

PROOF LEMMA 3.3. From the setup of the network described in section 2, observe that the density of the node deployment has lower and upper bounds in any region of the network. In particular, for the following analysis we assume that there are constants β_1, β_2 ($\beta_1 < \beta_2$) such that the number of nodes in any disk of radius $r \geq 1$ lies between $\beta_1 r^2$ and $\beta_2 r^2$.

The probability $1 - O(\frac{1}{d})$ can be rewritten as $1 - \gamma g(d)$ for $\gamma = O(\log^{-2.4-\frac{\log d}{\log \log d}} d)$ and a suitable function $g(d) = O(\log^{2.4} d)$. And the number of rounds $O(\log^{3.4} n)$ can be written as $\tau g(d)$ for a suitable $\tau = O(\log d)$.

Note that, if in the jth round of phase i a node x selects a node y to gossip, then $S_{i,j}(x) \subseteq S_{i,k}(y)$, $\forall k > j$. And this property holds transitively. So, all we need to prove is that there would be a sequence of gossip selections taking the message from x to y within $g(d) = O(\log^{2.4} d)$ rounds with probability at least $1 - \gamma g(d)$. Our induction hypothesis is that the result holds for distances upto $d^{3/4}$.

First note that for the base case of r equal to some constant, any constant probability $1 - \gamma g(r)$ of the value from x reaching y can be obtained with constant k number of selections by x. This constant will depend on β_2, the upper bound on density since there can be $\beta_2 d^2$ nodes that are nearer to x than y.

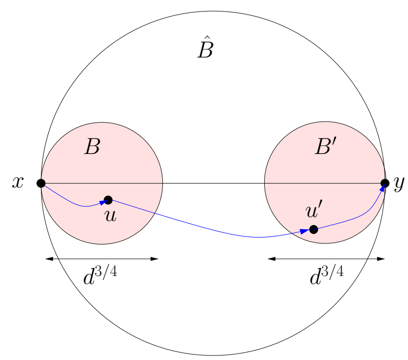

Fig. 9. In a time interval τ, the long link uu' exists with high probability, and links xu and $u'y$ exist with corresponding high probability.

Consider the disk $\hat{B}$ of diameter d containing both x and y. Inside $\hat{B}$, we take two disks B and B' of diameter $d^{3/4}$ containing x and y respectively. Our induction hypothesis is that the result holds for pairs of nodes $d^{3/4}$ apart. Thus, g is the recursive function $g(r) = 1 + 2g(r^{3/4})$. It can be shown that $g(r) = O(\log^{2.4} r)$.

We divide the time interval $\tau g(d)$ into intervals $\tau g(d^{3/4})$, τ and $\tau g(d^{3/4})$. We need to show, that with high probability, some node u from B selects some node u' from B' to gossip with in a time interval of length τ. The rest follows by induction. The probability that in τ rounds, no node from B selects any node from B' to gossip with, is given by

$$\left(1 - c\beta_1 \frac{k^{3/2}}{4k^3}\right)^{\tau \beta_1 k^{3/2}} \leq \left(\frac{1}{e}\right)^{c\tau\beta_1^2/4}.$$

We select τ such that $\left(\frac{1}{e}\right)^{c\tau\beta_1^2/4} = \gamma$. Then the probability that some node u in B selects some node u' in B' in an interval of τ rounds is at least $1 - \gamma$. In other words, assuming that u has the message at the end of $\tau g(d^{3/4})$ rounds, the probability that some $u' \in B'$ receives the message in the τ is at least $1 - \gamma$.

By induction hypothesis, u receives the message from x with probability $1 - \gamma g(d^{3/4})$ in the first $\tau g(d^{3/4})$ rounds. And y receives the message from u' with probability $1 -$

$\gamma g(d^{3/4})$ in another $\tau g(d^{3/4})$ rounds after u'. Thus, the probability that in $\tau g(d)$ rounds the message propagates from x to y is at least $1 - \gamma - 2\gamma g(d^{3/4}) = 1 - \gamma g(d)$.

Since $\gamma = O(\log^{-2.4 - \frac{\log d}{\log \log d}} d)$ and $\tau = O(\frac{-1}{\beta_1^2 c} \log \gamma)$, we have $\tau = O(\log d)$. Therefore, in $\tau g(d) = O(\log^{3.4} d)$ rounds the message travels from x to y with a probability of $1 - O(\frac{1}{d})$. □