How *a priori* should chances be in Statistical Mechanics?

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Outline of talk:

1. My account of Humean objective chance (HOC) & its application to classical SM
2. [quickly] The least *a priori* approach: frequentism. (Hemmo & Shenker 2012)
3. W. Myrvold's *epistemic chance* account
4. HOC as the Goldilocks level of aprioricity?
5. Which HOC?

Statistical Mechanics

- Classical Statistical Mechanics (CSM) seeks to derive or explain TD-like behavior of macroscopic systems
  - [Starting from classical Newtonian mechanics.]

- Core idea: TD-like behaviour is not guaranteed, ever; instead we try to show that it is how systems will behave, *with very high probability.*
  - But given the underlying deterministic mechanics, what can *'probability'* mean here?

Three desiderata

1. SM probabilities should be well-defined.
2. SM probabilities should be able to underwrite - in at least a *weak* sense - real uses of SM in predicting how systems behave (e.g. approach to equilibrium, fluctuations, etc.) and with what frequencies.
3. SM probabilities should not make disastrous retrodictions about the past.

Aims of Statistical Mechanics

Derive the behavior of systems like gases, in particular the Law of approach to equilibrium and their high-probability conformity with the Second Law of Thermodynamics, in terms of the dynamical laws that govern the behavior of the system's micro constituents

Boltzmann's Law (BL). Assume that at time $t = t_0$ the Boltzmann entropy $S_B(t_0)$ of an isolated system is low. It is then *highly probable* that at any time at $t_1 > t_0$ we have $S_B(t_1) > S_B(t_0)$.

Challenge: elucidate what is meant by 'highly probable', and argue that BL is true.

The Puzzles

- What does “probability” mean here?
- How can these probabilities be *objective* probabilities, if CSM is based on a probability-free, deterministic theory?
- If these probabilities are *not* objective, how can they explain why we can *rely on* TD-like behaviour?

3 ideas to explore

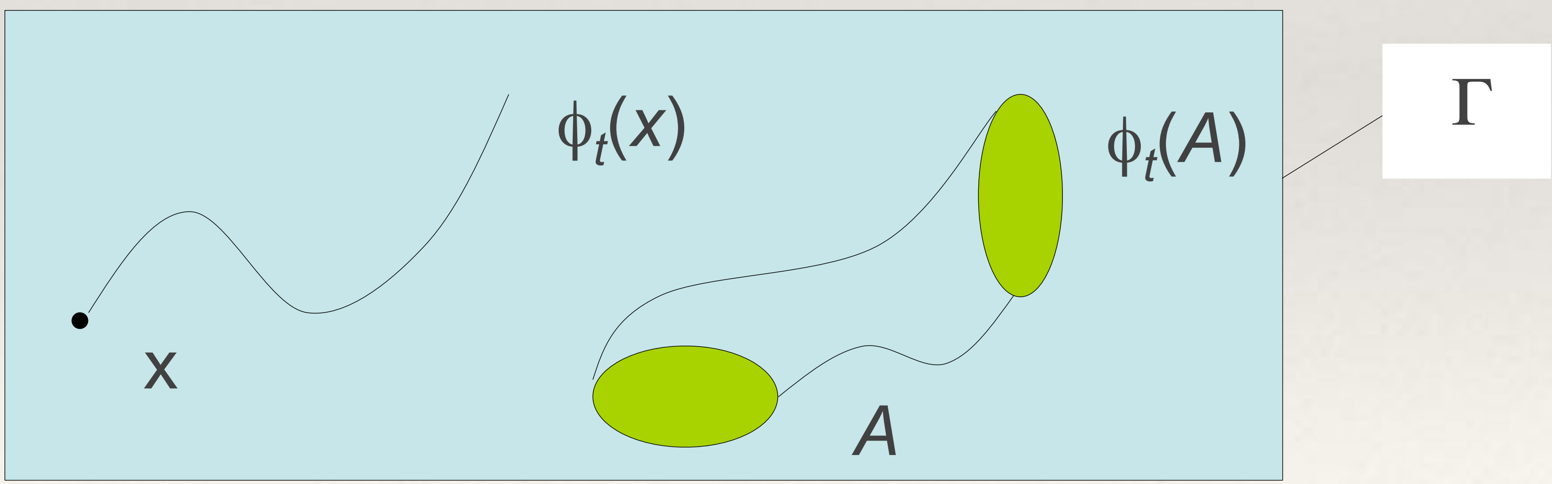
- 1) SM probabilities are Humean Objective Chances (Frigg & Hoefer; Albert & Loewer)
- 2) SM probabilities are determined by (actual) *frequencies of macrostate transitions* (Hemmo & Shenker)
- 3) SM probabilities are *epistemic chances* (Myrvold)

Boltzmannian Statistical Mechanics

Fine grained state: a point x in Γ, the 6n-dimesional phase space of the system.

