Coreference, negation, and modal subordination[1]

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Handout

You can find the handout here:
`https://patrl.keybase.pub/handouts/rutgers-handout.pdf`

1 *Roadmap*

> "Very generally, the dynamic view on meaning comes to this: the meaning of a sentence is the change an utterance of it brings about, and the meanings of non-sentential expressions consist in their contributions to this change."
> (Groenendijk, Stokhof & Veltman 1996: p. 3)

In this talk, I'll be discussing presupposition projection and anaphora in dynamic semantics — well-trodden ground indeed.

Broadly, the question I'll be asking is: *what kind of logical properties should our dynamic semantics have*?

The problems I'll raise can be thought of as instances where the mainline approaches to dynamic semantics stray *too far from the classical*.

We'll aim to *reinstate* some classical equivalences, and as we'll see, the result will be (arguably) a simplification of the theory and superior empirical coverage.

This talk has two halves:

- In the first half, I'll motivate a dynamic fragment — DAS* — which validates Double Negation Elimination (DNE). This will be necessary for understanding some basic observations regarding anaphora in disjunctive sentences.
- In the second half, I'll take up the central themes of DAS*, and incorporate them into a non-eliminative update semantics UPD, with the resources for handling epistemic modality and modal subordination.

The treatments of negation and modal subordination will work in tandem to account for some novel data involving anaphora in conjunctive possibility statements.

We'll begin with an overview of what we know about anaphora and presupposition projection in disjunctive sentences.[2]

[2] See also Simons (1996, 2005) for an overview.

2 What we can learn from bathrooms

2.1 Disjunction and presupposition projection

In the literature on presupposition and anaphora, much ink has been spilled on so-called *bathroom sentences*.[3]

[3] Attributed by Roberts (1989: p. 702) to Barbara Partee, p.c.

There are two variants of bathroom sentence: presuppositional and anaphoric. We'll start with the presuppositional case:

(1) Either there is no bathroom, or the bathroom is upstairs.

Note that, despite the fact that the second disjunct contains a presupposition trigger, *the bathroom*, the disjunction as a whole is presuppositionless.

Compare (2a) and (2b):

(2) a. Either this house has been renovated, or the bathroom is upstairs.
    b. Either the bathroom is upstairs, or this house has been renovated.

Neither (2a) nor (2b) are presuppositionless — an assertion of (2a) or (2b) requires accommodation of the information that *there is a bathroom*.[4]

[4] Of course, approaches to presupposition projection in the Heimian tradition predict that (2a) presupposes something weaker — namely, either this house hasn't been renovated, or there is a bathroom. This is the proviso problem, in a familiar guise (Geurts 1996). Since proviso is a problem for the theory of *accommodation*, we'll abstract away from this issue.

These observations regarding what disjunctive sentences presuppose are special cases of the Heim-Karttunen projection generalization for disjunctive sentences.

In the Heimian tradition, the idea is that the second disjunct is interpreted in the context of the *negation* of the first (Beaver 2001).

(3) Either there is no bathroom,
    or (there isn't no bathroom and) the bathroom is upstairs.

2.2 Disjunction and anaphora

As Partee emphasizes, we find a parallel in the domain of *anaphora*.[5]

[5] One worry that we won't address until the very end of the talk, is that presupposition projection is arguably symmetric, whereas anaphora is not. See §5.1.

(4) Either there is no bathroom, or it's upstairs.

One pertinent fact is that disjunctive sentences are typically externally and internally static (Groenendijk & Stokhof 1991, Simons 1996):[6]

(5) # Either Jones owns a^1 bicycle or it$_1$'s broken. Simons 1996: p. 245

(6) # Either Jones owns a^1 bicycle, or he walks. It$_1$'s a cheap one.

It's tempting to offer the same explanation for (4) as we did for (4) — namely, the second disjunct is interpreted in the context of the *negation* of the first:

(7) Either there is no bathroom, or (there isn't no bathroom and) it's upstairs.

This seems like a reasonable explanation, but as we'll discuss, many dynamic theories (Heim 1982, Kamp 1981, Groenendijk & Stokhof 1991) hit a roadblock here.

Essentially, the problem boils down to the invalidity of DNE in first-generation dynamic theories; they erroneously predict anaphora to be unsuccessful in the following:

(8) There isn't no^1 bathroom. It$_1$'s upstairs.

This is a major bug, and is arguably an instance where dynamic semantics strays too far from the classical.

There have already been a number of attempts to fix this bad prediction, departing to a greater or lesser extent from orthodox Dynamic Semantics (DS) — see, e.g., Krahmer & Muskens (1995), Rothschild (2017), Gotham (2019), Mandelkern (2020).

One of our goals today will be to present a fresh take on this issue, directly inspired by Krahmer & Muskens's approach, grounded in Charlow's (2014, 2019a) Dynamic Alternative Semantics (DAS).

2.3 *Possible bathroom sentences*

A variant of bathroom sentences which (to my knowledge) has not been discussed in the literature — *possible bathroom sentences*.

The patterns we observe in Partee's bathroom sentences replicate in conjunctions of modalized sentences. As before, these come in both presuppositional and anaphoric variants.[7]

[6] The demonstration of internal/external staticity is of course contingent on the indefinite taking narrow scope within the disjunction.

[7] If you find the following sentences odd, they improve with the connective *but*. Since *but* has the same projection properties as *and* (and is indeed truth-conditionally equivalent), the same points still hold regardless.

(9) a. It's possible there is no bathroom,
and it's possible the bathroom is upstairs.
b. It's possible there is no bathroom,
and it's possible it's upstairs.

(10) a. Maybe there's no bathroom and maybe the bathroom is upstairs.
b. Maybe there's no bathroom and maybe it's upstairs.

N.b. that epistemic modals are externally static (Groenendijk, Stokhof & Veltman 1996):

(11) # It's possible there's a^1 bathroom. It$_1$'s upstairs.

Possible bathroom sentences pose a serious problem to theories of presupposition projection and anaphora.

To see why, consider the following facts.

Presupposition projection in conjunctive sentences

According to the Heim-Karttunen projection generalizations, the second conjunct in a conjunctive sentence is interpreted in the context of the first conjunct.

