Sections 11.1-2: Parametric Curves - Worksheet

#100. Find an equation of the tangent line to the given parametric curve at the point defined by the given value of t.

(a) $\begin{cases} x = 5t^2 - 7 \\ y = t^4 - 3t \end{cases}$, $t = -1$

(b) $\begin{cases} x = e^{4t} - e^t + 2 \\ y = t - 3e^{2t} \end{cases}$, $t = 0$.

(c) $\begin{cases} x = \sec(3t) \\ y = \cot(2t - \pi) \end{cases}$, $t = \frac{\pi}{12}$.

(d) $\begin{cases} x = \arcsin(2t) \\ y = \arctan(4t) \end{cases}$, $t = \frac{1}{4}$.

#101. Find all points on the following parametric curves where the tangent line is (i) horizontal, and (ii) vertical.

(a) $\begin{cases} x = t^4 - 8t^2 \\ y = t^2 - 6t \end{cases}$

(b) $\begin{cases} x = \sin(2t) + 1 \\ y = \cos(t) \end{cases}$, $0 \leqslant t < 2\pi$.

(c) $\begin{cases} x = 3t - t^3 \\ y = t^2 + 4t + 3 \end{cases}$

(d) $\begin{cases} x = 4t - e^{2t} \\ y = t^2 - 18\ln|t| \end{cases}$

#102. Consider the ellipse of equation $x^2 + 4y^2 = 4$.

(a) Find a parametrization of the ellipse.

(b) Find the area enclosed by the ellipse.

(c) Find the area of the surface obtained by revolving the top-half of the ellipse about the x-axis.

#103. For each of the following parametric curves: (i) find the arc length, (ii) set-up (but do not evaluate) an integral that computes the area of the surface obtained by revolving the curve about the x-axis and (iii) set-up (but do not evaluate) an integral that computes the area of the surface obtained by revolving the curve about the y-axis.

(a) $\begin{cases} x = e^{4t} \\ y = e^{5t} \end{cases}$, $0 \leqslant t \leqslant 1$.

(b) $\begin{cases} x = \ln(t) \\ y = \sin^{-1}(t) \end{cases}$, $\frac{1}{2} \leqslant t \leqslant \frac{1}{\sqrt{2}}$.

(c) $\begin{cases} x = t^3 - t \\ y = \sqrt{3}t^2 \end{cases}$, $0 \leqslant t \leqslant 1$.

(d) $\begin{cases} x = \ln(t) \\ y = t + \dfrac{1}{4t} \end{cases}$, $1 \leqslant t \leqslant 2$.

(e) $\begin{cases} x = \tan^{-1}(t) \\ y = \frac{1}{2}\ln(t^2 + 1) \end{cases}$, $0 \leqslant t \leqslant 1$.

(f) $\begin{cases} x = t + \sin(t) \\ y = 1 - \cos(t) \end{cases}$, $\pi \leqslant t \leqslant 2\pi$.

#104. Consider the parametric curve $\begin{cases} x = \sqrt{4 - t} \\ y = \sqrt{4 + t} \end{cases}$, $0 \leqslant t \leqslant 2$.

(a) Find the length of the curve.

(b) Find the area of the surface obtained by revolving the curve about the y-axis.

#105. Consider the parametric curve $x = \sin^3(4t)$, $y = \cos^3(4t)$, $0 \leqslant t \leqslant \frac{\pi}{8}$.

(a) Find an equation of the tangent line to the curve at the point corresponding to $t = \frac{\pi}{16}$.

(b) Find the length of the curve.

(c) Find the area of the surface obtained by revolving the curve about the x-axis.

(d) Find the area of the surface obtained by revolving the curve about the line $x = -2$.

#106. Consider the parametric curve $x = \ln(\sec(t) + \tan(t))$, $y = \sec(t)$, $0 \leqslant t \leqslant \frac{\pi}{4}$.

(a) Find the length of the curve.

(b) Find the area of the surface obtained by revolving the curve about the y-axis.

#107. Consider the curve with parametric equations $x = 2t$, $y = 3\ln(t) + 2$, $\dfrac{3}{2} \leqslant t \leqslant 3$.

(a) Calculate the length of the curve.

(b) Calculate the area of the surface of revolution obtained by revolving the curve about the y-axis.

(c) Set-up (but do not evaluate) an integral that computes the area of the surface of revolution obtained by revolving the curve about the x-axis.