Measure: Γ is equipped with the 'standard' Lebesgue measure μ.

Dynamics: measure-preserving automorphism on the phase space: ϕ_t: $\Gamma \rightarrow \Gamma$, given by Hamilton's equations of motion.



- **Macrostates**: $M_i \subseteq \Gamma$, $i = 1, \ldots, k$, where the M_i form a partition of Γ.

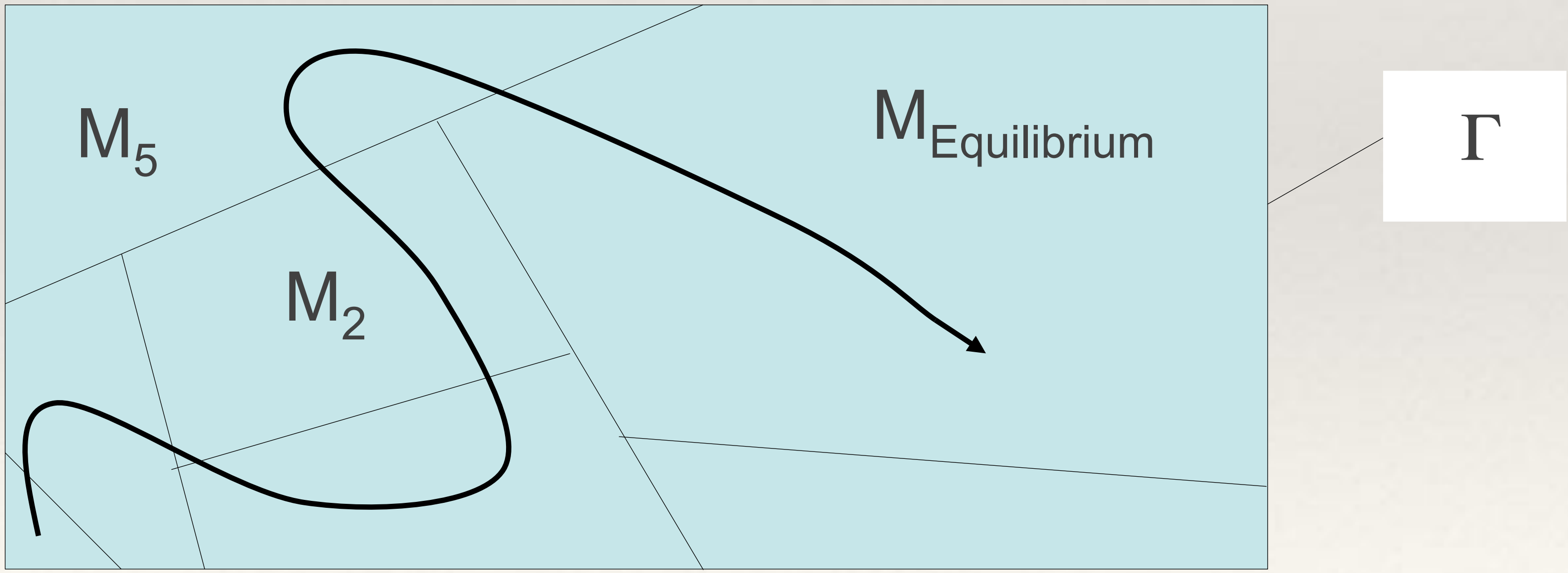
- **Boltzmann Entropy of a macrostate**:

$$S_B(M_i) := k \log[\mu(M_i)].$$

- **Boltzmann Entropy of the system**:

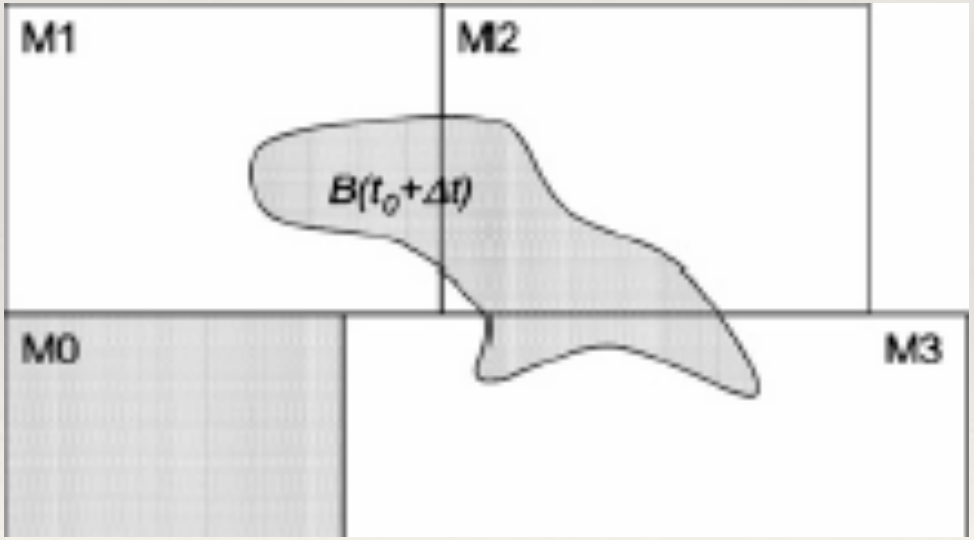
$$S_B(t) := k \log[\mu(M_t)],$$

- where M_t is the system's macrostate at time t.
- **Equilibrium**: the largest macro state.
- **Approach to equilibrium**: is understood as the motion from a small to a large region.



Evolution of a macrostate

- Macrostate M_0 evolves under ϕ_t into $B(t_0 + \Delta t)$.
- Any *measure* or *probability distribution* over M_0 evolves into a new measure over $B(t_0 + \Delta t)$.



Enter Probability

The fundamental postulate introducing probabilities is a "static" probability rule. Suppose the system is in macrostate M_i, and A is a measurable subset of M_i. Then

$$SP: \quad \mathrm{P}(x \in A) = \mu(A) \, / \, \mu(M_i) \, ,$$

In words: the probability that the system's microstate is in A is equal to the Lebesgue measure of A normed to the measure of the macrostate M_i.

Static to Dynamic probs

One could take these probabilities as literally correct. But they are in any case not directly testable.

Alternative: use them to define macro-state transition probabilities.

Let M_{t+} be the set of all points in M_t which will evolve under ϕ_t to a macrostate of higher entropy after time Δt.

Then the probability that the system evolves to a state of higher entropy after time Δt is given by:

P(entropy ↑) = $\mu(M_{t+})$ / $\mu(M_t)$.

One goal of SM is to show, for macrostates M_t that are not equilibrium states, that this probability is extremely high!

Note 1: these probabilities are about *measurable* state-transitions, observable events.

Note 2: After this is where things begin to go wrong, due to reversibility objections . . .

Past Hypothesis

We need: to be able to say that increase in entropy *toward the future* is highly probable, but that increase in entropy *toward the past* is **not**.

Past Hypothesis: stipulate that the system of interest came into being in a state of very low entropy.

The system of interest may be *the universe* (Albert, Loewer), or a “branch system” such as thermos-box of gas out in space (starting out in some low-entropy state such as all particles confined to the left half of the box ...).

Past Hypothesis Statistical Postulate

- Let M_t be the system's macro-state at time t. SP is valid for the Past State M_p , which obtains at time t_0 . For all times $t > t_0$ the probability at time t that the system's micro-state lies in A is
- $P_t(A) = \mu(A \cap R_t) \,/\, \mu(R_t)$
- where $R_t := M_t \cap \phi_{t-t0}(M_p)$.

- **In words:** the probability of being in *A* is the proportion of the current macrostate that overlaps both *A and* the [blob of the] Past State.

Summary: standard (Boltzmannian) probs in CSM

- For *initial* macrostate of system M_p, the probability of being in a sub-region A is given by the Lebesgue measure of A relative to the macrostate.
- For later times, if system observed to be in macrostate M_i then probability of being in region Q in M_i given by the L-measure of the overlap of Q with *the sub-region of* M_i *that evolved* ***from*** M_p
- And, the probability of being in macrostate M_i at time $t_0 + \Delta t$ is given by the L-measure of the intersection of blob $B(t_0 + \Delta t)$ with M_i relative to the L-measure of B as a whole.
    - *All of which does not tell us what 'probability'* means *here.*

How to interpret CSM Probs?