This can lead to *local satisfaction* if the first conjunct entails the presupposition of the second.

(12) There's a bathroom in this house, and the bathroom is upstairs.

Presupposition projection with epistemic modals

Epistemic modals are typically assumed to be *holes*; we accommodate that *there is a bathroom* from an assertion of (14).[8]

(14) It's possible [that the bathroom is downstairs].

Taken together, these facts predict that in a sentence of the form $\ulcorner \Diamond\, \phi_\pi \wedge \Diamond\, \psi_\rho \urcorner$, $\Diamond\, \psi_\rho$ presupposes ρ, and is interpreted in the context of $\Diamond\, \phi_\pi$.

Classical theories of presupposition projection therefore just straight up can't account for possible bathroom sentences; we need a richer theory.

I'll suggest that what we don't necessarily need to tweak the theory of presupposition projection, but rather the semantics of epistemic modals.

Our simple-minded translation of an epistemic modal into a sentential oper-

[8] We might suspect that epistemic modals pattern with doxastic attitude verbs, which give rise to conditional presuppositions that are strengthened by the logic of proviso (see Mandelkern 2016, Grove 2019 for recent discussion). This is a little hard to see, because epistemic modals are typically speaker-oriented in English. (13) however seems like a case of local satisfaction, so we probably want to treat epistemic modals as filters rather than holes.

(13) If you think this house has a bathroom,
is it possible that the bathroom is downstairs?

This issue won't affect the logic of the argument here.

ator ignores a crucial factor in the interpretation of modalized sentences — *modal subordination* (Roberts 1989, van Rooij 2005).

Before we manage to address this, to even have a hope of accounting for anaphoric bathroom sentences, we need to address the problem of DNE in DS.

3 *Negation in dynamic semantics*

The solution i'll adopt to deal with DNE in DS is directly inspired by Krahmer & Muskens's (1995) bilateral approach — we'll crucially distinguish between the positive and negative information encoded by a sentence.

In this section, I'll outline a Dynamic Alternative Semantics (DAS) based on Charlow's (2014, 2019a) monadic rendering of Groenendijk & Stokhof (1991).[9] The main difference is as follows:

[9] This is a good fit, since, on independent grounds, Charlow's theory ends up distinguishing between the positive and negative information encoded by a sentence, due to the fact that output assignments are paired with "pure" propositions.

- Given a sentential meaning, first generation dynamic theories such as Groenendijk & Stokhof (1991) distinguish between assignments in/not in the output.
- DAS distinguishes between *true-tagged* assignments and *false-tagged* assignments, in addition to assignments simply absent from the output.

I'll show that DAS, like first-generation dynamic theories, fails to validate DNE, but this will form the basis for our refined fragment.

We'll subsequently tweak DAS in order to re-validate DNE, and derivatively, account for anaphoric bathroom sentences — we'll call the revised fragment DAS*.

We'll subsequently come back to possible bathroom sentences, borrowing the central ideas of DAS* into an update semantics.

A existential statement in DAS, after Charlow (2014):

(15) $$[\![\text{some}^1 \text{ philosopher left}]\!] = \lambda g\ .\ \{\ (\mathsf{left}\ x, g^{[1\to x]}\ \mid \mathsf{philosopher}\ x)\ \}$$

Equivalently:

(16) $$\begin{aligned} = \lambda g\ .\ & \{\ (\top, g^{[1\to x]}) \mid \mathsf{philosopher}\ x \land \mathsf{left}\ x\ \} \\ & \cup \{\ (\bot, g^{[1\to x]}) \mid \mathsf{philosopher}\ x \land \neg\ (\mathsf{left}\ x)\ \} \end{aligned}$$

The extra information encoded in a DAS proposition is encoded in the type signature, which we'll abbreviate as T:[10]

[10] Here, g is the type of assignments, $\{\,.\,\}$ is the constructor for set types, and $*$ is the constructor for pair types, so T is the type of a function from assignments to sets of pairs of truth-values and assignments.

(17) $\mathsf{D} \coloneqq \mathsf{g} \to \{\, \mathsf{g} \,\}$ classical DS

$\mathsf{T} \coloneqq \mathsf{g} \to \{\, \mathsf{t} * \mathsf{g} \,\}$ dynamic alternative semantics

N.b. in discussing DNE, we'll stick with a purely extensional fragment. Later, in order to deal with possible bathroom sentences, we'll introduce possible worlds, but the explanatory strategy for DNE will remain intact.

3.1 *An initial DAS fragment*

Predicates map individuals to dynamic propositions.

(18) $[\![\text{vaped}]\!] \coloneqq \lambda x g \,.\, \{\, (\mathsf{vaped}\ x, g) \,\}$ $\qquad \mathsf{e} \to \mathsf{T}$

Indefinites compose with dynamic predicates, and induce an *indeterminate* output via a set of alternatives.[11]

[11] Charlow (2014, 2019b) *derives* this entry for indefinites via monadic bind. The details of Charlow's analysis, which are important in accounting for *exceptional scope*, are not relevant to the main point here however.

(19) $[\![\text{some}^n \text{ philosopher}]\!] \coloneqq \lambda k g \,.\, \bigcup_{x \in \mathsf{philosopher}} k\ x\ g^{[n \to x]}$ $\qquad (\mathsf{e} \to \mathsf{T}) \to \mathsf{T}$

Pronouns also compose with dynamic predicates, introducing input sensitivity.

(20) $[\![\text{she}_1]\!] \coloneqq \lambda k g \,.\, k\ g_1\ g$ $\qquad (\mathsf{e} \to \mathsf{T}) \to \mathsf{T}$

Conjunction involves feeding the outputs of the first conjunct pointwise into the second, and gathering up the results (i.e., relational composition). The contained truth-values are conjoined.

(21) $\mathsf{and} \coloneqq \lambda m \,.\, \lambda n \,.\, \lambda g \,.\, \{\, (t \wedge u, g'') \mid (u, g'') \in (n, g') \mid (t, g') \in m\ g \,\}$ $\qquad \mathsf{T} \to \mathsf{T} \to \mathsf{T}$

Finally, this brings us to destructive negation in DAS; this is easy to define in a way parallel to negation in orthodox dynamic theories; the outputs of m are existentially closed, meaning that any anaphoric information introduced cannot be passed on further.

