On Couplings for Kinetic Langevin Diffusions

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- **Target.** $\mu_{\text{target}}(dx) \propto e^{-U(x)}\,dx$ on $\mathbb{R}^d$, with $U \in C^1$.

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- Noise enters *only the velocity* ⇒ hypoelliptic.

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- A *square-root speedup* in ill-conditioned problems; the motivation for studying kinetic Langevin and its splitting discretizations.

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From SDE to MCMC: splitting schemes

- Decompose the generator into three solvable flows:

$$\underbrace{O_h}_{\text{OU step (exact)}}, \quad \underbrace{\theta_h^{(\mathrm{A})}}_{\text{kinematic}}, \quad \underbrace{\theta_h^{(\mathrm{B})}}_{\text{kick by } -\nabla U}.$$

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- Second-order weakly accurate.
- One step is a deterministic map of two Gaussian increments $\xi_k = (\xi_k^{(1)}, \xi_k^{(2)}) \sim \mathcal{N}(0, \mathrm{Id}_{2d})$:

$$Z_{k+1}^h = \Psi_{Z_k^h}(\xi_{k+1}), \qquad Z_k^h = (X_k^h, V_k^h).$$

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For which initial laws ν does OBABO attain the **acceleration** $t_{\mathrm{mix}}(\nu, \varepsilon) \asymp \kappa^{1/2} \log(1/\varepsilon)$?

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Construct a coupling of the two chains and bound $\mathbb{P}$(disagree).

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2. Wasserstein-to-TV regularisation.

$$d_{\mathsf{TV}}\big(\delta_z P^n, \delta_{\tilde{z}} P^n\big) \;\leq\; C_n\,|z-\tilde{z}| \;\Longrightarrow\; d_{\mathsf{TV}}(\nu P^n, \tilde{\nu} P^n) \;\leq\; C_n\, W_1(\nu, \tilde{\nu}),$$

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High-acceptance localisation[4] transfers these bounds to the Metropolis-adjusted chain.

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[4] (B.-R. & Oberdörster (2024), EJP)

Markovian vs. non-Markovian couplings

Let $\mathcal{F}_k = \sigma\big((Z_j^h, \tilde{Z}_j^h) : 0 \leq j \leq k\big)$.

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- **Non-Markovian**: the joint chain is not adapted as a Markov chain to $(\mathcal{F}_k)$; the driving noises $(\tilde{\xi}_k)$ may depend on the entire sequence (ξ_j) at once.

A brief history of couplings for *hypoelliptic* SDEs

- **Stochastic oscillator.**[5]
  First hypoelliptic coupling: switching between synchronous and antithetic drivers.

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  Hybrid sync/reflection in phase space, non-convex U.
  - Discrete-time:[8] step-size-restricted analogues for OBABO and variants.

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- **TV mixing via Wasserstein-to-TV regularisation.**[9]
  Hypoelliptic smoothing transfers Wasserstein to TV control.

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[9] (Roberts & Rosenthal (2002); Madras & Sezer (2010); Monmarché (2021); Gouraud et al. (2025); Chak & Monmarché (2025))

Prior work: Chak & Monmarché (2025)

Theorem (Chak & Monmarché, 2025, Thm. 2.2)

Let $U \in C^2(\mathbb{R}^d)$ *satisfy*

- *bounded Hessian:* $|\nabla^2 U(x)\, v| \leq L\, |v|$ *for all* v, x*,*
- *Hessian-Lipschitz (Frobenius):* $\|\nabla^2 U(x) - \nabla^2 U(y)\|_F \leq L_H\, |x - y|$*.*

There exist $\beta, h_0 > 0$ *depending on* L, L_H *such that for all* $h \leq h_0$*,* $n \geq 2$ *with* $hn \leq 1$*:*

$$d_{\mathrm{TV}}\big(\pi_n(\delta_z),\, \pi_n(\delta_{\tilde{z}})\big) \;\leq\; \beta\Big[\tfrac{1}{(hn)^{3/2}} + \tfrac{1}{(hn)^{1/2}} + (hn)^{1/2}\Big]\, |\tilde{z} - z|.$$

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- **Mechanism.** An explicit *coalescence map* $\Psi^n_{z,\tilde z}$ built from an ansatz, with bound $d_{\mathrm{TV}}(\pi_n(\delta_z), \pi_n(\delta_{\tilde z})) \le d_{\mathrm{TV}}(\mathrm{Law}(\xi), \mathrm{Law}(\Psi^n_{z,\tilde z}(\xi)))$ closed by [1].