Now we'll look at four ways to attempt this:

- Hoefer-Frigg *pragmatic Humean Best System* account
- Loewer & Albert's Mentaculus/imperialist HOC Hemmo & Shenker's *frequentism*
- Myrvold's *epistemic chance*

HOC in general

1) The Principal Principle (PP) tells us most of what we know about objective chance

2) Objective chances are not primitive modal facts, powers or propensities, but rather facts entailed by the patterns in the events and processes in the actual world (often called the "Humean Mosaic"). (Humean Supervenience)

See: Lewis (1986, 1994); Hoefer (2019)

HOC in general

- The *objective chances* in a world are the chance-rules (or “laws”) that are to be found in a “Best System” of such rules:
  - The Best System is extracted from the entire history of actual events in the world (“Humean Mosaic”)
  - The Best System has the best combination of 3 qualities: *simplicity, strength* and *fit with the actual event patterns*.
    - Humean: no modality or powers in Humean Mosaic.

Hoefer (& Frigg) HOC

- In tradition of Lewisian Best Systems accounts of laws and chances, **but:**
- Best System account of *chances only* (leave laws to one side)
- Pragmatic: user-friendly, not seeking 1 or few, fundamental-physical, chance rules.

Hoefer (& Frigg) HOC

- Chances are Pr(outcomes in setups) - chance rules, not "laws"
- Chances are [entailed by] patterns in the actual events in the full Humean mosaic (all levels of scale);
- Patterns such as to make the adoption of credences identical to the chances rational, in the absence of better information
- Best System account of chances, not (necessarily) laws.

Positive account cont'd

- *Stochasticity Postulate*: it is a fortunate fact about events in our world that, all over the place, at all sorts of levels (but especially at relatively micro-levels), nicely stochastic- looking distributions of events are to be found.
- Some OC.s are explicable as entailed by the structure of the chance setup + stochastic-lookingness of inputs (I.C.s and B.C.s) from the micro-level: *Stochastic Nomological Machines*.
    - *Note close relation to Myrvold's epistemic chance*
- Chances comprise a Best System; **analogy:** advice of the benevolent Super-scientist.

Summing up

- Chances are **constituted** by the existence of patterns in the Humean mosaic of events in the world. These patterns are such as to make the adoption of credences identical to the chances rational in the absence of better information, if one is obliged to make guesses or bets concerning the outcomes of chance set-ups. One aspect of the mosaic of events in our world is this: It is a fact about actual events that, at many levels of scale (but especially micro-scale), events look "stochastic" or "random", with a certain stable distribution over time. Some stable, macrocscopic chances that supervene on the overall pattern are explicable as regularities guaranteed by the structure of the assumed chance set-up, together with this micro- stochastic-lookingness (SP). Not all genuine objective chances have to be derivable from the SP, however. The right sort of stability and randomness of outcome-distribution, over all history, for a well-defined chance set-up, is enough to constitute an objective chance. Moreover, set-ups with few actual outcomes, but the right sort of similarities to other set-ups having clear objective chances (e.g. symmetries, similar dynamics, etc.) can be ascribed objective chances also: these chances supervene on aspects of the Humean mosaic at a higher level of abstraction than just the outcome frequencies. The full set of objective chances in our world is thus a sort of Best System of many kinds of chances, at various levels of scale and with varying kinds of support in the Humean base. What unifies all the chances is their ability to play the role of guiding credence, as codified in the Principal Principle.

Hoefer-Frigg approach

- HOC Best System candidates arguably **may** contain macrostate transition chances, supervening on *macro-level facts alone* (as do the simple coin-flip chances)
- Best System candidates may also contain *static* probabilities (like PHSP), allowing *micro-derived* macrostate transition chances, as seen above (possibly restricted to certain kinds of systems)

If a candidate contains both, the two should be consistent (enough).

Hoefer-Frigg approach

Could a good candidate contain a micro-level-based chance rule (like BSM's), *independent* of whether the Mosaic contains particle trajectories?

- The micro-level-based chance rule may be incorporated into the System *instrumentalistically,* to allow assignment of macrostate transition chances (pragmatic/instrumentalist approach)
- Its *simplicity* and/or *fit* may far outstrip the macro-derived chances.

Does a Best System for our world contain BSM-type chances?