(22) $\mathsf{not}\ m \coloneqq \lambda g \,.\, \{\, \neg \exists g' [(\top, g') \in m\ g], g \,\}$ $\qquad \mathsf{T} \to \mathsf{T}$

This of course accounts for the fact that negation renders indefinites inaccessible as antecedents for subsequent pronouns:

(23) #It's not true that any[1] philosopher left. She_1's sitting over there.

But, this entry leads to the familiar problem of DNE — It's already easy to see that *two* negations won't allow for subsequent anaphora.

In fact, the inner negations cancel out, and the result is a closure operator; (24) tags the input *true* if a philosopher vapes, and *false* otherwise, but does not change it.

(24) $[\![\text{it's not true that no philosopher vapes}]\!]$ $\qquad$ T

$$= \lambda g \,.\, \{\, (\exists x[\mathsf{philosopher}\ x \wedge \mathsf{vapes}\ x], g) \,\}$$

This has no hope of being useful in an analysis of bathroom sentences.

3.2 *Why not externally dynamic negation?*

One thing we might want to consider is just making negation externally dynamic. This is easy to define in DAS:

(25) $\mathsf{not_{dy}} \coloneqq \lambda m \,.\, \lambda g \,.\, \{\, (\neg\, t, g') \mid (t, g') \in m\ g \,\}$ $\qquad$ $\mathsf{T} \to \mathsf{T}$

There's an independent reason why having such an entry for negation might be conceptually desirable — it's a straightforward lifting of *classical* negation into a dynamic setting; each truth value t, such that $(t, *)$ is in the output of $m\ g$ is negated, and the results are gathered up.[12]

Unfortunately, this will fail to fulfill one of our empirical desiderata — when an indefinite occurs in the scope of a negative operator, it is inaccessible for subsequent anaphora; with not_{dy}, indefinites always outscope negation.

(28) a. $\mathsf{not_{dy}}\ ([\![\text{some}^1\ \text{philosopher vapes}]\!])$

b. $= \mathsf{not_{dy}}\ (\lambda g \,.\, \{\, (\mathsf{vapes}\ x, g^{[1\to x]}) \mid \mathsf{philosopher}\ x \,\})$

c. $= \lambda g \,.\, \{\, (\neg\, (\mathsf{vapes}\ x), g^{[1\to x]}) \mid \mathsf{philosopher}\ x \,\}$

Perhaps counter-intuitively, I'm going to argue that we can get away with retaining not_{dy} as an entry for natural language negation, but this will necessitate tweaking the semantics of indefinites we've been assuming.

3.3 *A revised dynamic alternative semantics:* DAS*

In this section, we'll revise our initial fragment in some substantive ways; we'll call the new fragment DAS*.

[12] Showing this is a little bit involved, but the basic observation is that T is simply the result of applying a general recipe for dynamic types to t; let's call this *type constructor* S.

(26) $\mathsf{S\ a} \coloneqq \mathsf{g} \to \{\, \mathsf{a} * \mathsf{g} \,\}$

S is a *functor*, which just means that we can lift functions of type $\mathsf{a} \to \mathsf{b}$ into functions of type $\mathsf{S\ a} \to \mathsf{S\ a}$, in a logically well-behaved way. The relevant lifting operation, map, is given below:

(27) $\mathsf{map}\ f\ m \coloneqq \lambda g \,.\, \bigcup_{(x,g')\in m\ g} \{\, (f\ x, g') \,\}$ $\qquad$ $(\mathsf{a} \to \mathsf{b}) \to \mathsf{S\ a} \to \mathsf{S\ b}$

We can derive $[\![\text{not}_{dy}]\!]$ by applying map to classical negation.

Let's think a little more about the properties of our system that give rise to the bad predictions with externally dynamic negation.

Intuitively, this is because the semantics for indefinites doesn't discriminate between true- and false-tagged assignments in the output. Assume that $\mathsf{linguist} = \{\, \mathsf{andy}, \mathsf{dani}, \mathsf{yasu} \,\}$, and only Dani vapes.

(29) a. $[\![\text{some}^1 \text{ linguist vapes}]\!] = \lambda g\ .\ \{\, (\mathsf{vapes}\ x, g^{[1\to x]}) \mid \mathsf{linguist}\ x \,\}$
b. $= \lambda g\ .\ \{\, (\bot, g^{[1\to a]}), (\top, g^{[1\to d]}), (\bot, g^{[1\to y]}) \,\}$

In a positive context, the presence of the false-tagged assignments doesn't matter. To be more concrete, this is because, if we conceive of a context c as a set of assignments, the update rule in DAS is as follows:

Definition 3.1 (Update in DAS)**.** Given a set of assignments c, and a sentence ϕ, the update of c by ϕ, written $c[\phi]$ is defined as follows:

$$c[\phi] \coloneqq \bigcup_{g\in c} \{\, g' \mid (\top, g') \in [\![\phi]\!]\ g \,\}$$

Externally dynamic negation simply flips the polarity of the output assignments, resulting in a linguist Discourse Referent (DR) who doesn't vape:

(30) a. $\mathsf{not}_{\mathsf{dy}}\,([\![\text{a}^1 \text{ linguist vapes}]\!]) = \lambda g\ .\ \{\, (\neg\,(\mathsf{vapes}\ x), g^{[1\to x]}) \mid \mathsf{linguist}\ x \,\}$
b. $= \{\, (\top, g^{[1\to a]}), (\bot, g^{[1\to d]}), (\top, g^{[1\to y]}) \,\}$

We can address this problem by refining the semantics of indefinites, such that they only introduce discourse referent in what we can informally call the positive dimension.

In order to do this, we'll introduce a couple of derivative notions.