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- **Our take.** Read $\Psi^n_{z,\tilde z}$ as the *noise transport* of a non-Markovian coupling; *optimise* the trajectory and *realise* the coupling.

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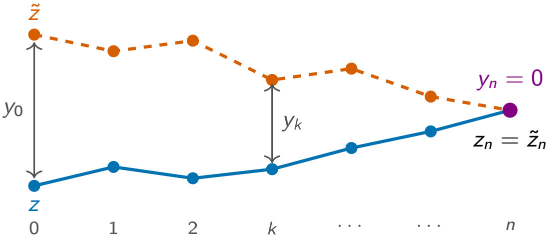
Coalescence map



Chain map. OBABO from z driven by noise $\xi = (\xi_1, \ldots, \xi_n)$:

$$z = z_0 \to z_1 \to \cdots \to z_n =: \Psi_z^n(\xi).$$

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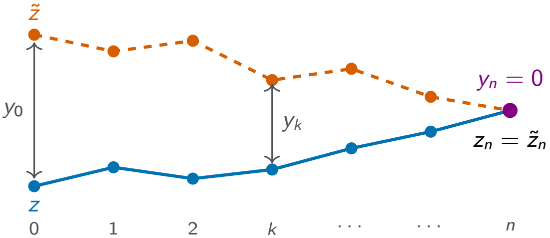

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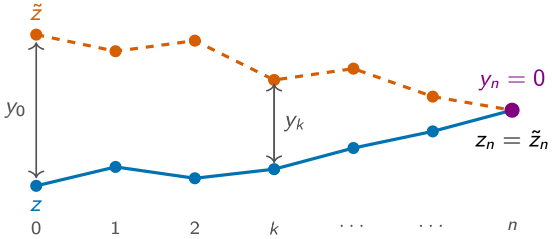

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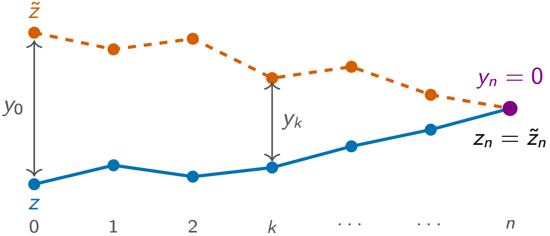

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Optimization. The interior $y_1, \ldots, y_{n-1}$ is free, and later picked by LQ optimization.

General lemma: TV bound between a reference and a perturbed Gaussian

Lemma (B.-R. & Eberle, 2023, Lem. 15)

Let $\xi \sim \mathcal{N}(0, \mathrm{Id}_d)$ *and* $\Phi\colon \mathbb{R}^d \to \mathbb{R}^d$ *be a* C^1 *diffeomorphism. Then*

$$d_{\mathsf{TV}}\big(\mathrm{Law}(\xi), \mathrm{Law}(\Phi(\xi))\big) \ \leq \ \sqrt{2\,\mathrm{KL}},$$

where $\mathrm{KL} \ = \ \mathbb{E}\big[\tfrac{1}{2}|\Phi(\xi) - \xi|^2 \ + \ \mathrm{tr}\big(D\Phi(\xi) - \mathrm{Id}\big) \ - \ \log|\det D\Phi(\xi)|\big]\,.$

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Sketch.

- Pinsker gives the first inequality;
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□

Remark (OBABO). For the OBABO coalescence map (next slide), $D\Psi^n_{z,\tilde z}$ is block lower-triangular with identity diagonal: $\mathrm{tr}(D\Psi - \mathrm{Id}) \equiv 0$, $\det D\Psi \equiv 1$, and $d_{\mathsf{TV}} \le \mathbb{E}[|\Psi^n_{z,\tilde z}(\xi) - \xi|^2]^{1/2}$.