- Arguably, *yes*. Evidence is hard to come by, but not impossible; e.g.: perhaps studies of fluctuations and/or approach-to-equilibrium for low-*N* dilute gases . . .
- In other words: whatever empirical evidence in the ordinary sense supports BSM, equally supports the existence of BSM chances in the Humean Best System
- *Flexibility* of the Pragmatic Best System approach makes successful capture of SM probabilities plausible

Albert & Loewer's approach to SM chances

- Suppose (or pretend) that the world is fundamentally "governed" by deterministic laws such as those of classical mechanics
- Suppose further that it is *true* that a very early state of the world can be said to have been a *very low-entropy macrostate*, the 'past state'
- A Lewisian Best System for such a world could be much stronger, without much loss of simplicity, by adding the PHSP as applied to the whole universe.
- <u>Mentaculus claim</u>: From these posits, arguably all the chances we might want, AND all the laws of the special sciences, would flow as consequences.

Albert & Loewer's approach to SM chances

- Suppose (or pretend) that the world is fundamentally "governed" by deterministic laws such as those of classical mechanics
    - Unless Bohmian mechanics somehow is true, we know this can't be right in a literal sense. But Lewisian laws are supposed to be truths about the *actual* Mosaic, not idealisations thereof.
- Suppose further that it is *true* that a very early state of the world can be said to have been a *very low-entropy macrostate*, the 'past state'
    - Even in CM worlds, if they have gravity and are infinite, it is not clear that one can apply the Boltzmannian framework coherently

Albert & Loewer's approach to SM chances

- A Lewisian Best System for such a world could be much stronger, without much loss of simplicity, by adding the PHSP as applied to the whole universe.
  - As Frigg and I explicit *simplicity*, this is not true; calculation simplicity counts for something, and nobody can calculate anything starting from a universal state. Nor can it be proven mathematically (I think) that we can count (with high probability) on local branch systems behaving *as if* a local past state + Lebesgue measure were true.
- Mentaculus claim: From these posits, arguably all the chances we might want, AND all the laws of the special sciences, would flow as consequences.

Hemmo & Shenker: *frequentism*

- Reject *a priori* measures in CSM as representing real probabilities (Lebesgue; 'typicality' measures).
- Macrostate transitions primary; their *observed frequencies* determine correct measure μ
  - H & S: "When a measure is chosen in order to calculate the transition probability from an initial macrostate [M_0] to a given macrostate [M_i] after a given time interval t, according to the Probability Rule … the measure is chosen on the empirical basis of observed transition probabilities." (2012 p. 156.)

Hemmo & Shenker: *frequentism*

- $P(M_1, t_1 \mid M_0, t_0) = \mu(B(t_1) \cap M_1)$
- Macrostate transitions primary; their *observed frequencies* determine correct measure μ
  - *Past* frequencies - not necessarily observed by us. (?)
  - Mathematical technique needed for finding a reasonable μ based on frequency data;
  - *Generally we won't have static probabilities… unless the empirically determined μ happens to always turn out to match (e.g.) Lebesgue measure.*

H&S frequentism: concerns

- Need *lots* of identical copies of CSM systems...
  - ... for *each distinct phase space?*
    - 1x3.2m box, energy E , 10^{23} molecules;
    - 1x3.2m box, energy E , $10^{23} + 11$ molecules;
    - 2x2x2m box, energy E + 3 erg, 10^{23} - 36 molecules;
    - . . .
  - world unlikely to oblige

