Foreign Exchange Intervention Redux

Roberto Chang*

Rutgers University and NBER

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Abstract

Received wisdom posits that sterilized foreign exchange intervention can be effective by altering the currency composition of assets held by the public. This paper proposes an alternative channel: sterilized intervention may (or may not) have real effects because it changes the net credit position of the central bank vis a vis financial intermediaries, thereby affecting external debt limits. This argument is developed in the context of an open economy model with domestic banks subject to occasionally binding collateral constraints. Intervention has real effects if and only if it occurs when the constraints bind; at such times, a sterilized sale of official reserves relaxes the constraints by reducing the central bank's debt to domestic banks, freeing resources for the latter to increase the supply of credit to domestic agents. The analysis yields several noteworthy implications for intervention policy, official reserves accumulation, and the interaction between intervention and monetary policy.

*Email: chang@econ.rutgers.edu

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1 Introduction

Arguably, no issue in International Macroeconomics exhibits more dissonance between academic research and policy practice than foreign exchange intervention. The dominant view from academia is that sterilized foreign exchange (FX) intervention has a tiny, if any, impact on real variables, which makes it virtually useless as an independent macroeconomic policy tool. Indeed, a large body of empirical literature has struggled to find a consistent link between FX intervention and macroeconomic aggregates, including exchange rates.[1] From a theory perspective, this is hardly surprising, especially since modern dynamic macroeconomic models often predict that FX intervention should be irrelevant (Backus and Kehoe 1989).

Policymakers, on the other hand, have ignored the prescriptions from research and have intervened, frequently and intensely, in the foreign exchange market. FX intervention has become prominent and noticeable following the global financial crisis in advanced economies, while in emerging ones FX intervention was the norm already before the crisis, and even in countries committed to inflation targeting. Interestingly, central bankers reportedly believe that FX intervention is effective as a policy tool, and that it has been used successfully.[2]

The purpose of this paper is to develop a recent perspective on FX intervention which, among other advantages, can help reconciling the contrasting views of academics and policy makers. Following Céspedes, Chang, and Velasco (2017), I adopt the view that FX intervention can and should be seen as a specific instance of the so called "unconventional" central bank policies reviewed, for example, in Gertler and Kiyotaki (2010). This view strongly indicates that a useful analysis of FX intervention requires a framework that allows for financial frictions and institutions, for otherwise unconventional policies turn out to be irrelevant (as in Wallace 1981 or, as already mentioned, Backus and Kehoe 1989).

Accordingly, I analyze FX intervention in an extension of Chang and Velasco's (2017) model

[1] For instance, citing Obstfeld (1982) and Sarno and Taylor (2001), Taylor (2014, p. 369) writes: "the evidence is often weak and a source of ongoing controversy". A more recent survey of the empirical literature is Menhoff (2013).

[2] See Chutasripanish and Yetman (2015). Also, Adler and Tovar (2011).

of a small open economy. In that economy, financial intermediaries or banks borrow from the world market and, in turn, extend credit to domestic households or the government, subject to an external debt limit. The model is intended to be standard and as simple as possible to help exposition, so as to isolate two features that turn out to be central. The first one is the specification of sterilized intervention. Sterilized FX interventions are operations in which the central bank buys (or sells) official reserves of foreign exchange, and at the same time it sells (or buys) an offsetting amount of securities, such as "sterilization bonds". This implies that the central bank issues sterilization bonds, or more generally reduces its net credit position, when it purchases reserves, and cancels the bonds when it sells reserves.

The second aspect of our model is that domestic banks face an external debt limit that may or may not bind in equilibrium. This is key because, as I show, FX intervention has no impact on macroeconomic aggregates if it occurs when that limit does not bind. Conversely, as I also show, FX intervention does affect equilibrium real outcomes if it takes place at times of binding financial constraints.

More precisely, sterilized FX intervention can affect equilibrium because the associated sterilization operations relax or tighten financial constraints. When the central bank sells foreign exchange, sterilization means that the central bank retires sterilization bonds (or, more generally, increases its net credit position *vis a vis* domestic banks). If financial constraints do not bind, domestic banks accommodate this change by simply borrowing less from the world market, and equilibrium is left undisrupted. But when financial constraints do bind, the fall in the central bank's demand for credit associated with sterilization frees resources for banks, allowing them to increase the supply of loans to the domestic private sector. The result is that loan interest rates fall, and aggregate demand expands.

This view of the mechanism through which sterilized FX intervention works differs sharply from alternative ones, and in particular from those of currently dominant portfolio balance models. Such models assume that domestic currency bonds and foreign currency bonds are imperfect substitutes and, as a result, uncovered interest parity holds up to a risk premium that depends on the ratio of domestic currency bonds to foreign currency bonds in the hands of the public. Sterilized FX intervention affects that ratio and hence, the risk premium, which in turn requires macroeconomic adjustments. In contrast, the mechanism proposed in this paper does not rely on imperfect asset substitutability nor on differences in currency denomination. In fact, and in order to stress the point, I show that FX intervention can be an effective policy tool (when financial constraints bind) under perfect asset substitutability and even if the economy is "financially dollarized".

As a significant additional payoff, our exploration of the model highlights a close link between sterilized intervention and the cost-benefit analysis of official reserves accumulation. Under the natural assumption that the central bank cannot issue foreign currency, maintaining a large stock of foreign exchange enhances the ability of the central bank to stimulate the economy, by selling reserves, when financial constraints become binding. This is obviously beneficial and intuitive. But in this model there is also a cost of holding reserves, namely, that larger reserves also imply larger outstanding quantities of sterilization bonds, the financing of which may place banks closer to their credit limits, making them more vulnerable to adverse exogenous shocks. A main trade-off the emerges: large amounts of official reserves allow the central bank to respond more effectively, via FX intervention, when financial constraints are hit, at the cost of those constraints being hit more frequently.

Our analysis yields several lessons for FX intervention rules and their relation to conventional monetary policy. Notably, a policy of selling reserves when the exchange rate is weak and buying them when the exchange rate is strong can relax financial constraints when they bind, but also leads to intervention when the constraints do not bind, which can be counterproductive. A policy of intervention based on credit spreads is superior, as it is only activated when financial constraints bind. Also, the question of whether sterilized FX intervention can be an independent policy tool and complement conventional monetary policy has an affirmative answer in our model. But the fact that financial constraints bind only occasionally is crucial and means, in particular, that one must go beyond the analysis of linear models or linear approximations around the steady state.

Finally, our approach yields several other appealing insights. Specifically, it is consistent with the empirical difficulty to find significant macroeconomic effects of FX intervention in the data, since intervention has real impact only at times of binding constraints, which may be infrequent. It also indicates how intervention can be welfare improving. And it sheds light on the role of the so called *quasi fiscal* deficits that central banks derive from intervention.

This paper builds on and contributes to a large literature on FX intervention. For useful surveys, see Sarno and Taylor (2001), Menkoff (2013), Adler and Tovar (2011), and Ostry, Ghosh, and Chamon (2016). Until the powerful critique by Backus and Kehoe (1989), the literature was dominated by models derived from the optimal portfolio choices of investors that viewed domestic currency assets and foreign currency assets as imperfect substitutes. Recently the portfolio balance approach has experienced a revival, led by Benes, Berg, Portillo, and Vavra (2015), and followed by Vargas, González and Rodriguez (2013), Montoro and Ortiz (2016), and Cavallino (2017).

The newer portfolio balance models are similar to ours in that FX intervention can have real effects because of the interaction of sterilization operations with financial frictions. They differ substantially along some important details, however. For example, Benes et al. (2015) and Vargas et al. (2013) impose that banks pay portfolio management costs similar to those in Edwards and Vegh (1997). They make assumptions about those costs that make domestic currency bonds and foreign currency bonds imperfect substitutes for the banks, which leads to the same kind of uncovered interest parity condition *cum* risk premium that was the hallmark of the older portfolio balance approach. This indicates that, while the newer models have been successful in providing satisfactory theoretical underpinnings to the portfolio balance approach, they still have to be reconciled with the same evidence as older models. In comparison, in the model of this paper, financial frictions only bite sometimes and not others, which makes a significant difference in the results. For one thing, under the assumption that financial constraints are not binding in the steady state, our model implies that FX intervention is irrelevant for shocks that are not large enough to drive the economy to the financially constrained region. As already noted, this aspect of the model consistent with the scarcity of empirical evidence of nontrivial effects of sterilized intervention on macro variables.