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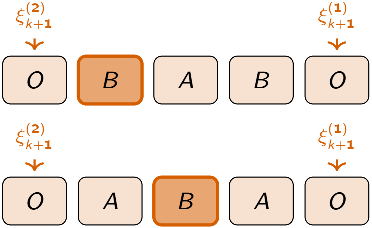

$\xi^{(2)}_{k+1}$ $\xi^{(1)}_{k+1}$

OBABO O B A B O q_{k+1} is *affine* in $\xi^{(1)}$

$\xi^{(2)}_{k+1}$ $\xi^{(1)}_{k+1}$

OABAO O A B A O q_{k+1} is *nonlinear* in $\xi^{(1)}$

Quantitative TV bound for OBABO

Theorem (B.-R.–Cox–Schieven)

Let $\gamma, h > 0$, $n \in \mathbb{N}$, *and* $U \in C^2(\mathbb{R}^d)$ *with* ∇U *L-Lipschitz. Then for all* $z, \tilde{z} \in \mathbb{R}^{2d}$*:*

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- **Realized by an explicit non-Markovian coupling**.

The optimal noise-transport trajectory (force-free)

Force-free bound. For $\nabla U \equiv 0$, Lemma 15 reduces to a controlled L^2 cost on the gap trajectory y_k:

$$d_{\mathsf{TV}} \;\leq\; \tfrac{1}{2}\left(\sum_{k=1}^{n} |E_k|^2\right)^{1/2}, \qquad E_{k+1} = -L_h^{-1}(y_{k+1} - A_h y_k).$$

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LQ-optimal trajectory. Minimum-energy control $\min_E \|E\|^2$ s.t. $y_0{=}\Delta z$, $y_n{=}0$. Closed form via controllability Gramian:

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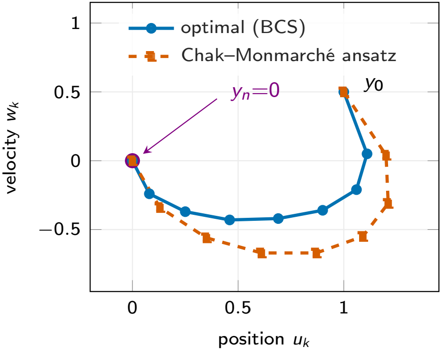
General U: use this y and treat ∇U as a Lipschitz perturbation.



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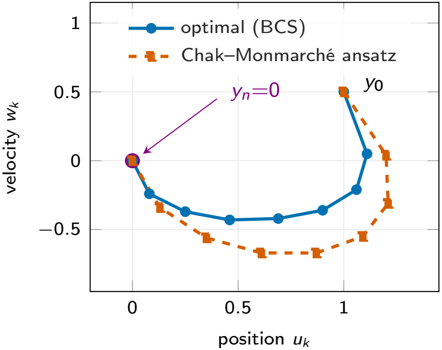

Design principle (EGZ & BCS). Construct for $\nabla U \equiv 0$, then treat ∇U as a Lipschitz perturbation.

The non-Markovian coupling

- Sample $\xi \sim \mathcal{N}(0, \mathrm{Id}_{2dn})$; proposal $\tilde{\xi}_\star := \Psi^n_{z,\tilde{z}}(\xi)$; maximally couple ξ to $\tilde{\xi}_\star$.
- On accept: $\tilde{\xi} = \tilde{\xi}_\star$ — chains meet at time n. On reject: $\tilde{\xi}$ drawn independently — chains evolve independently.
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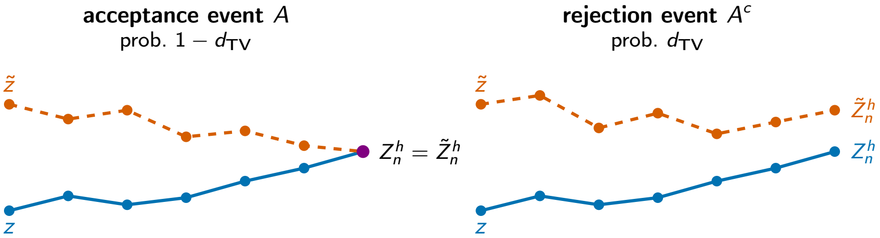