Positive and negative extension

Positive and negative extension operators simply filter out the false- and true-tagged assignments from an output set, respectively.[13]

(31) Positive and negative extension (def)
a. $p^+ \coloneqq \{\, (\top, g') \mid (\top, g') \in p \,\}$ $\qquad + : \{\, \mathtt{g}, \mathtt{t} \,\} \to \{\, \mathtt{g}, \mathtt{t} \,\}$
b. $p^- \coloneqq \{\, (\bot, g') \mid (\bot, g') \in p \,\}$ $\qquad - : \{\, \mathtt{g}, \mathtt{t} \,\} \to \{\, \mathtt{g}, \mathtt{t} \,\}$

Positive collapse

All of the heavy-lifting in the revised semantics for inefinites will be performed by the *positive collapse* operator:

[13] We'll only be making use of the positive extension for now, but the negative extension will come in handy for giving a concise semantics for disjunction.

Positive collapse

The positive collapse operations takes a DAS* proposition m, and gives back a new proposition which just returns that true-tagged assignments in $m\ g$ if there are any, and $(\bot, g)$ otherwise.

$$m^{\dagger} \coloneqq \lambda g\ .\begin{cases}(m\ g)^{+} & (m\ g)^{+} \neq \varnothing \\ \{\,(\bot, g)\,\} & \text{otherwise}\end{cases} \qquad \mathsf{T} \rightarrow \mathsf{T}$$

Indefinites redefined

We can now redefine indefinites in terms of our original semantics + positive collapse.

Indefinites in DAS*

Indefinites in DAS* have the same semantics as indefinites in DAS, only after the indefinite has composed with its scope k, the resulting DAS* proposition is subject to positive collapse.

$$[\![\text{some}^{1}\ \text{linguist}]\!] = \lambda k\ .\left(\lambda g\ .\bigcup_{x \in \mathsf{ling}} k\ x\ g^{[1 \rightarrow x]}\right)^{\dagger}$$

The revised entry in action

Let's first check that the revised entry accounts for the same data as destructive negation was designed to capture — namely, the fact that negation renders indefinites inaccessible for subsequent anaphora.

- $\mathsf{linguists} = \{\,\mathsf{andy}, \mathsf{dani}, \mathsf{paul}\,\}$
- Only Andy and Dani vape.

The revised semantics for indefinites makes no difference in a positive context, since false-tagged assignments have no effect on update anyway.

(32) a. $[\![\text{some}^{1}\ \text{linguist vapes}]\!] = (\lambda g\ .\ \{\,(\top, g^{[1\rightarrow a]}), (\top, g^{[1\rightarrow d]}), (\bot, g^{[1\rightarrow y]})\,\})^{\dagger}$
b. $= \lambda g\ .\ \{\,(\top, g^{[1\rightarrow a]}), (\top, g^{[1\rightarrow d]})\,\}$

Now, if we apply *externally dynamic negation*, this time we fail to erroneously introduce a DR, since there are no false-tagged assignments to be flipped; in fact, the sentence is predicted to be false in the given context.[14]

[14] A sentence ϕ is g*−true* in DAS* iff $\exists g'[(\top, g') \in [\![\phi]\!]\ g]$

(33) $\mathsf{not}_{\mathsf{dy}}\,(\llbracket \text{some}^1 \text{ linguist vapes} \rrbracket = \lambda g\,.\,\{\,(\bot, g^{[1\to a]}), (\bot, g^{[1\to d]})\,\})$

If the set of linguists is the same, and nobody vapes, we predict the existential statement to be false, and the negated statement to be true, without introducing any DRS. So far so good!

(34) *Context: nobody vapes*

a. $\llbracket \text{some}^1 \text{ linguist vapes} \rrbracket = \lambda g\,.\,\{\,(\bot, g)\,\}$

b. $\mathsf{not}_{\mathsf{dy}}\,(\llbracket \text{some}^1 \text{ linguist vapes} \rrbracket) = \lambda g\,.\,\{\,(\top, g)\,\}$

Now the moment of truth; if only Andy and Dani vape, a second negation flips the polarity of the false-tagged assignments, thus *re*-introducing the DR.

(35) *Only Andy and Dani vape*

a. $\llbracket \text{some}^1 \text{ linguist vapes} \rrbracket = \lambda g\,.\,\{\,(\top, g^{[1\to a]}), (\top, g^{[1\to d]})\,\}$

b. $\mathsf{not}_{\mathsf{dy}}\,(\llbracket \text{some}^1 \text{ linguist vapes} \rrbracket) = \lambda g\,.\,\{\,(\bot, g^{[1\to a]}), (\bot, g^{[1\to d]})\,\}$

c. $\mathit{not}_{\mathsf{dy}}\,((\mathsf{not}_{\mathsf{dy}}\ \llbracket \text{some}^1 \text{ linguist vapes} \rrbracket)) = \lambda g\,.\,\{\,(\top, g^{[1\to a]}), (\top, g^{[1\to d]})\,\}$

We've a dynamic theory in which (i) negation roofs the dynamic scope of indefinites, and (ii) double-negation elimination is validated![15]

[15] In order to maintain sensible predictions for conjunctive sentences, conjnction must be redefined in terms of positive collapse. See the first appendix for details.

Disjunction and anaphoric bathroom sentences

In order to give a concise entry for disjunction which accounts for bathroom sentences, as below, we'll define the negative counterpart of positive collapse — *negative collapse*.

(36) Either there's no^1 bathroom or it$_1$'s upstairs.

Negative collapse

Negative collapse retains false-tagged outputs, if there are any, and if not returns the input assignment paired with *true*.

$$m^{\ddagger} := \begin{cases} (m\ g)^{-} & (m\ g)^{-} \coloneqq \varnothing \\ \{\,(\top, g)\,\} & \text{otherwise} \end{cases} \qquad \mathsf{T} \to \mathsf{T}$$

We can now give a concise entry for disjunction which accounts for bathroom sentences:

Disjunction in DAS*

The entry below takes the union of (i) the output of the first disjunct, and (ii) the result of feeding the output of the *negative collapse* of the first disjunct into the second disjunct pointwise, and gathering up the results.

$$\mathsf{or} \coloneqq \lambda n \,.\, \lambda m \,.\, \lambda g \,.\, m\, g \cup \bigcup_{g' \in m^{\ddagger}\, g} n\, g' \qquad \mathsf{T} \to \mathsf{T} \to \mathsf{T}$$

Note that this correctly predicts that disjunction is internally static, since only false-tagged assignments are fed into the second disjunct.