Section 2 of this paper presents the model that serves as the basis for our discussion. A baseline version of the model assumes complete price flexibility and financial dollarization. In that baseline version, Section 3 discusses FX intervention and reserves accumulation. Nominal price rigidities and, hence, a nontrivial role for monetary policy are introduced in Section 4. That section examines the interaction between monetary policy and FX intervention. Section 5 shows how the assumption of financial dollarization can be relaxed, with only minor changes in our arguments. Section 6 concludes. An Appendix collects some peripheral technical derivations.

2 A Model of FX Intervention and Reserves Accumulation

To convey our ideas regarding intervention policy, I extend the model of Chang and Velasco (2017) to a stochastic setting, emphasizing the mechanics of sterilized intervention and how intervention policy interacts with financial constraints that bind only occasionally. This section develops a baseline version of the model that, in order to focus on the essentials, imposes very restrictive assumptions: it assumes perfectly flexible prices (implying that conventional monetary policy has no bite) and complete *financial dollarization* (i.e. that all financial assets are denominated in foreign currency). These assumptions not only simplify the analysis but also underscore that the mechanism by which FX intervention works does not depend on the currency denomination of assets or the interaction with other monetary policy tools. Of course, realistic models might require allowing for nominal price rigidities, powerful monetary policy, and differences in the currency denomination of assets. But these can be added at relatively little extra cost later, as shown in sections 4 and 5.

2.1 Commodities and Production

We consider an infinitely lived, small open economy. In each period there are two internationally traded goods, home and foreign. The price of the foreign good in terms of a world currency (called *"dollar"*) is fixed at one.

The home good is the usual Dixit-Stiglitz aggregate of varieties, with elasticity of substitution ϵ. Each variety is produced by one of a continuum of monopolistically competitive firms indexed by i in $[0, 1]$. In period t, firm i produces variety i via $y_{it} = An_{it}$, where n_{it} denotes labor input, and A a productivity term kept constant for ease of exposition. Firms take wages as given, and hence nominal marginal cost in period t is common to all, given by:

$$MC_t = W_t/A \tag{1}$$

where W_t is the nominal wage, that is, the wage expressed in terms of a domestic currency (*"peso"* hereon).

For now, we assume flexible prices, meaning that in every period all firms set the peso price for their produce after observing that period's exogenous shocks. All varieties then carry the same price in equilibrium, given by the usual markup rule:

$$P_{ht} = \left(1 - \frac{1}{\epsilon}\right) MC_t \tag{2}$$

P_{ht} is also the price of the domestic home aggregate good. That aggregate is sold at home and abroad. The foreign part of demand is given simply by a function xe_t^{χ} of its relative price, the *real exchange rate:*

$$e_t \equiv \frac{E_t}{P_{ht}}$$

with E_t denoting the nominal exchange rate (pesos per dollar), and x and χ positive parameters.

Home demand for the domestic aggregate good is derived from the demand for final consumption. The latter is denoted by c_t and assumed to be a Cobb Douglas function of the domestic composite good and foreign goods. The Law of One Price is assumed, implying that the peso price of foreign goods is given by E_t. Then the price of final consumption (the CPI) is

$$P_t = P_{h,t}^{\alpha} E_t^{1-\alpha}$$

where α is a parameter between zero and one.

The implied demand for the home aggregate is $c_{ht} = \alpha e_t^{1-\alpha} c_t$, and therefore the market home output clears if

$$y_t = \alpha e_t^{(1-\alpha)} c_t + x e_t^{\chi}, \tag{3}$$

2.2 Banks

There is a large number of domestic financial intermediaries, or banks, which borrow from the rest of the world and lend to either households or the government, subject to financial frictions.

A representative bank starts a period t with an amount of capital or net worth of k_t dollars. This amount is, as we will see, raised from domestic households in exchange for a share of the bank's next period profits. Given k_t, the bank borrows d_t dollars from foreigners, at a gross interest rate of $R_t^* \geq 0$, which the bank takes as given.

Because of financial frictions, external borrowing is restricted by a collateral constraint

$$d_t \leq \theta k_t$$

where θ is a constant. As noted in the literature, this kind of constraint can be rationalized in various ways. [3]

The resources raised by the bank finance loans to the domestic private sector, l_t, or the

[3] For example, one may assume that, after raising d_t, the banker can "abscond" with the funds at a cost of θ times equity. Knowing this, lenders will not extend more credit than θk_t.

This being said, the exact form of the collateral constraint may or may not have significant impact on the analysis. For example, one might consider an alternative specification such as $d_t \leq \theta(k_t + b_t)$, on the basis that domestic banks government bonds could be pledge government bonds as collateral to foreigners. It is not hard to see that our analysis below remains qualitatively the same if $\theta < 1$. But there may be important quantitative differences. (Thanks to José De Gregorio for this observation.)

purchase of bonds issued from the central bank, b_t. Private loans and central bank bonds are perfect substitutes and carry the same interest rate, ϱ_t, between periods t and $t+1$.

Observe that, for now, loans and bonds, and the interest rate, are all assumed to be denominated in dollars. This case of financial dollarization may be realistic for some countries and not for others; however, it is the simplest assumption to start with. More importantly, it emphasizes that the basic mechanism by which FX intervention works in our setting does not rely on differences in currency denomination. Once that mechanism is laid out, section 5 turns to its interaction with peso denominated loans and bonds.

The typical bank's balance sheet therefore requires that:

$$b_t + l_t = k_t + d_t.$$

and the bank's profits are given by

$$\pi_{t+1} = (1 + \varrho_t)(l_t + b_t) - R_t^* d_t,$$

Under our maintained assumptions, profits are realized in period $t+1$ but they are known as of period t. The bank's problem, therefore, is simply to choose b_t, d_t, and l_t to maximize π_{t+1} subject to the collateral constraint.

The solution is simple. Combining the preceding two equations, profits can be written as

$$\pi_{t+1} = R_t^* k_t + (1 + \varrho_t - R_t^*)(l_t + b_t)$$

i.e. profits are a sum of a "normal " return on equity plus an excess return on domestic credit. Hence, if $1 + \varrho_t = R_t^*$, there are no supranormal returns, and the bank's optimal policy is indeterminate as long as $b_t + l_t = k_t + d_t$ and $d_t \leq \theta k_t$. If $1 + \varrho_t > R_t^*$, on the other hand, the bank lends as much as it can. The collateral constraint then binds, so that $d_t = \theta k_t$, and $b_t + l_t = (1 + \theta) k_t$.

Finally, the return to equity is denoted by $(1+\omega_t)R_t^* \equiv \pi_{t+1}/k_t$, and given by:

$$\frac{\pi_{t+1}}{k_t} = R_t^* + (1+\varrho_t - R_t^*)(1+\theta) \equiv (1+\omega_t)R_t^*$$

2.3 Central Bank, Intervention, and Reserves Accumulation

The essence of sterilized FX intervention is that, whenever a central bank sells or buys foreign exchange, it also buys or sells a matching amount of securities. This can be implemented in many different ways, and the menu of alternatives depends in practice on institutional aspects of each economy, such as the kind of securities that are involved in sterilization. But again, and as emphasized in the literature, the defining aspect of sterilized intervention is that it involves a simultaneous change in official reserves and the net credit position of the central bank.

Accordingly, in what follows we assume that sterilized FX intervention means that the central bank buys or sells official reserves (dollars) and, at the same time, issues or retires a corresponding quantity of its own bonds (which therefore might be referred to as *sterilization bonds*). While highly stylized, this assumption is the same as in the recent papers of Benes et al. (2015) and Vargas et al. (2013). It also corresponds closely to actual practice in some countries. For example, Vargas et al. (2013) discuss the Colombian experience en some detail, and how the practice of FX intervention led Colombia's government to issue sterilization bonds. The same specification is incorporated into modern textbooks such as Feenstra and Taylor (2014).

In our model, as will become apparent, FX intervention can affect equilibria when and only when the matching sterilizing operation relaxes or tightens the external credit constraint. This argument, stressed in Céspedes, Chang, and Velasco (2017), differs from older ones, in particular with the traditional portfolio balance view. That view started from the assumption that sterilization operations involved securities denominated in domestic currency, and therefore FX intervention must change the ratio of foreign currency assets to domestic currency assets in private hands. If, in addition, securities denominated in different currencies were imperfect substitutes, restoring equilibrium required a change in relative rates of return. Such an argument is obviously not applicable to our model, as we have assumed that all securities are denominated in dollars and are perfect substitutes. But this is only to emphasize that the mechanism by which FX intervention works is not a portfolio balance one.