H&S frequentism: concerns

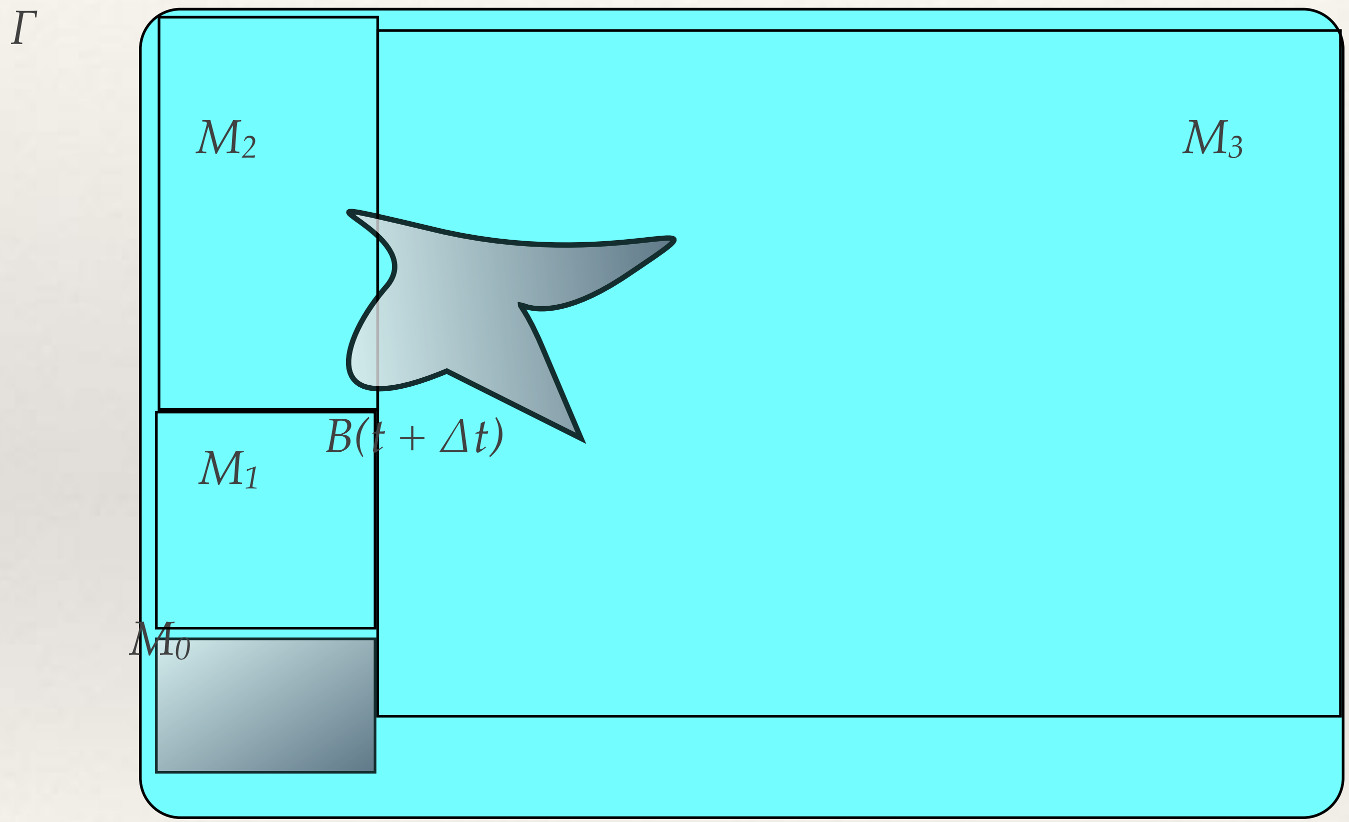
- Need *lots* of identical copies of CSM systems…
  - ◇ Response: ‘extend’ measure μ from a phase space with plenty of examples (good frequencies) to ‘nearby’ phase spaces.
  - If we find a recipe to extend results from one phase space to a ‘nearby neighbour’, what do we do if ∃ two or more neighbours, and the recipe yields conflicting extensions?
  - ⟹ Need to go very pragmatic, look for meta- or high-level frequency *patterns* … 3/4 way to my account.

H&S frequentism: concerns

- more concerns:
  - World could give us *too many* (∞) copies, ⟹ frequencies always undefined (∞/∞), or 0
  - No probabilities in early days of universe
    - Possibly solvable by helping oneself to frequencies over *all* universe-history -- again, bringing one closer to my account.
  - Further technical questions regarding Δt intervals & more...