(37) #Either there's a^1 bathroom or it$_1$'s upstairs.

Now the main scenario of interest:

(38) Either there's no^1 bathroom or it$_1$'s upstairs.

(39) Context: *There is exactly one bathroom* (b) *and it's upstairs.*
a. $[\![\text{there isn't a bathroom}^1]\!]\ g_{\varnothing} = \{\, (\bot, [1 \to \mathsf{b}])\,\}$
b. $[\![\text{there isn't a bathroom}^1]\!]^{\ddagger}\ g_{\varnothing} = \{\, (\bot, [1 \to \mathsf{b}])\,\}$
c. $[\![\text{it}_1\text{'s upstairs}]\!]\ [1 \to \mathsf{b}] = \{\, (\top, [1 \to \mathsf{b}])\,\}$
d. $([\![\text{there isn't a bathroom}^1]\!]\ \ ([\![\text{or}]\!]\ \ [\![\text{it}_1\text{'s upstairs}]\!]))\ g_{\varnothing}$
$= \{\, (t, g) \mid (t, g) \in (\{\, (\bot, [1 \to \mathsf{b}])\,\} \cup \{\, (\top, [1 \to \mathsf{b}])\,\})\,\}$
$= \{\, (\bot, [1 \to \mathsf{b}]), (\top, [1 \to \mathsf{b}])\,\}$

Note that we predict that *negating* the disjunctive sentence in this context should reintroduce a bathroom DR. This seems correct:

(40) It's false that there's neither a^1 bathroom, nor is it$_1$ upstairs.
It$_1$'s (in fact) in the basement.

We've resolved the problem of anaphoric bathroom sentences using a fairly austere dynamic semantics, minimally extending Groenendijk & Stokhof 1991.

In the next section, we'll start talking about how to lift the central ideas of DAS* into an update semantics.

4 *Epistemic modals and possible bathroom sentences*

DAS*, while accounting for anaphoric possible bathroom sentences, doesn't have the expressive power to deal with epistemic modals.

An influential account of epistemic modals in DS: Veltman's (1996) *test semantics*.

Veltman's idea: a modalized sentence such as (41) tentatively updates the context set with the information that *it's raining*; if the update results in a non-absurd information state, the test is passed, and the original context set is returned.

(41) It might be raining.

Independent motivation for a test semantics — it accounts for so-called *epistemic contradictions* (see also Yalcin 2007).

(42) # It might be raining and it's not raining.

(43) # It's not raining and it might be raining.

Veltman shows that we require the expressive power of an *update semantics* to give a test semantics for epistemic modals, i.e., a dynamic semantics in which sentences denote transitions between information states.

In the following, I'll show how to upgrade DAS* into a fragment I'll call Updating Dynamic Semantics (UPD) (inspired by Groenendijk, Stokhof & Veltman) while keeping the account of anaphoric bathroom sentences intact.

4.1 *From DAS* to UPD*

In DAS*, a dynamic proposition was a function from an input assignment, to a set of truth-value-assignment pairs.

In UPD, a dynamic proposition is a function from a *assignment-context-set* pair, to a truth-value-world-assignment tuple.

(44)

$\mathtt{D}$	$\coloneqq \mathtt{g} \to \{\, \mathtt{g} \,\}$	classical DS
$\mathtt{T}$	$\coloneqq \mathtt{g} \to \{\, \mathtt{t} * \mathtt{g} \,\}$	dynamic alternative semantics
$\mathtt{U}$	$\coloneqq (\{\, \mathtt{s} \,\} * \mathtt{g}) \to \{\, \mathtt{t} * (\mathtt{s} * \mathtt{g}) \,\}$	updating dynamic semantics

Here's an example of a sentential meaning in UPD:

(45) $[\![\text{Xavier left}]\!] \coloneqq \lambda(c, g)\,.\;\{\, (\mathsf{left}_w\; x, w, g) \mid w \in c \,\}$ $\quad \mathtt{U}$

To illustrate, we'll frequently apply the update denoted by a given sentence to an idealized *initial state*, consisting of a set of possible worlds representing an ignorance context, and the *initial assignment* $g_\varnothing$.[16]

[16] The initial assignment is the unique assignment $g_\varnothing$ whose domain is the empty set.

In this particular instance, let's assume that the initial context consists of four worlds $c = \{ w_n \mid 0 \leq n < 4 \}$, where Xavier left in the even worlds but not in the odd worlds. The result of applying the update denoted by (45) to the initial state is shown below.

(46) $[\![\text{Xavier left}]\!]\ (c, g_\varnothing) = \{ (\top, w_0, g_\varnothing), (\bot, w_1, g_\varnothing), (\top, w_2, g_\varnothing), (\bot, w_3, g_\varnothing) \}$

Note that this essentially expresses the same information as the corresponding update in classical update semantics (Stalnaker 1976, Veltman 1996). Since there are no indefinites, the update is fully deterministic and hence eliminative.

Negation and existentials in UPD

We maintain our externally dynamic entry for negation, lifted into an UPD setting:

Negation in UPD

Just as in DAS*, negation in UPD is classical, and hence externally dynamic.

$$\begin{aligned} \mathsf{not}_{\mathsf{dy}}\ u &:= \lambda(c, g)\ .\ \{ (\neg\, t, w, g) \mid (t, w, g) \in u\ (c, g) \} \\ \mathsf{not}_{\mathsf{dy}} &: \mathsf{U} \to \mathsf{U} \end{aligned}$$

Existential statements in UPD introduce a DR alongside true-tagged worlds only; this can be cashed out in terms of a Charlow-esque entry for indefinites and a positive closure operator, but we skip the compositional niceties here.

Existential statements in UPD

$$\begin{aligned} [\![\text{someone}^1\ \text{left}]\!] := \lambda(c, g)\ .\ & \{ (\top, w, g^{[1 \to x]}) \mid \mathsf{left}_w\ x, w \in c, x \in \mathsf{dom} \} \\ & \cup \{ (\bot, w, g) \mid \neg\exists x[\mathsf{left}_w\ x], w \in c \} \end{aligned}$$

As the reader can verify for themselves, these entries for existential statements and negation inherit all of the same advantages as their corresponding entries in DAS*; namely, DNE is validated.