Note that we assume that sterilization bonds are held solely by domestic agents, banks in this case. This assumption is natural and realistic, and no different from what is usually imposed in the literature. But it is a crucial part of our argument. If the central bank could freely sell sterilization bonds to the rest of the world, then the economy as a whole would effectively face no external collateral constraint. The key aspect of our assumptions is that sterilization bonds add to the economy's overall external debt, which has a limit. One can presumably adapt our analysis to alternative scenarios as long as they imply that sterilization bonds interact with financial frictions.

As mentioned, central bank bonds are assumed to yield the same interest rate as private loans, ϱ_t. Reserves, on the other hand, are assumed to be invested abroad, at the external interest rate R^*_t. In this setup, the central bank makes operational losses (the so-called *quasifiscal deficit*) if $1 + \varrho_t > R^*_t$. For the time being we assume that such losses, if any, are financed via a lump sum tax on households; one implication is that the net worth of the central bank is constant, and normalized here to zero for convenience. These assumptions are prevalent in the literature, but it should be noted that they are not trivial either for the theory or in practice. Further research is clearly warranted on this issue; I offer further thoughts in the closing section.

Our maintained assumptions now ensure that, if f_t denotes the central bank's international reserves, the central bank's balance sheet is simply given by $f_t = b_t$, and that the central bank's quasifiscal deficit in period t is given by

$$T_t = (1 + \varrho_{t-1} - R^*_{t-1})b_{t-1}$$

Hence there is a tight link between foreign exchange intervention and the amount of central bank bonds: selling foreign exchange reserves is a fall in f_t, which then amounts to a reduction in b_t; conversely, accumulating reserves leads to an increase in b_t.

Finally, it seems natural to assume that the central bank cannot issue international currency. In this setting, this requires imposing that official central bank reserves have a lower bound, which we assume to be zero: $f_t = b_t \geq 0$.

2.4 Households

The economy has a representative household with preferences that depend on consumption and labor effort, and given by the expected value of $\sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$, with

$$U(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{\eta}{1+\phi} n^{1+\phi}$$

where σ and ϕ positive parameters.[4]

In each period t, the household decides how much to consume and to work, how much to borrow from domestic banks, and much much equity to send to the banks. The period's budget constraint, expressed in dollars, is:

$$e_t^{-\alpha} c_t + k_t - l_t = (1+\omega_{t-1}) R_{t-1}^* k_{t-1} - (1+\varrho_{t-1}) l_{t-1} \; + e_t^{-\alpha} w_t n_t + v_t + z_t - T_t,$$

where $w_t = W_t/P_t$ is the real wage, v_t denotes (dollar) profits from domestic firms and banks, and T_t denotes to the lump sum taxes needed to finance the central bank's quasifiscal deficit. Finally, z_t is an exogenous endowment of foreign goods (dollars), which can be thought of as income earned from the ownership of a natural resource, as oil or commodities. The LHS gathers the value of the household's expenditure in consumption and new equity purchases, minus new bank loans. The RHS includes the return on equity, minus the repayment of bank loans, plus income net of taxes.

Finally, we follow Chang and Velasco (2017) in assuming that there is an exogenous limit,

[4] And as usual, if $\sigma = 1$, $u(c) = \log(c)$.

referred to as the domestic *equity constraint,* to how much bank equity the household can hold:

$$k_t \leq \widetilde{k}$$

with $\widetilde{k} > 0$ is some constant. The *equity constraint* reflects, presumably, some domestic distortions that we do not model here.

The household's optimal plan is straightforward. Optimal labor supply is given by

$$w_t c_t^{-\sigma} = \eta n_t^{\phi} \tag{4}$$

Assuming that the household borrows a positive amount, which will be the case in equilibrium, the usual Euler condition must hold:

$$c_t^{-\sigma} = \beta E_t c_{t+1}^{-\sigma} R_{t+1}$$

where we have defined the consumption interest rate by

$$R_{t+1} = (1 + \varrho_t) \left(\frac{e_{t+1}}{e_t} \right)^{\alpha} \tag{5}$$

Finally, the equity constraint binds in period t if and only if the return on equity, $(1+\omega_t)R_t^*$, exceeds the cost of domestic loans, $1 + \varrho_t$. As the reader can check, in equilibrium this will be case if and only if $1+\varrho_t > R_t^*$. But this means that the equity constraint and the bank's external debt constraint must bind under exactly the same circumstances. This simplifies the analysis considerably, since it allows us to impose, without loss of generality, that $k_t = \tilde{k}$ always, and that the constraint $d_t \leq \theta \tilde{k}$ binds if $1 + \varrho_t > R_t^*$ and is slack if $1 + \varrho_t = R_t^*$.

2.5 Equilibrium

We assume that parameter values are such that financial frictions do not bind in the non stochastic steady state. As is well known, in order to be able to apply approximation techniques around that steady state, some additional assumptions must be imposed to ensure stationarity (Schmitt Grohe and Uribe 2017). Here we assume that the external cost of credit, R_t^* , is given by the world interest rate, denoted by $\bar{R}^*$ and taken as exogenous and constant (for simplicity), plus a spread term that depends on the amount of bank credit $l_t = k_t + d_t - b_t$:

$$\begin{aligned} R_t^* &= \bar{R}^* + \tilde{\Psi}(e^{l_t - \bar{l}} - 1) \\ &= \bar{R}^* + \tilde{\Psi}(e^{(d_t - \bar{d}) - (b_t - \bar{b})} - 1) \end{aligned}$$

where $\bar{l}$, $\bar{d}$ and $\bar{b}$ are the steady state values of domestic loans, external, debt and reserves, respectively, and $\tilde{\Psi}$ is an elasticity coefficient.

Two brief comments on the above specification are warranted. First, that the $R_t^* - \bar{R}^*$ spread increases with domestic loans implies that it increases with the economy's external debt net of reserves. This seems defensible: in fact, the (negative of the) quantity $d_t - b_t$ corresponds to measures of international liquidity emphasized in Chang and Velasco (2000) and elsewhere. Second, we assume that $\bar{l}$ is given exogenously. This differs somewhat from the literature, which usually imposes an exogenous $\bar{d}$. This is because we want to allow for FX intervention policies for the management of reserves and central bank debt, with implications for the steady state value of reserves $\bar{b}$. It will become apparent that the assumption of an exogenously given $\bar{l}$ yields a cleaner analysis than an exogenous $\bar{d}$. Since whether taking $\bar{l}$ or $\bar{d}$ as exogenous is arbitrary and only needed for technical reasons, we stick with exogenous $\bar{l}$.

Under flexible prices, one can combine the optimal markup rule (2) and the labor supply condition (4) to arrive at the equilibrium aggregate supply condition:

$$e_t^{-(1-\alpha)} c_t^{-\sigma} = (1 - \frac{1}{\epsilon}) \eta y_t^{\phi} / A^{1+\phi}$$

In turn, the external resource constraint can be written as

$$(1-\alpha)e_t^{-\alpha}c_t - [z_t + \varkappa e_t^{\chi-1}] = d_t - b_t - R_t^*(d_{t-1} - b_{t-1}) \tag{6}$$

which says that the trade deficit in period t must be financed by increasing external debt or reducing central bank debt, i.e., selling international reserves. As emphasized by Chang and Velasco (2017), this constraint is a key aspect of the model, given that the collateral constraints require

$$\begin{aligned} d_t &= \theta\tilde{k} \quad \text{if } 1+\varrho_t > R_t^* \\ &\leq \theta\tilde{k} \quad \text{if } 1+\varrho_t = R_t^* \end{aligned}$$

It may aid intuition to express the external resource constraint (6) as

$$TD_t + r_t^*(d_{t-1} - b_{t-1}) = (d_t - d_{t-1}) + \Delta_t$$

where we have defined the trade deficit TD_t as the LHS of (6), $r_t^* = R_t^* - 1$ as the net rate of interest on the external debt, and $\Delta_t = -(b_t - b_{t-1}) = f_{t-1} - f_t$ as the size of foreign exchange sales of the central bank in period t. Thus written, the LHS is the current account deficit, the sum of the trade deficit and the service of the net foreign debt. The preceding equation then emphasizes that a current account deficit is financed either via additional foreign debt or via sales of official reserves; the latter imply a fall in the quantity of sterilization bonds. It also stresses that, if reserves cannot be negative, $\Delta_t \leq f_{t-1}$, i.e. foreign exchange operations in each period are limited by the inherited level of reserves.

