Modeling Social Influence

How does an agent get influenced by its neighbors?

- Diseases
- Information, ideas, innovation
- Social behaviors (e.g., smoking, obesity, voting)
- Opinions (for or against a policy)
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Two main factors

- **Contagion models**
- Network structures
Opinion Dynamics: Continuous Model

Each agent has a real-valued opinion variable \( x(v) \in [-1, 1] \)
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- Edge \((i, j)\) carries a weight \(w_{ij}\).
- At time \(t\), every agent updates its opinion by (weighted) influence from neighbors.
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Research questions:
- Does the network converge?
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Research questions:
- Does the network converge?
- If so, what is the converged state?
Opinion Dynamics: French-DeGroot Model [1956]

Each agent $i$ has a real-valued opinion variable $v_i \in [-1, 1]$
- $-1$: against; $1$: supportive.
- At time 0 nodes have initial opinions.
- Edge $(i, j)$ carries a weight $w_{ij} \geq 0$.
- At time $t$, every agent updates its opinion by (weighted) influence from neighbors.

$$v_i(t + 1) = \sum_j w_{ij} v_j(t).$$

Research questions:
- Does the network converge? Yes.
- If so, what is the converged state? Global consensus.
Opinion Dynamics: Friedkin-Johnsen Model [1990]

Each agent $i$ has an opinion variable $v_i \in [-1, 1]$
- $-1$: against; 1: supportive.
- At time 0 nodes have initial opinions $u_i$.
- Edge $(i, j)$ carries a weight $w_{ij} \geq 0$.
- At time $t$, every agent updates its opinion by (weighted) influence from neighbors.

$$v_i(t + 1) = (1 - \lambda_i) \sum_j w_{ij} v_j(t) + \lambda_i u_i.$$

Research questions:
- Does the network converge? Yes under favorable conditions on $W$.
- If so, what is the converged state? no consensus.
Opinion Dynamics and Polarization

Social influence leads to homogeneity. Yet, the real world is not homogeneous.
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- In reality, negative or repulsive influence – boomerang effect [AG14, HJK53], backfire effect [Nyhan10].
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What if we have negative weights?
Structure Balance in Signed Networks [Heider46]

Positive ties: friendship; negative ties: hostility.

Only triangles with even negative ties are stable.
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Global property: A stable network consists of two groups, where edges within the group are positive, and edges across the group are negative.
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Global property: A **stable network** consists of two groups, where edges within the group are positive, and edges across the group are negative.
Model on network dynamics when a graph is not balanced?
Our Results: Co-Evolution Model [WLG22]

- Opinion dynamics: DeGroot model + negative ties

$s V(t+1) = V(t) + W(t)$
$s W(t+1) = W(t) + V(t)$

Matrix Riccati Equation:
$s W' = WW^T + C$
$s C = V(0)V(0)^T - W(0)W(0)^T$
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- Opinion dynamics: DeGroot model + negative ties
- Tie dynamics: tie strength ↑ if two nodes agree with each other. (Schelling’s model of residential segregation [Schelling 71])
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$$
\begin{align*}
V(t+1) &= V(t) + W(t)V(t) \\
W(t+1) &= W(t) + V(t)V(t)^T.
\end{align*}
$$

(1) Matrix Riccati Equation: $W'(t) = WW^T + C$, $C = V(0)V(0)^T - W(0)W(0)^T$.

Main result:
- The network converges to structural balance, unless $|V(t)| \rightarrow 0$.
- Community membership can be solved from the initial states.
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Co-Evolution Dynamics

Random initial weights.

Iteration=0

Iteration=15

Iteration=30
Co-Evolution Dynamics: Karate Club

- Only two nodes: #10 and #33 have initial (opposite) opinions.
- All edges start with small positive weights.
Co-Evolution Dynamics: Political Blog Network

- 20% nodes carry ground truth opinions.
- All edges start with small positive weights.

Avg 97.21% prediction accuracy.
Summary and Acknowledgement

- Modeling: social media platforms.
- Algorithmic perspective: promote truth learning, reduce polarization
Acknowledgement

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