Positive update and conjunction

In order to analyze possible bathroom sentences, we'll need to say something about conjunction. This receives a parallel entry to the one in DAS* (see the appendix) — we define it in terms of a derivative notion: the *positive update* operator.[17]

[17] We can of course, also define disjunction in terms of *negative* update, and thereby capture anaphoric bathroom sentences in the current setting.

Positive update

positive update applies an update to an input state, and returns a *set of context-assignment pairs*, corresponding to the different ways in which the input state could be updated.

$$m[u]^+ \coloneqq \bigcup_{(*,g)\in(u\ m)^+} \{ (\{ w \mid (w,g) \in (u\ m)^+ \}, g) \}$$

$$[.]^+ \quad : (\mathsf{c},\mathsf{g}) \to \mathsf{U} \to \{ (\mathsf{c},\mathsf{g}) \}$$

It will be useful to illustrate briefly how positive update works.

- Initial state: $c \coloneqq \{ w_{xy}, w_x, w_y, w_\varnothing \}$, and the initial assignment $g_\varnothing$.
- We'll compute $(c, g_\varnothing)[\text{someone}^1 \text{ left}]^+$

(47) **Step 1: feed the initial state into update**

$$[\![\text{someone}^1 \text{ left}]\!]\ (c, g_\varnothing) = \left\{ \begin{array}{c} (\top, w_{xy}, [1 \to x]), (\top, w_x, [1 \to x]), \\ (\top, w_{xy}, [1 \to y]), (\top, w_y, [1 \to y]), \\ (\bot, w_\varnothing, g_\varnothing) \end{array} \right\}$$

Next we retrieve just the true-tagged world-assignment pairs. Note that assignments are associated with multiple worlds.

(48) **Step 2: retrieve true-tagged world-assignment pairs**

$$(47)^+ = \left\{ \begin{array}{c} (w_{xy}, [1 \to x]), (w_x, [1 \to x]) \\ (w_{xy}, [1 \to y]), (w_y, [1 \to y]) \end{array} \right\}$$

Finally, we return the set of (g', c') pairs, s.t. each world $w \in c'$ is paired with g' in the positive extension. This gives us the result of performing the positive update of "someone left".

(49) **Step 3: pair each assignment with the worlds it's paired with**

$$\bigcup_{(*,g')\in(48)} \{ (\{ w \mid (w,g') \in (48) \}, g') \} = \left\{ \begin{array}{c} (\{ w_{xy}, w_x \}, [1 \to x]) \\ (\{ w_{xy}, w_y \}, [1 \to y]) \end{array} \right\}$$

Note that the way in which a sentence with an indefinite updates an input state is *indeterminate*.

Conjunction can be defined in terms of positive update:

Conjunction in UPD

The definition of conjunction in UPD parallels the one given for DAS*; each state in the output of the positive update by the first conjunct is fed into the second, and the results are gathered up.

$$u \text{ and } u' := \lambda m \,.\, \bigcup_{m' \in m[u]^+} (u'\ m')$$

$$\text{and} : \mathsf{U} \to \mathsf{U} \to \mathsf{U}$$

4.2 *Epistemic modals and modal subordination in UPD*

Veltman's test semantics

Since UPD is non-distributive with respect to *worldly* content, we can straightforwardly import Veltman's test semantics for epistemic modals into the current setting.

Epistemic modals in UPD

An epistemic modal tests whether the positive extension of the contained sentence is empty. If it isn't, the test is passed, and the worlds in the input context are true-tagged; if the test fails, the worlds in the input context are false tagged.

$$\text{poss } u := \lambda(c, g) \,.\, \begin{cases} \{ (\top, w, g) \mid w \in c \} & (u\ (c, g))^+ \neq \varnothing \\ \{ (\bot, w, g) \mid w \in c \} & \text{otherwise} \end{cases} \qquad \mathsf{U} \to \mathsf{U}$$

To illustrate, let's consider our previous initial state $(\{ w_{xy}, w_x, w_y, w_\varnothing \}, g_\varnothing)$.

We'll compute the result of applying the update denoted by (50) to the initial state.

(50) Maybe someone[1] left.

First, we tentatively update the initial state with the complement of the modal.

(51) $$[\![\text{someone}^1 \text{ left}]\!]\ (c, g_\varnothing) = \left\{ \begin{array}{c} (\top, w_{xy}, [1 \to x]), (\top, w_x, [1 \to x]) \\ (\top, w_{xy}, [1 \to y]), (\top, w_y[1 \to y]) \\ (\bot, w_\varnothing, g_\varnothing) \end{array} \right\}$$

Since the positive extension of the resulting output set is clearly non-empty, the modalized sentence simply tags every world in the input context *true*.

(52) $(\text{poss } [\![\text{someone}^1 \text{ left}]\!])\,(c, g_\varnothing)$

$$= \left\{ \quad (\top, w_{xy}, g_\varnothing), (\top, w_x, g_\varnothing), (\top, w_y, g_\varnothing), (\top, w_\varnothing, g_\varnothing) \quad \right\}$$

This treatment of epistemic modals inherits the usual advantages of Veltman's test semantics, i.e., a natural treatment of *epistemic contradictions* (see Veltman 1996, Groenendijk, Stokhof & Veltman 1996 for details).

Modal subordination

Our entry for epistemic modals clearly predicts that they are externally static operators. This is a good prediction:

(53) It's possible that a^1 philosopher is attending this talk.
She$_1$'s sitting at the back.

However, there are certain discourses where anaphora is licensed involving a subsequent modalized assertion; Geurts (2019) refers to such cases as "piggy-back anaphora":

(54) It's possible that a^1 philosopher is attending this talk.
She$_1$ might be sitting at the back.

What is to blame here is clearly *modal subordination* (Roberts 1989) — the *might* is interpreted relative to the information introduced by the complement of *possible*.

As well as being independently necessary, in order to capture anaphora in cases like (54), modal subordination will be crucial in explaining bathroom sentences.

Our system is still not quite expressive enough to deal with modal subordination, but we have almost all the pieces we need.