Finally, if the collateral constraints bind so that $d_t = d_t = \theta\tilde{k}$, we have that $TD_t = \Delta_t - r_t^*(d_{t-1} - b_{t-1})$. This says that, in the absence of foreign exchange intervention, the trade deficit is predetermined. One implication, noted in Chang and Velasco (2017), is that adverse shocks must be fully offset within the period by a fall in absorption (consumption) or real exchange depreciation. In particular, if there is a temporary fall in exogenous exports z_t, consumption smoothing would require an increase in external borrowing, which is not feasible. Some consumption smoothing, on the other hand, can be achieved by an increase in Δ_t, that is, a sale of reserves. In this sense, foreign exchange intervention can relax binding financial constraints,as we will explore.

Equilibrium is pinned down once we specify a rule for the evolution of b_t, that is, a foreign exchange intervention policy. An analysis of alternative policies provided next.

3 Reserves Accumulation and Intervention

The first subsection discusses a crucial property of the model: that sterilized intervention can have real effects if and only if it relaxes binding financial constraints. A second subsection examines the implications of various intervention rules in a calibrated version of the model. In addition of illustrating how the model works under frequently observed policy rules, the exercise helps identifying additional aspects of the model and policy implications.[5]

3.1 General Considerations

As in Céspedes, Chang, and Velasco (2017), sterilized intervention is irrelevant in our model unless it occurs at times of binding collateral constraints (or make financial frictions bind if they would have not). For a more precise statement, fix any equilibrium, which we will denote with carets. As the interested reader can check, the equilibrium conditions[6] can be written so that $\hat{d}_t$ and $\hat{b}_t$ appear only in three of them. The first one is the collateral constraint, which

[5]For a more analytically oriented discussion of the transmission mechanisms in this kind of model, the interested reader is referred to Chang and Velasco (2017).

[6]For notational simplicity, this paper follows the convention that the "t" subscripts index date event pairs, that is, c_t denotes consumption at t conditional on the whole history of exogenous shocks up to that point. With that understanding, our discussion, particularly in this section, applies without change to stochastic models. An alternative notation would have been to write something like $c_t = c(s^t)$, and so on for each variable, where s^t is the history of shocks up to t. I see little gain here, however, in using the more precise but also more cumbersome notation.

can be rewritten as:

$$\hat{l}_t = (1+\theta)\tilde{k} - \hat{b}_t \ \text{ if } \ 1+\hat{\varrho}_t > R_t^*$$
$$\hat{l}_t \leq (1+\theta)\tilde{k} - \hat{b}_t \quad \text{ if } \ 1+\hat{\varrho}_t = R_t^*$$

The second one is the bank's balance sheet, which requires:

$$\hat{d}_t = \hat{b}_t + \hat{l}_t - \tilde{k}$$

The third one is the FX intervention rule. We allow the central bank to set b_t as any function of past, present, or future expected values of any variables in the model, as long as the rule pins down an equilibrium implying a well defined process for official reserves, $\{\hat{b}_0, \hat{b}_1, ...\}$.[7]

Now consider any different policy rule that (possibly in conjunction with the original equilibrium) implies an alternative process for reserves, $\{b'_0, b'_1, ...\}$, that coincides with $\{\hat{b}_0, \hat{b}_1, ...\}$ at all times except at some given date t. If the collateral constraint did not bind at t in the original equilibrium, and does not bind under the new policy (i.e. $\hat{l}_t \leq (1+\theta)\tilde{k} - b'_t$), then the policy leaves the original equilibrium unchanged, except that $d'_t = b'_t + \hat{l}_t - \tilde{k}$, i.e. that there is a change in external debt that exactly offsets the change in reserves. This is of course feasible, since the collateral constraint does not bind in equilibrium, and it is also intuitive: if the central bank sells reserves, the supply of stabilization bonds increases by the amount of the sale. Domestic banks can finance the increased holdings of bonds by borrowing abroad, without disrupting the domestic supply of loans, as long as their credit limit is slack.

Conversely, to affect equilibria, a change in intervention policy must involve a change in b_t at some t in which either the collateral constraint binds or a nonbinding constraint becomes binding under the new policy. It also becomes apparent that, when collateral constraints

[7]More precisely, the policy rule together with the rest of the model implies an equilibrium in which the stochastic process for reserves is $\{\hat{b}_0, \hat{b}_1, ...\}$.

bind, the central bank can stimulate the economy by selling foreign exchange. By doing so, it redeems central bank bonds, making room for domestic banks to increase credit to households. In this sense, and as emphasized by Céspedes, Chang, and Velasco (2017), sterilized intervention "works" because the sterilizing operation relaxes the external collateral constraint.[8]

The alert reader might recognize that the propositions just stated are extensions of those in Backus and Kehoe (1989). For a large class of models, Backus and Kehoe identified conditions under which sterilized intervention would not affect equilibria. But they also allowed for the possibility that sterilized intervention might not be irrelevant if those conditions were not met. Our analysis proceeds further, by asking what are the implications of intervention when they can matter.[9]

Intuitively, the economy benefits if the central bank sells foreign exchange reserves when financial constraints bind. This provides a rationale for the accumulation of official reserves if, as we have assumed, foreign exchange reserves cannot be negative. In other words, our analysis of intervention has implications for the discussion of observed reserves accumulation in emerging economies and elsewhere.

One such implication relates to the costs of accumulating reserves. Why would the central bank not accumulate a very large amount of foreign exchange in normal times, so as to be ready to act if financial constraints suddenly bind? In our model, reserves accumulation involves two kinds of costs. The first one has been recognized in the literature: to finance the accumulation of reserves, the central bank borrows from domestic banks with an interest cost that adds to the quasifiscal deficit. In our model, however, in normal times (i.e. when financial constraints

[8] The mechanism through which intervention works is similar to that in Benes et al. (2015) and Vargas et el. (2013). Those papers assume a financial transaction technology that implies that a reduction of the supply of sterilization bonds, associated with a sale of official reserves, must induce domestic banks to also decrease the supply of loans to households, resulting in an increase in the interest cost of domestic loans. Note that one consequence is that central bank FX sales must always be contractionary in those models. In the model here, in contrast, central bank FX sales either leave the supply of loans unchanged (if financial constraints do not bind) or increase it (if they do). And, crucially, the circumstances under which FX purchases stimulate domestic credit are exactly those in which the economy is credit constrained.

[9] In contrast, Backus and Kehoe stopped their analysis after stating that, when intervention can matter, its real effects depend on accompanying assumptions about fiscal policy. Here we make progress by making specific assumptions on the quasi fiscal deficit.

do not bind), the interest cost is fully offset by the interest earned on reserves. A second source of costs is new, to my knowledge, and potentially more significant: the accumulation of central bank reserves induces domestic banks to increase their own external debt and, hence, place themselves nearer to their foreign credit limit. It then becomes more likely that, in response to adverse shocks, the limit becomes binding.

So our model features a novel trade-off in reserves accumulation: larger official FX reserves are necessary for the central bank to be ready to stimulate the economy at times of binding financial constraints; but the financing of those reserves induces domestic banks to increase international borrowing, making the economy less resilient to shocks. Exploring the implications of such a trade-off is beyond the scope of the present paper, but should be a fruitful avenue for future research.

3.2 Numerical Illustrations

To illustrate our main ideas, this subsection develops a calibrated version of the model. I stress that the objective of this subsection is to expand and clarify our discussion, rather than empirical realism. Hence we choose some parameter values on the basis of just simplicity and convenience.

Details of the calibration are given in the Appendix; here we only mention salient aspects. A period is a quarter. In steady state, the world interest rate is four percent per year. The steady state values of y, e, and c are all one, and the trade surplus to GDP ratio is one percent. In the absence of foreign exchange intervention, an implication is a steady state debt to (annual) GDP ratio of twenty five percent, which accords well with usual values in the literature (e.g. Schmitt Grohe-Uribe 2017).

The final important aspect of the calibration is the debt limit $\theta\tilde{k}$. For our discussion, I set it at a very stringent value, so that in steady state the economy is not financially constrained, but close to being so. This is because my purpose is to illustrate the workings of the model, with emphasis on the role of financial constraints.