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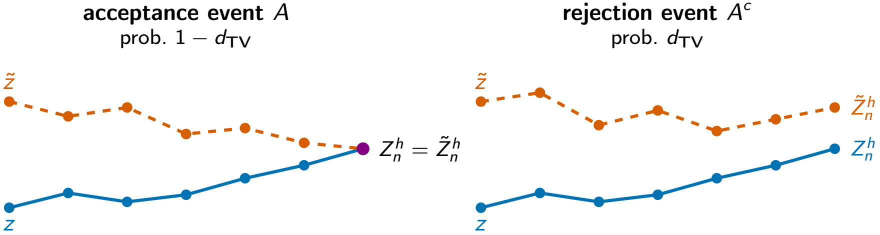

- **Non-Markovian**, designed for horizon n: $\tilde{\xi}$ depends on the whole ξ at once; intermediate chains generally disagree.

Are Markovian couplings enough?

- Test problem: **quadratic potential** $U(x) = \alpha|x|^2$, $\alpha \geq 0$, overdamped regime $\gamma^2 > 4\alpha$.

$$dX_t = V_t\,dt, \qquad dV_t = -\alpha X_t\,dt - \gamma V_t\,dt + \sqrt{2\gamma}\,dW_t.$$

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- Question: can a *Markovian* coupling match this rate?

No Markovian coupling matches the TV decay

Theorem (B.-R.–Cox–Schieven, continuous time)

Under $\gamma^2{>}4\alpha{\geq}0$*: (upper) if* $\lambda_-\Delta x{=}\Delta v$*,* $\Delta x{\neq}0$*, then* $d_{\mathsf{TV}} \leq Ce^{\lambda_- t}|\Delta z|$*; (lower) for every Markovian coupling* μ *and every* Δz*,* $\mu(Z_t \neq \tilde{Z}_t) \geq c_\mu \min\big(t^{-1/2}, e^{\lambda_+ t}\big)$ *for* $t \geq t_\mu$*.*

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Same setting. For every $h > 0$ *and Markovian* μ_h *(*c *indep. of* μ_h*):*

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- Extends (Banerjee & Kendall (2016)) (Kolmogorov diffusion) to kinetic Langevin.

The canonical Markovian candidate: iterated one-shot

- For linear Markov chains $Z_{k+1} = A_k Z_k + B_k \xi_{k+1}$:
  - Greedy: maximize the meeting probability *at the next step* by reflection coupling.
  - Satisfies the *now-equals-forever* property: $Z_k^h = \tilde{Z}_k^h \Rightarrow Z_j^h = \tilde{Z}_j^h$ for $j \geq k$.

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- For kinetic Langevin: necessarily suboptimal (by Theorem above). By how much?

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Exact meeting probability for iterated one-shot

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- Used as a sharp *lower bound* on $\mathbb{P}(Z_n \neq \tilde{Z}_n)$, not a TV upper bound.
- Free kinetic Langevin ($\alpha = 0$, $\Delta z = (\Delta x, -\gamma \Delta x)$): $\mathbb{P}(Z_k^h \neq \tilde{Z}_k^h) \geq c\, h^{-1} e^{\lambda_- hk}$ — *saturates* the $h^{-1} e^{\lambda_- hk}$ term in the impossibility bound.

Summary and outlook

- Sharp TV bound for OBABO via an LQ-optimal coalescence map; realised by an explicit non-Markovian rejection sampler. Drops Hessian-Lipschitz and $hn \leq 1$ of (Chak & Monmarché (2025)).
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Open.

- ⋆ **Acceleration** at rate $\kappa^{-1/2}$: *reduced* to proving W_1 contraction at this rate.
- Underdamped regime $\gamma^2 < 4\alpha$ (rigidity uses real eigenvalues).

Related directions.

- KL / Rényi divergences[1]; other splittings[2]; Metropolis adjustment[3].

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[1] (B.-R., Mitra & Wibisono (2026))
[2] (Schuh & Whalley (2025))
[3] (B.-R. & Oberdörster (2024))

Thank you!

On Couplings for Kinetic Langevin Diffusions

joint work with **Sonja Cox** and **Roy Schieven**

Questions?

Thanks to **Pierre Monmarché**, **Andreas Eberle**, and **Stefan Oberdörster**
for fruitful discussions.