It's worth quoting Groenendijk, Stokhof & Veltman (1996) directly on this point:[18]

> "To make an analysis like this work the framework needs to be extended in such a way that within the update procedures intermediate hypothetical states are remembered, rather than immediately forgotten. Roughly speaking, if the next sentence is in the indicative mood, such hypothetical states can be removed from memory; if the next sentence is a modal statement, this signals that if such hypothetical states are in memory, they can be put to use, where the particular modality involved, determines the way in which they should be used."
>
> (Groenendijk, Stokhof & Veltman 1996: p. 25)

[18] Despite clearly indicating how to cash out modal subordination in their framework, Groenendijk, Stokhof & Veltman do not actually do so.

Accounting for piggyback anaphora

We'll simply revise our entry for an epistemic modal such that it introduces a discourse referent corresponding to an output of the positive update triggered by the modal's complement.

Epistemic modals as DR introducers

As a first attempt, we'll assume that an epistemic modal indexed n, as well as performing Veltman's test, introduces a DR corresponding to a state in the output of the positive update by the contained sentence.

$$\mathsf{poss}^n\ u$$

$$\coloneqq \lambda(c,g)\ .\ \begin{cases} \{\, (\top, w, g^{[n \to p]}) \mid p \in (c,g)[u]^+, w \in c) \,\} & (u\ (c,g))^+ \neq \varnothing \\ \{\, (\bot, w, g) \mid w \ \in c \,\} & \text{otherwise} \end{cases}$$

This suggests a natural account of modal subordination as involving an *anaphoric* epistemic modal, which tests an input state relative to an update of g_n with the modal's complement, where n is a state DR introduced by a previous epistemic modal.[19]

[19] For convenience, we'll give two separate lexical entries for (i) DR-introducing epistemic modals, and (ii) anaphoric epistemic modals. It is of course possible to accommodate both properties within a single lexical entry, it just gets a little cumbersome.

Anaphoric epistemic modals

An anaphoric epistemic modal is anaphoric on a *state* u; it performs a test on the contained sentence by interpreting it in the context of u.

$$\mathsf{poss}_n\ u \coloneqq \lambda(c,g)\ .\ \begin{cases} \{\, (\top, w, g) \mid w \in c \,\} & (u\ g_n)^+ \neq \varnothing \\ \{\, (\bot, w, g) \mid w \in c \,\} & \text{otherwise} \end{cases}$$

As the reader can verify, this straightforwardly accounts for piggyback anaphora in cases like the following:[20]

(55) someone^1 might^2 be in the room. They$_2$ might$_2$ be sitting down. $\Diamond > \exists$

[20] The careful reader will note that this account of modal subordination only accounts for cases involving successive possibility statements; if, e.g., the second modal has a universal force, this account will not work. The universal modal needs access to the whole update denoted by the complement of the first modal, or alternatively, the conglomerate of states in the output of the positive update. I leave a more realistic account of modal subordination to future work.

Advantages of UPD

Existing theories which use dynamic mechanisms in order to treat modal subordination as anaphora, such as Kibble 1994 and others, face a problem with the sentences of the kind in (56).

This is the correlate of the double-negation problem in the domain of modal subordination.

(56) It's not impossible that anyone is here. They might be in the closet.
.

UPD accounts for such instances of modal subordination, since negation is externally dynamic, assuming the following LF:

(57) Not [not [possible^1 [anyone^2 is here]]]
and [might$_1$ [they$_2$ are in the closet]]

We can also formulate the *bathroom* correlate of this accessibility issue. Modal anaphora is predicted to be unsuccessful here by orthodox theories for exactly the same reason; UPD resolves this problem.

(58) Either it's impossible that anyone is here,
or it's possible that they're in the closet.

(59) [not [possible^1 [anyone^2 is here]]]
or [possible$_1$ [they$_2$ are in the closet]]

Possible bathroom sentences

Here, we'll suggest that possible bathroom sentences such as (60) also involve a kind of modal subordination.

(60) There might^1 be no bathroom, and it might$_1$ be upstairs.

The intuition behind our account will be that the first occurrence of *might* introduces an indeterminate state DR 1, but as well as ranging over states in the *positive* update of the modal's complement, 1 also ranges over states in the *negative* update of the modal's complement.

Epistemic modals in UPD (revised)

The only difference here is that epistemic modals introduce a DR corresponding to a state in *either* the positive or negative update of the contained sentence. We maintain the same treatment of anaphoric epistemic modals as before.

$\mathsf{poss}^n\ u$

$$:= \lambda(c, g)\ .\ \begin{cases} \{\, (\top, w, g^{[1 \to s]}) \mid w \in c, p \in c[u]^+ \cup c[u]^- \,\} & (c, g)[u]^+ \neq \varnothing \\ \{\, (\bot, w, g) \,\} & \text{otherwise} \end{cases}$$

Let's illustrate with a concrete example, (60).

- We'll take the initial state s to consist of the set worlds $\{ w_b, w_\varnothing \}$, where subscripts indicate the presence/absence of the unique bathroom b, and the initial assignment $g_\varnothing$.

First, let's compute the positive and negative updates of the initial state with the first modal's prejacent – this gives us the states that the modal DR ranges over:

(61) a. $s[\llbracket\text{there is no}^2\text{ bathroom}\rrbracket]^+ = \left\{ \ (\{ w_\varnothing \}, g_\varnothing) \ \right\}$

b. $s[\llbracket\text{there is no}^2\text{ bathroom}\rrbracket]^- = \left\{ \ (\{ w_b \}, [2 \to b]) \ \right\}$

c. $(61a) \cup (61b) = \left\{ \begin{array}{c} (\{ w_\varnothing \}, g_\varnothing) \\ (\{ w_b \}, [2 \to b]) \end{array} \right\}$

Now we can compute the *positive update* of the initial state by the first conjunct. Note that the positive update is *indeterminate*, and varies according to the state DR introduced by the modal.

(62) $s\ \llbracket\text{there might}^1\text{ be no}^2\text{ bathroom}\rrbracket]^+ = \left\{ \begin{array}{c} ① (\{ w_b, w_\varnothing \}, [1 \to \boxed{(\{ w_\varnothing \}, g_\varnothing)}]) \\ ② (\{ w_b, w_\varnothing \}, [1 \to \boxed{(\{ w_b \}, [2 \to b])}]) \end{array} \right\}$

The second conjunct involves an anaphoric epistemic modal, which licenses piggyback anaphora.