Having calibrated the model, finding numerical solutions requires nonlinear procedures. For the experiments reported here, I solved the model via the remarkably useful Occbin procedures developed by Guerrieri and Iacovello (2015). Occbin adapts Dynare to approximate our model regarded as having different regimes, given by times of binding and nonbinding constraints. In response to exogenous shocks, the transition between regimes is endogenous and part of the computation. See Guerrieri and Iacovello (2015) for details, as well as commentary on the accuracy of the resulting approximations.

To obtain a feel for the model in the absence of foreign exchange intervention. Figure 1[10] displays impulse responses to a purely temporary fall in the exogenous endowment z; one can think of this shock as a fall in the world price of an export commodity. The broken (green) lines give the impulse responses in the absence of financial constraints. In that case, as clear from the figure, a purely temporary fall in z would be accommodated primarily by borrowing from the rest of the world. This would allow the economy to spread the cost over time, smoothing the response of consumption. The exchange rate would depreciate, reflecting the fall in the derived demand for nontradables, but only by a small amount. Finally, the interest rate on loans (ϱ_t) would essentially remain the same (it increases minimally only because the increase in the debt raises the spread $R_t^* - \bar{R}^*$ through the debt elastic mechanism, which is negligible).

With occasionally binding financial constraints, the impulse responses are given by solid (blue) lines. External debt increases to the credit limit, which binds for thirteen periods. The binding constraint implies that, in response to the fall in z, consumption contracts substantially more than without the constraint. As households would like to borrow more, the consumption based interest rate R_{t+1} must increase. For this to happen, there is a large increase in the loan interest rate ϱ_t, as shown in the figure. Note that the size of this increase is enhanced by the behavior of the exchange rate, since there is a real depreciation on impact (reflecting the fall in the demand for nontradables), and a subsequent appreciation that reduces the consumption based interest rate (see (5)).

[10] All figures are found at the end of the paper.

Hence the model implies that binding financial constraints amplify the real impact of adverse external shocks. It bears stressing that the assumption in Figure 1 is that the fall in z is large enough so that the debt constraint becomes binding. If it does not, the impulse responses just coincide with the ones without financial constraints (in the figure, the solid and dashed lines would have coincided if the fall in z had been small enough).

Figure 2 shows the first one thousand periods of a typical simulation. The figure illustrates two aspects of the calibration. First, the value of the debt limit, given by $\theta\tilde{k}$, combines with the stochastic process for exogenous shocks to give the frequency with which financial constraints bind. For the figure, I assume i.i.d. shocks with standard deviation of one percent. Then $\theta\tilde{k}$ is set so that the collateral constraint binds about one fourth of the time. This may be too frequent for realism, but again my purpose here is to illustrate the workings of the model.

Second, the figure emphasizes that times of binding constraints are also times of high volatility in consumption, the real exchange rate, and interest spreads.

We turn to the impact of intervention policy. To start, assume that intervention is simply given by an exogenous autorregresive process with a zero lower bound:

$$b_t = Max\{0, (1-\rho_b)\bar{b} + \rho_b b_{t-1} + \varepsilon_{bt}\}$$

where ε_{bt} is an i.i.d. process, which could be interpreted as an unanticipated central bank purchase of reserves. Here, $\bar{b}$ is the steady state stock of reserves. For ease of exposition, we assume that $0 < \bar{b} < (1+\theta)\tilde{k} - \bar{l}$, that is, that in the steady state foreign reserves are strictly positive and the external constraint does not bind. (Note that we have not provided any rationale for this policy rule. Instead, we study its implications hoping to obtain insight about the way intervention may or may not work.)

Under the above assumption on $\bar{b}$, and intuitively, small FX operations (i.e. values of ε_{bt} of small absolute value) do not affect real equilibria, and there are matched one for one by changes in d_t. To preserve space we omit the (boring) impulse responses.

With sufficiently large ε_{bt}, the implications are asymmetric. A large negative ε_{bt} amounts to a large sale of official reserves. But reserves are bounded below by zero, so the central bank runs out of reserves. This is the only real consequence in the model, however: the fall of reserves is completely offset by a decrease in external debt, leaving domestic credit untouched.

In contrast, a sufficiently large unanticipated purchase of reserves (positive ε_{bt}) brings the economy to the financially constrained region. Figure 3 depicts the implications. As in the previous figures, the dashed green lines depict impulse responses to a positive ε_{bt} in the absence of financial constraints. In this case, as shown in the figure, the accumulation of reserves would be exactly matched by an increase in the external debt of the banks, with no other real effect. In contrast, the solid blue lines are the responses taking into account financial constraints. The central bank intervention requires an increase in the amount of stabilization bonds, leading domestic banks to borrow abroad up to the credit limit. In this case, the economy remains financially constrained for two periods. Because of the credit limit, loans to domestic households must fall, which explains the fall in consumption, the increase in the loan interest rate, and the real exchange rate depreciation. Finally, the real depreciation is responsible for the output increase on impact. In short, the large purchase of FX reserves leads to the exhaustion of external credit and a domestic credit crunch.

This discussion illustrates the main trade-off associated with the average level of reserves $\bar{b}$. A low $\bar{b}$ raises the possibility that the central bank runs out of reserves. A high $\bar{b}$, on the other hand, requires external credit and uses up some of the country's credit limit, making the economy more likely to fall into the financially constrained region in response to exogenous shocks.[11]

To illustrate further, Figure 4 shows how the response of external debt to an unanticipated purchase of reserves depends on the average value of reserves $\bar{b}$. The dashed line corresponds to a lower average level of reserves (lower $\bar{b}$) than the solid line. In each case, the figure shows the

[11] This argument is reminiscent of that of Alfaro and Kanczuk (2006) in the context of sovereign debt. In their model, increased official reserve levels may reduce the amount of sovereign debt that is sustainable. The mechanisms in that paper, however, are quite different to ours, and they do not bear on the issue of FX intervention.

response of debt relative to its steady state value, which depends on $\bar{b}$ (since $\bar{d} = \bar{l} - \bar{b}$). The purchase of reserves is of the same magnitude and results in the external constraint binding in both cases. However, as shown, with lower $\bar{b}$, external debt can expand by more before hitting the credit limit. In addition, the economy exits the constrained region faster than with higher $\bar{b}$.

The above considerations help understanding the implications of intervention rules that respond to endogenous variables, such as exchange rates. Consider, for instance, an intervention rule of the form:

$$b_t = Max\{0, (1 - \rho_b)\bar{b} + \rho_b b_{t-1} - \upsilon_e(e_t - \bar{e})\} \tag{7}$$

Assuming $\upsilon_e \geq 0$, the rule has the central bank buying foreign exchange when the exchange rate is stronger than its steady state value, and selling it when the exchange rate is abnormally weak. The size of the response is given by the coefficient υ_e.

By now, it should be apparent how the policy can aid stabilization in the face of adverse shocks that make financial constraints bind. For the policy prescribes that, in such events, the central bank sell reserves in response to the real depreciation. The resulting fall in the quantity of stabilization bonds frees domestic banks to extend additional credit to households, who helps them smooth consumption. This is depicted in Figure 5. In the figure, the dashed green lines are impulse responses to a fall in z assuming that the FX intervention rule does not respond to the exchange rate ($\upsilon_e = 0$) . In fact, there is no FX intervention at all in that case, even if the shock is assumed to be large enough that the economy hits the credit constraint, as in the figure. Domestic credit increases, but not enough to satisfy the increased demand for credit. Consumption then falls, the exchange rate depreciates, and the interest rate on loans goes up.

The solid blue lines assume that $\upsilon_e > 0$. Now the policy rule prescribes a sale of reserves, since the fall in z leads to real depreciation. As it does so, the central bank retires stabilization bonds, freeing resources for domestic banks to increase loans to households. The figure shows that the fall in consumption is then less acute, and the adjustments in the real exchange rate and interest rates less sharp.

We see, therefore, that an FX intervention rule of the form (7) can stimulate the economy when financial constraints become binding. This is beneficial insofar as domestic residents would be willing, at those times, to borrow more than they can, at the external rate of interest. Rules of this kind, however, also have pitfalls. In particular, they prescribes intervention in response to exchange rate movements even when financial constraints do *not* bind. In our model as we have seen, this is best ineffective and, at worst, it can be detrimental.