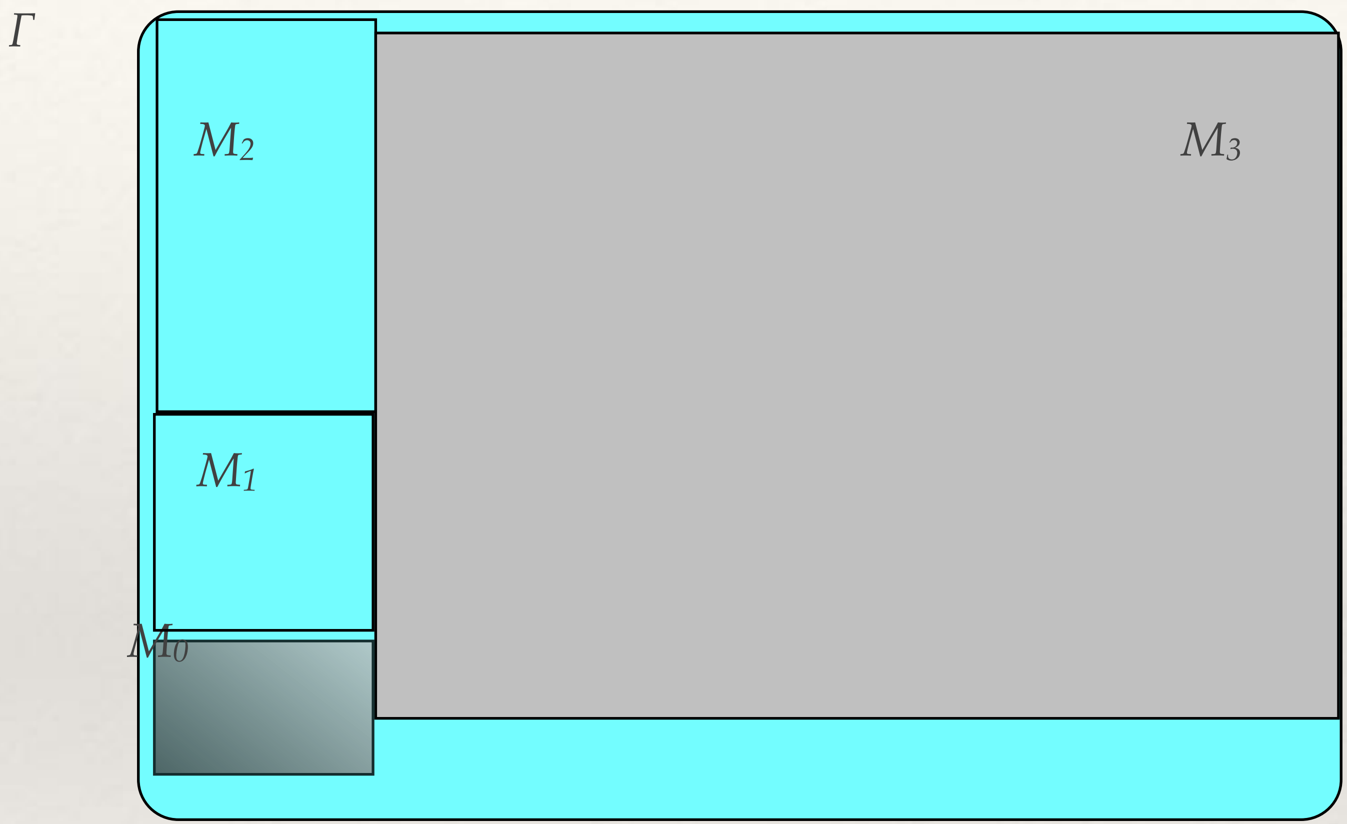
Myrvold: *epistemic chance*

- Imagine a class C of *reasonable subjective initial credence distributions* on initial macrostate M_0



Myrvold: *epistemic chance*

- Because of the nature of the dynamics (ergodic? mixing?) after evolving forward the members of C *long enough*, they will all make *essentially* the same predictions *re* measurable properties. (analogy to Poincaré's wheel, method of arbitrary functions).
- C can't actually contain *arbitrary* distributions



Any old distribution becomes densely filibrated in *eq*. macrostate, hence makes same predictions as uniform measure.

Myrvold: *epistemic chance*

- "The ingredients that we start with are:
  - A class C of credence-functions about states of affairs at time t_0 that is the class of credences that a reasonable agent could have, in light of information that is accessible to the agent;
  - A dynamical map T that maps states at time t_0 to states at time t_1, which induces a map of probability distributions over states at time t_0 to distributions over states at time t_1;
  - A set A of propositions about states of affairs at time t_1, and
  - A tolerance threshold for differences in probabilities below which we regard two probabilities as essentially the same. ...

Myrvold: *epistemic chance*

- Myrvold's proposal in words: If there is a proposition *S* about future events such that the time evolute of any of the distributions given in C assigns *essentially the same* probability to *S*, then *S* has an *epistemic chance* equal to that probability.

Epistemic chance: virtues

- No claim that all agents should have the same credences over regions of phase space
- Objectivity comes *from the dynamics*
- If *C* is specified carefully, disastrous retrodictions won't arise
- Epistemic element consonant with partly-epistemic nature of TD.