(63) a. $\llbracket\text{it}_2\text{ might}_1\text{ be upstairs}\rrbracket\ ① = \left\{ \begin{array}{l} (\bot, w_b, [1 \to \boxed{(\{ w_\varnothing \}, g_\varnothing)}]), \\ (\bot, w_\varnothing, [1 \to \boxed{(\{ w_\varnothing \}, g_\varnothing)}]) \end{array} \right\}$

b. $\llbracket\text{it}_2\text{ might}_1\text{ be upstairs}\rrbracket\ ① = \left\{ \begin{array}{l} (\top, w_b, [1 \to \boxed{(\{ w_b \}, [2 \to b])}]), \\ (\bot, w_\varnothing, [1 \to \boxed{(\{ w_b \}, [2 \to b])}]) \end{array} \right\}$

Feeding the positive update of the initial state by the first conjunct into the second, pointwise, and gathering up the results in the following positive update. Piggyback anaphora is successful, due to the state DR which includes an assignment mapping 2 to the unique bathroom.

(64) $s[\llbracket\text{there might be no bathroom and it might be upstairs}\rrbracket]^+$

$= \{ (\{ w_b, w_\varnothing \}, [1 \to \boxed{(\{ w_b \}, [2 \to b])}]) \}$

5 Problems and prospects

5.1 Unifying disjunctive and conjunctive possibility statements

The parallels between disjunctive bathroom sentences and possible bathroom sentences are striking

As discussed by Schlenker (2008: p. 185), there is in fact reason to believe that projection in disjunctive sentences is symmetric.[21]

(65) This house has no bathroom or (else) the bathroom is well hidden.

(66) The bathroom is well hidden or (else) this house has no bathroom.

We find the exact same evidence for symmetry in possible bathroom sentences:

(67) It's possible this house has no bathroom,
and it's possible the bathroom is well hidden.

(68) It's possible the bathroom is well hidden,
and it's possible this house has no bathroom.

Our entries for disjunction, based on Beaver's (2001) asymmetric entry, of course don't capture (66).

Following the discussion of Schlenker (2008) there are different directions in accounting for (65) and (66); we can either invoke local accommodation (Heim 1983) or a symmetric entry for disjunction.

Likewise, we don't account for (68), but for an entirely different reason — since modal subordination for us involves familiar anaphoric mechanisms, we don't have an account of apparent cases of *cataphoric* modal subordination.[22]

This speaks to a larger issue with the analysis — intuitively, sentences of the form "P or Q" and "possibly P and possibly Q", raise possibilities, and it is this commonality which is responsible for their similarity with respect to projection/anaphora.

It's questionable that our analysis does this intuition justice.

One potential direction is in pursuing the line of reasoning in Zimmermann (2000), Geurts (2005), who reduce disjunction to conjunctive possibility statements.

[21] The data here are from Schlenker 2008: p. 185.

[22] We can't, e.g., posit a symmetric entry for conjunction, since this would sacrifice an account of left-to-right asymmetries in conjunctive sentences more generally.

We might also want to consider recasting the story in terms of an explanatory theory of presupposition projection, such as Schlenker 2008, in which projection no longer receives a strictly localist explanation.[23]

[23] See especially Schlenker's discussion of *unless*.

5.2 DNE, uniqueness, and universal readings

One potential worry for the analysis comes from uniqueness effects: as discussed by Gotham (2019), anaphora from under double negation seems to give rise to a uniqueness inference.

(69) a. John owns a shirt^1. It$_1$'s hanging up. The rest are in the closet.
 b. John doesn't own no shirt^1. It$_1$'s hanging up. ??The rest are in the closet.

I don't have much to add here, but note there seem to be exceptions to uniqueness, involving maximal reference, as illustrated in (70).[24]

(70) John doesn't own no shirt^1. They're in the closet.

[24] I'm grateful to Simon Charlow (p.c.) for this data.

Indeed, Gotham does not have a principled explanation for the uniqueness effect, and it seems reasonable to conclude that this phenomenon is still poorly understood.

See also Krahmer & Muskens (1995), who argue that bathroom sentences have *universal* readings. I don't take a stance on this issue here.

One final note is that, as Krahmer & Muskens's note, the crucial distinction between the positive and negative information associated with a sentence is a hallmark of trivalent semantics.

An interesting recent approach to the problem of DNE is that of Mandelkern (2020), who uses trivalent semantics to ensure that indefinites only introduce a DR if there is a verifier; the explanatory strategy is extremely reminiscent of the one here.

I leave it to future work to unearth the commonalities between the approach outlined here and Mandelkern's static semantics.

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A Conjunction in DAS*

We need to tweak the entry for conjunction in DAS* in order to avoid some bad results.

As it stands, we predict that anaphora should succeed in sentences like (71).[25]

(71) # It's false [that there's no^1 bathroom, and it$_1$'s upstairs].

This is because our existing entry for conjunction feeds each assignment that the first conjunct outputs, be it true-tagged, or false-tagged into the second conjunct.

It follows that a false-tagged assignment outputted by the first conjunct can license anaphora in the second, and the result of the entire conjunction can be flipped by negation, resulting in a true sentence.

Conjunction in DAS*

The solution we adopt here is straightforward: conjunction only feeds outputs in the *positive collapse* of the first conjunct into the second.

$$[\![\text{and}]\!] \coloneqq \lambda n \,.\, \lambda m \,.\, \lambda g \,.\, \bigcup_{(t,g')\in m^{\dagger}\, g} \{\, (t \wedge u, g'') \mid (u, g'') \in n\, g' \,\} \\ : \mathsf{T} \to \mathsf{T} \to \mathsf{T}$$

This ensures that, in the problematic example (71), DR 1 is eliminated by the positive collapse of the first conjunct, and anaphora will fail.

[25] Mayr's (2020) recent SuB presentation claims that similar sentences are in fact acceptable. Since I haven't been able to replicate these claims, I'll assume that (71) should indeed be ruled out.