To see this, suppose that financial constraints do *not* bind, and the economy is hit by an unanticipated increase in z. The economy can then afford more consumption, which could be beneficial, at least in principle. The intervention rule introduces a concern, however: since the exchange rate must appreciate, the central bank will accumulate reserves, according to the rule.[12] If the accumulation of reserves is small, the economy remains financially unconstrained, although sterilization brings the economy closer to its credit limit, making it more vulnerable to subsequent adverse shocks, as we have seen. More damagingly, if the increase in reserves is large enough, the financial constraint become binding. In order to accommodate the sterilization bonds of the central bank, domestic banks must then reduce loans to households. In other words, foreign exchange purchases in response to real appreciation can end up crowding private credit out.

This is depicted in Figure 6. As before, dashed lines are impulse responses when there are no financial constraints. An unanticipated increase in z induces the representative household to consume more and borrow less. Given the increase in consumption demand, the exchange rate appreciates. Following the intervention rule (7), the central bank then buys foreign exchange. In the figure, the increase in the quantity of sterilization bonds more than compensates for the fall in the private demand for credit, and external debt increases. In the absence of financial constraints, increased external borrowing does not affect the cost of domestic loans.

In the presence of financial constraints, however, the FX intervention rule makes the economy hit the external constraint, which remains binding for several periods. To finance the increased

[12] Note that this aspect of the model is consistent with evidence (e.g. Chinn 2017) that reserves accumulation is associated with a larger current account balance.

supply central bank sterilization bonds, domestic credit falls by more than in the absence of financial constraints. This means that domestic consumption must fall relative to the financially unconstrained case; this is accomplished via an increase in the interest rate on domestic loans, as shown in the figure (solid blue lines). The weaker response of consumption also explains why the exchange rate appreciates by less than in the unconstrained case. Hence the FX intervention policy looks like it succeeds at stabilizing the exchange rate. But this is the case only because it generates a credit crunch.

The disadvantage of a FX intervention rule that responds to the exchange rate is, therefore, that it prescribes intervention even when not justified by binding financial constraints. This suggests a superior strategy: intervention should occur in response to interest rate spreads. A suitable rule might be:

$$b_t = Max\{0, (1-\rho_b)\bar{b} + \rho_b b_{t-1} - \upsilon_\varrho(1+\varrho_t - R_t^*)\} \tag{8}$$

with $\upsilon_\varrho \geq 0$ giving the elasticity of central bank sales to widening spreads. Under this rule, that the central bank sells foreign exchange, relaxing financial constraints, when the loan interest rate increases above the cost of international credit. This means that FX sales occur when financial constraints bind. When financial constraints do not bind, however, the spread is zero in our model, so that no intervention is called for (over and above what is required to bring the level of reserves back to its steady state value $\bar{b}$).

Responses to a fall in z with the above rule are given in Figure 7. The fall in z raises the households' demand for credit, which banks attempt to meet by borrowing abroad. As the credit limit is hit, the spread of the domestic loan rate over the foreign interest rate widens. The intervention rule then implies that the central bank sells reserves. The associate reduction in stabilization bonds then allows banks to expand domestic further. This helps stabilizing credit spreads, consumption, output, and the exchange rate.

The responses in Figure 7 are similar in shape to the ones in Figure 6, and the intuition is also very close. The main difference is variable to which FX intervention reacts to (the exchange rate in Figure 6, credit spreads in Figure 7). But this difference is crucial: when financial constraints do not bind, there is active FX intervention with the exchange rate-based policy, but none with the spreads-based policy.

This subsection indicates that the analysis of sterilized intervention should pay close attention to the interplay between intervention, official reserves, and occasionally binding financial constraints. Such a focus promises to deliver useful insights and potentially valuable lessons for policy. We have seen, for example, that an intervention rule that responds to the exchange rate can be improved upon by a rule that reacts to credit spreads. Further study of the properties and consequences of intervention rules should prove fruitful for future research.

4 Nominal Rigidities and Monetary Policy

As claimed earlier, our analysis of sterilized intervention is easily amended to study its interaction with conventional monetary policy. To show how, in this section I drop the assumption of nominal price flexibility, and instead adopt the well known Calvo pricing protocol. Because this specification is well known, I only give a brief description here, and refer interested readers to Gali (2015) for details.

In any given period, an individual producer can set a new price for her product only with some probability $(1-\theta) < 1$. Because producers cannot set prices every period, they do not set the static optimal markup when they can, and equation (2) is dropped. Instead, producers able to change prices choose them so that the markup over marginal cost is optimal, on average, for the random interval of time until they can change prices again. As shown in Gali (2015), to a first order approximation, domestic inflation, denoted by $\pi_{ht} = \log P_{ht} - \log P_{ht-1}$, is then given by

$$\pi_{ht} = \beta E_t \pi_{h,t+1} + \lambda(\log mc_t - \mu) \tag{9}$$

where $mc_t = MC_t/P_{ht}$ denotes marginal cost in terms of domestic goods, $\mu = \log(1 - \frac{1}{\epsilon})$ is its steady state value (in logs), and the coefficient λ is given by

$$\lambda = \frac{(1-\theta)}{\theta}(1-\beta\theta)$$

Domestic inflation now depends on current and future real marginal costs. In turn, real marginal costs in our model are determined by technology, as given by (1), and optimal labor supply (4):

$$mc_t = \frac{(W_t/A)}{P_{ht}} = \eta e_t^{1-\alpha} c_t^{\sigma} y_t^{\phi} / A^{1+\phi} \tag{10}$$

Solving the model now requires one more equation, which is given by a monetary policy rule. Our model is cashless but, as discussed by Woodford (2003), this is not an issue if monetary policy is given by an appropriate interest rate rule of the Taylor type. As advocated by Romer (2000), here we assume that the central bank sets policy in order to steer the expected real interest rate:

$$i_t \equiv E_t R_{t+1} = E_t(1+\varrho_t)\left(\frac{e_{t+1}}{e_t}\right)^{\alpha}$$

Then we posit a rule of the Taylor type, such as:

$$i_t = \log R_t^* + \phi_\pi \pi_t + u_{mt} \tag{11}$$

To get a sense of the implications, Figure 8 displays impulse responses to a contractionary monetary shock (positive u_{mt}), assumed to be large enough to place the economy in the financially constrained region. The dashed green lines assume no financial constraints, while the solid blue lines take binding constraints into account. In both cases, the shock directly raises the expected consumption based interest rate (by assumption), and therefore consumption growth. In response, consumption must fall on impact.[13] Households attempt to cushion the blow by borrowing from domestic banks. Domestic loans (not shown) then increase in both

[13] Note that, for this experiment I assumed that that the coefficient of risk aversion is 2. This is because, under log utility, there is no impact on the level of debt.

cases; but this mechanism is limited if there are financial constraints. As the figure shows, the external credit constraint is reached on impact: if there had been no constraints (dashed lines), consumption would fall less and external debt would increase more than in the presence of constraints (solid lines). To ration credit in the case of binding constraints, the interest rate on domestic loans, $1 + \varrho_t$, rises above and over the world interest rate.

This exercise emphasizes that not only monetary policy is powerful in this model, but also that binding financial constraints can exacerbate the impact of monetary shocks on domestic demand. One may note, on the other hand, that the exchange rate appreciates, but binding financial constraints reduce the extent of the appreciation. As a consequence, domestic inflation and output fall by less and that the policy rate increases than more than in the absence of financial constraints. The intuition is that, when there are no financial constraints, the loan rate ϱ_t is pinned down by the external rate R_t^*, so that a given raise in the expected rate i_t is accomplished solely via an increase in the expected depreciation rate. In contrast, when financial constraints bind, the raise in i_t is accomplished partly with an increase in the interest rate spread, requiring a comparatively smaller increase in expected depreciation, and consequently a smaller appreciation on impact.

We might now ask about the role of sterilized foreign exchange intervention. Our first observation is that, as in the model with flexible prices, intervention does not have real effects if it occurs at times of nonbinding constraints. The argument is virtually the same as in subsection 3.1, except that the relevant system of equilibrium equations excludes (2) and includes (9), (10), and (11). The intuition is unaltered: if the collateral constraint does not bind at t, any change in b_t (which leaves the constraint still not binding) is offset one for one by a change in d_t, without any impact on equilibrium.

A notable implication is that, independently of monetary policy, intervention policy does not affect real allocations for shocks that are small enough so as not to make financial constraints bind. This is clear under intervention rules such as (7) or (8). If intervention is triggered by abnormally high credit spreads, as with (8), there is no intervention at all as long as constraints do not bind. With an intervention rule that responds to the exchange rate, as with (7), shocks that do not result in binding financial constraints do trigger sales or purchases of reserves, but ones that are fully accommodated by changes in external debt d_t, with no other real impact.