Epistemic chance: concerns & Q's

1. What should count as a "reasonable" IC distribution?
   - Tension: *C* must exclude a lot of possible distributions, but do so
     - without the criterion of exclusion turning into something that looks like: 'anything that would spoil the convergence on propositions *S*.'
     - and without narrowing down *C* to very few members

Epistemic chance: concerns & Q's

1. What should count as a "reasonable" IC distrib.
   - Possible exclusions:
     - *not:* 'distributions that assign high density to micro-regions of phase space that evolve anti-entropically for significant time periods.'
       - looks like cheating;
       - leaves *in 'ultra-thermodynamic'* distributions
     - *not:* arbitrarily odd-shaped distributions (sinusoidal in even-numbered dimensions; Mickey-face-shaped; ...
     - *not:* strongly spiked around small regions;
       - we <u>need</u> those shapes, to encode behaviour towards past...

Epistemic chance: concerns & Q's

2. What should we say about a CSM-poss. world in which anti-TD behavior happens all over, even *reliably*?

    - Myrvold: "Our judgments about what sorts of processes occur in nature and our judgments about what sorts of credences are reasonable for well-informed agents are closely linked; if there were processes that could reliably prepare systems in states that lead to anti-thermodynamic behaviour, then it would not be unreasonable for an agent to attach non-negligible credence to the system having been prepared in such a state, and we would adjust our judgments about what are and are not reasonable credences accordingly."

Epistemic chance: concerns & Q's

2. What should we say about a CSM-poss. world in which anti-TD behavior happens all over, even *reliably*?

   - Concern: if *our experience of how things go in the world* strongly shapes *what is a reasonable initial credence function*, then... is our set C really doing the "work" of determining the [objective] epistemic chances, or is it instead facts about actual events?
   - Myrvold (in conversation): think of C as involving a 'dialogue' with the events in the world. . .

Epistemic chance: concerns & Q's

2. What should we say about a CSM-possible world in which anti-TD behavior happens all over, even *reliably*?

   - Concern 2: Myrvold's PP-analog principle just says: if you *know* the EC of A is *x* and you have no other, inadmissible evidence, then set your credence in A equal to *x*.
   - In order for the PP_{EC} to not give us horrible advice, we'll have to say that evidence of past widespread anti-entropic behavior is **inadmissible**. But it's just ordinary experience of past events, the very stuff that Lewis and other writers on chance have said must be admissible.
   - Any fix for these problems looks like it's going to *have* to give the world - i.e., contingent facts about frequencies and patterns - a big say in determining the SM chances.

HOC as Goldilocks

- Myrvold's ECs are hostage to the world's ICs being normal. In worlds where they are not, either PP_{EC} becomes a wrong principle (i.e, not one that is compelling as a constraint on rationality), or we save the day by the inadmissibility clause. But to take the latter course and allow the world to teach us by experience is to move a big step in the direction of HOC.
- It is also hostage to getting the class ***C*** of reasonable credence functions specified in a plausible, non-*ad hoc* manner... a problem imposed by wanting to keep everything epistemic and use *only* the dynamics, not physically contingent facts
    - HOC gets you all the epistemicity you need, via PP

HOC as Goldilocks

- Hemmo & Shenker ran into potential problems by giving contingent facts *too much* say, leaving no room for elegant systemization to play a role in determining μ
  - The easy, obvious moves to make, to fix their potential problems, all bring the view closer and closer to HOC

HOC as Goldilocks

- The Mentaculus program is beautiful and breathtakingly ambitious, but is also hostage to our world having the right sort of CM-like dynamics, and everything working out mathematically right *and also* having good-enough *fit* with the actual Mosaic
    - The way to alleviate these concerns is to sacrifice fundamentality of the chance law(s) in exchange for
        - more easily-argued applicability and utility
        - greater flexibility in fitting our Mosaic
        - immunity from the possibility that our world's ontology+laws are not even remotely like CM
    - I.e., go for my/Frigg-style HOC

Conclusions

- HOC can be seen as incorporating key elements of both epistemic chance (*via* SNMs and the PP) and frequentism
- On the other hand, epistemic chance and frequentism each can be seen as *omitting* a crucial element of the right understanding of objective chance, leading to problems
- The Mentaculus program shares those crucial elements with HOC, but pays a price in terms of *utility* and *flexibility* when compared to HOC, to purchase fundamentalist unity.