For large enough shocks, financial constraints bind and, as we have stressed, FX intervention does have real effects. In that kind of situation, intervention can complement conventional monetary policy. To illustrate, Figure 9 displays responses to a fall in z, assuming a Taylor rule like (11). In the figure, the dashed green lines depicts responses when there is no active intervention, while the solid blue lines give responses when intervention responds to spreads. The figure also assumes that, whether there is active intervention or not, financial constraints are present and become binding under the shock.

The figure shows that, without an active intervention response, the shock would raise the domestic demand for private loans. Banks would then borrow abroad up to the credit limit, and the loan interest rate would increase to ration credit. Consumption demand would fall, leading to a real exchange rate depreciation. The depreciation would imply an increase in the foreign demand for domestic output and an overall output increase. As a consequence, domestic inflation would increase. Then the Taylor rule would prescribe an increase in the policy interest rate.

With an intervention rule as (8), the increase in spreads prompts the central bank to sell reserves. As discussed, the corresponding fall in sterilization bonds allows for domestic loans to increase by more than in the absence of intervention. For this calibration, the intervention rule has negligible effects on the impact response of consumption, although it implies a smoother transition back to the steady state. More notably, the active intervention rule moderates the exchange rate depreciation, and hence the increases in output and domestic inflation.

Clearly, one could expand further on the specifics of this analysis and the consequences of different combinations of monetary rules and intervention policy. This is outside my main purpose here, however, which is to emphasize that our perspective on sterilized intervention can straightforwardly be combined with standard monetary policy analysis.

This being said, one notable and general lesson from our discussion is that, in the presence of financial frictions, the question of whether sterilized intervention can be an independent policy instrument has an unambiguously positive answer. But the answer differs substantially from others offered in the recent literature. Sterilized intervention is ineffective locally: it cannot benefit for shocks small enough that financial constraints do not bind. On the other hand, intervention can help when the constraints bind, and in that case it works by alleviating the external credit limit.

In short, nonlinearities are essential, and proper analysis of intervention requires going beyond current approaches that restrict attention to local approximations around the steady state.

5 The Role of Financial Dollarization

To this point we have assumed that the economy is "financially dollarized", in that all financial instruments are denominated in dollars. This is partly because some actual economies are financially dollarized, and partly to emphasize that our basic arguments do not depend on currency mismatches or debt denomination. Often, however, some securities are denominated in domestic currency (pesos) along with others that are foreign currency (dollar) denominated. In this section I show how to modify our model to allow for peso securities and argue that, while some additional effects are introduced, our line of reasoning remains largely untouched.

Assume now that domestic loans and central bank bonds are denominated in pesos, paying a gross interest rate R_t^n between periods t and $t+1$. What is crucial is that R_t^n is determined in period t : the arguments of previous sections obviously apply if return on peso securities were indexed to, say, the dollar. Under our new assumption, the *dollar* return on loans and bonds between t and $t+1$ depends on the realized rate of depreciation, and is given by

$$R_{t+1}^d = R_t^n \frac{E_t}{E_{t+1}}$$

Observe the notation: the subscript on R^d_{t+1} emphasizes that it is a random variable that becomes known only at $t+1$.

Because the dollar rate of return on domestic loans is unknown as of period t, we need to amend our analysis of the decision problems of domestic agents. To simplify things, we just assume from now on that domestic banks belong to households, which provide banks with equity $\tilde{k}$. Then the typical bank's problem is to maximize the discounted expected value of dollar profits:

$$E_t M_{t+1} \pi_{t+1}$$

where

$$\pi_{t+1} = R^d_{t+1}(l_t + b_t) - R^*_t d_t$$

subject to $b_t + l_t = \tilde{k} + d_t$ and the collateral constraint $d_t \leq \theta \tilde{k}$, where M_{t+1} is the household's discount factor for *dollar* payoffs, which we derive shortly.

The first order conditions to this problem imply that the collateral constraints now can be written as:

$$\begin{aligned} d_t &= \theta \tilde{k} \quad \text{if } E_t M_{t+1}(R^d_{t+1} - R^*_t) > 0 \\ &\leq \theta \tilde{k} \quad \text{if } E_t M_{t+1}(R^d_{t+1} - R^*_t) = 0 \end{aligned}$$

Note that these conditions are very similar to the ones we derived earlier, in the case of financial dollarization.

The analysis of the central bank is the same as before, observing only that the quasifiscal deficit in period t is now given by

$$T_t = (R^d_t - R^*_{t-1}) b_{t-1}$$

and hence it depends on the realized rate of depreciation.

Lastly, the household's problem is solved just as before, but now we need to take into account that the dollar interest rate on loans taken at t is R^d_{t+1} instead of $1+\varrho_t$, and hence it is uncertain as of period t. The Euler condition for loans then becomes:

$$c_t^{-\sigma} = \beta E_t c_{t+1}^{-\sigma} R^d_{t+1} \left(\frac{e_{t+1}}{e_t}\right)^{\alpha}$$

or

$$1 = E_t M_{t+1} R^d_{t+1}$$

which identifies the dollar discount factor as:

$$M_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\sigma} \left(\frac{e_{t+1}}{e_t}\right)^{\alpha}$$

The expected consumption-based real rate is $E_t R^d_{t+1} \left(\frac{e_{t+1}}{e_t}\right)^{\alpha}$. With these modifications, we can retrace the analysis above, without significant change.

To illustrate, Figure 10 presents impulse responses to a fall in z. The figure assumes a Taylor rule of the form (11), and an intervention rule similar to (8) but with $E_t R^d_{t+1} - R^*_t$ as the relevant spread. In the absence of financial constraints (dashed green lines), the shock would be accommodated by increased household borrowing, and an increase in the banks' external debt, without noticeable impact on real variables or inflation. Given the policy rules, the central bank does not change the policy interest rate nor intervenes in the foreign exchange market.

The shock is assumed to be large enough for external debt to hit the credit limit, however. As discussed before, adjustment then entails a larger fall in consumption, which requires an increase in the real interest rate. This is accomplished via a relatively large devaluation and, in this model, an increase in the nominal peso interest rate on loans. The monetary policy rate increases in response to rising inflation, and reserves fall because the intervention rule prescribes foreign exchange sales as credit spreads widen.

No major differences between this case and the one of the previous section emerge. In other words, assuming that domestic securities are denominated in pesos or dollars does not appear to have but a minor effect. In this model, the denomination of domestic securities only results in some unanticipated transfers between households and banks, which have little impact on equilibrium.

This being said, it must be noted also the presence of peso securities might result in currency *mismatches* interacting with credit constraints, resulting in potentially much more significant balance sheet effects. Such effects could presumably be added to our model. For instance, one might assume that the equity constraint is denominated in pesos, implying that $e_t^{\alpha} k_t \leq \tilde{k}$ rather than $k_t \leq \tilde{k}$. Then a real depreciation would cause a reduction in bank equity, tightening the debt limit. A plausible conjecture is that sterilized intervention might have a larger role in this context, but exploring this issue is outside the scope of the present paper.

6 Final Remarks

This paper has proposed an alternative perspective on the way sterilized foreign exchange intervention works, and developed several implications for theory and policy. As stressed in the introduction, this perspective can help reconcile theory and practice in compelling, intuitive ways.

As for the theory, we have seen that occasionally binding financial constraints imply that sterilized intervention can have real effects, but only at some specific times, if it relaxes the financial constraints when they bind. This result is quite consistent with standard theory, but it implies that sterilized intervention is not always irrelevant. And in fact, our analysis suggests that intervention can be powerful when it matters the most.

Our analysis also suggests that sterilized intervention may be irrelevant much, or even most, of the time. In this sense, it is no surprise that empirical evidence for significant effects of intervention has been elusive. Future empirical research should examine whether the impact of intervention depends on the incidence of financial constraints.

More generally, our analysis stresses that the impact of sterilized intervention may depend on the degree of financial frictions as well as the nature of financial institutions. This suggests that, empirically, the effectiveness of intervention should differ across countries, according to their degree of financial development.

As stressed in Céspedes, Chang, and Velasco (2017), the model in this paper suggests that it may be beneficial to sell reserves in response to an excessive depreciation, if "excessive depreciation" is to be understood as depreciation at times of binding constraints. On the other hand, there is no gain in fighting exchange rate *appreciation* over and beyond replenishing official reserves for the central bank to be ready to deal with future adverse shocks. In this sense, our analysis does not offer a justification of observed episodes of reserves accumulation that appear to be motivated by competitiveness reasons. Presumably one could extend our model in such a direction, but that extension is outside the scope of this paper.

For exposition, we made some very specific assumptions. One of them was that the central bank used its own sterilization bonds in sterilization operations. A little thought should convince the reader that this assumption is much less restrictive than it appears. For instance, suppose that, to sterilize purchases of official reserves, the central bank sells government debt instead of its own debt. The impact of this operation would be exactly the same as the one in this paper, assuming that government debt has to be absorbed by domestic banks. In fact, a useful way to look at this alternative may be to think of the "central bank" of our model as a consolidated entity encompassing the central bank plus the fiscal authority.

A second assumption worthy of additional comment is that sterilization bonds are held exclusively by domestic banks. This implies that intervention can have real effects by relaxing or tightening the external credit limit of the banks. But the assumption may appear unrealistic, especially in cases where central bank can sell debt to foreigners. It should not be too hard, however, to relax the assumption in realistic ways while preserving the essence of our analysis. For example, as in Montoro and Ortiz (2016) or Cavallino (2017), one might posit foreign investors that specialize in trading domestic securities, including stabilization bonds. If those investors are themselves constrained by some kind of market segmentation, financial imperfection, or credit limit, their presence and intermediation activities may not suffice for domestic agents to have unrestricted access to international credit. Assuming that the financial frictions are such that access is interrupted occasionally, sterilized intervention is likely to have real effects similar to the ones we have discussed, and for essentially the same reasons.

Finally, we had to commit to particular assumptions on the central bank quasifiscal deficit. If those assumptions were to be dropped, one would have to supply further detail about how the quasifiscal deficit is financed and, further, what determines the evolution and management of the central bank's net worth. These are not trivial issues, but best left for future research. Let us only remark that this question is related to the more general claim that unconventional policy may matter if there are frictions in the links between the central bank and fiscal authorities. See, for example, Benigno and Nistico 2015).

This paper focused on the transmission mechanism behind intervention, and suggested ways in which intervention may be beneficial in terms of welfare. But it did not attempt to characterize welfare maximizing policy, which is a promising avenue for future study.

A related question is that of optimal reserves management. Our discussion has identified a novel trade-off in accumulating reserves: larger reserves place the central bank in a better position to deal with suddenly binding financial constraints; but financing the stock of reserves may imply that the constraints bind more often. This indicates that the analysis of optimal reserves may involve this trade-off and combine insights from the recent macroprudential policy literature with the perspective on intervention discussed here.

Appendix

Here we provide details on the calibration used for the examples and illustrations. I assume that there is a steady state in which the external constraint does not bind. (It should be noted that this assumes that FX intervention policy is consistent with such a steady state.)

We denote steady state values with overbars. Then, $1+\bar{\varrho}=\bar{R}^{*}$ (which here denotes the steady state value of both $\bar{R}_t^{*}$ and R_t^{*}) because financial constraints do not bind. The Euler condition then requires that $\beta\bar{R}^{*}=1$, as usual.

The steady state values of y, c, and e must satisfy:

$$
\begin{aligned}
\bar{y} &= \alpha\bar{e}^{(1-\alpha)}\bar{c}+x\bar{e}^{\chi} \\
(1-\alpha)\bar{e}^{-\alpha}\bar{c}-[\bar{z}+\varkappa\bar{e}^{\chi-1}] &= -\bar{r}^{*}(\bar{d}-\bar{b}) \\
1/\eta(1-1/\varepsilon) &= \bar{e}^{(1-\alpha)}\bar{y}^{\phi}\bar{c}^{\sigma}/A^{1+\phi}
\end{aligned}
$$

where $\bar{r}^{*}=\bar{R}^{*}-1$

For calibration, I impose that the steady value of e be one, and that the trade balance surplus be one percent of output (it is common to impose balanced trade in the steady state, but Schmitt Grohe and Uribe 2017 argue in favor or a surplus of two percent of GDP; as a compromise, I impose one percent). Now, from the definition of trade surplus, this requires:

$$
\begin{aligned}
& [\bar{z}+\varkappa]-(1-\alpha)\bar{c} \\
= \; & \bar{z}+\bar{y}-\bar{c} \\
= \; & 0.01y
\end{aligned}
$$

the second equality following from market clearing ($\bar{y}=\alpha\bar{c}+\varkappa$)

Optimal labor supply reduces to

$$\Theta=\bar{c}^{\sigma}\bar{y}^{\phi}/A^{1+\phi}$$

where

$$\Theta = 1/(1 - \frac{1}{\epsilon})\eta$$

I choose parameters so that $\bar{y} = \bar{c} = 1$ as well. For the market clearing condition to be satisfied, this will require $\varkappa = 1 - \alpha$. Also, for optimal output,

$$A = \Theta^{-(1/1+\phi)} = \left((1 - \frac{1}{\epsilon})\eta\right)^{1/1+\phi}$$

and

$$\bar{z} = 0.01$$

Finally, for the country budget constraint to hold, we need that

$$0.01 = \bar{z} = \bar{r}^*(\bar{d} - \bar{b})$$

This restricts $(\bar{d} - \bar{b})$. The usual assumption is that $\bar{b} = 0$; if so, $\bar{d} = \bar{z}/\bar{r}^*$. If we assume $\bar{r}^* = 0.01$,then $\bar{d} = 1$. (Note that this is the ratio of debt to *quarterly* output. So, it corresponds to 0.25 in terms of the usual debt/annual GDP ratio, and so it is in the ballpark.)

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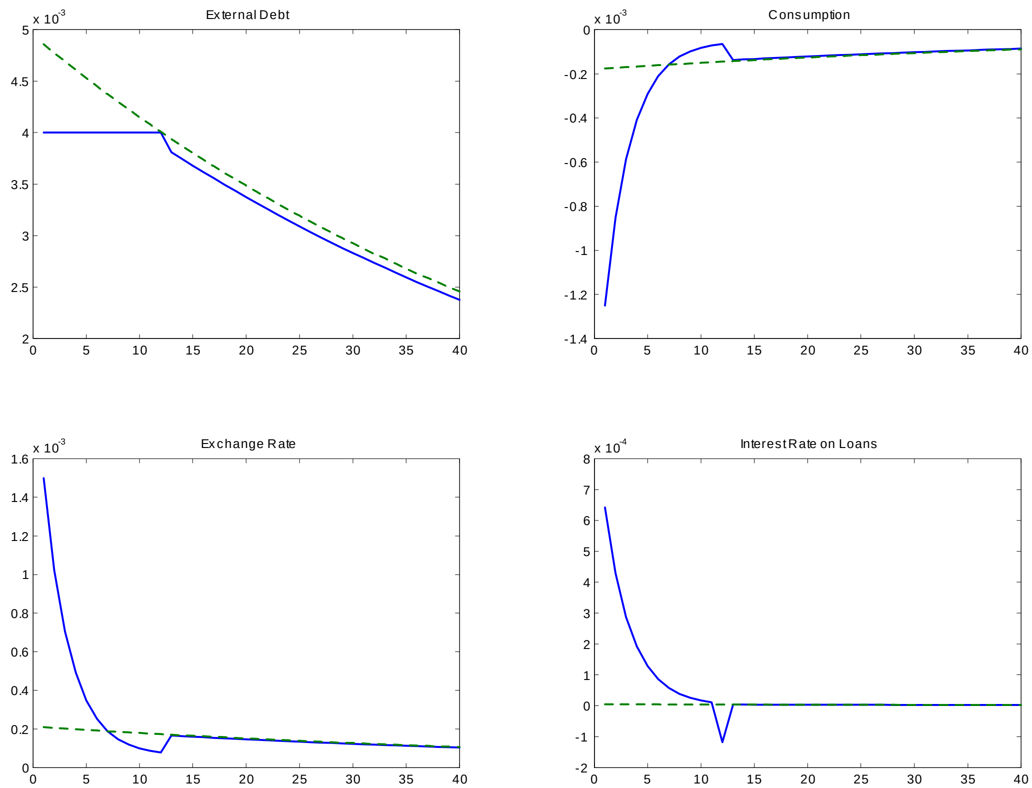
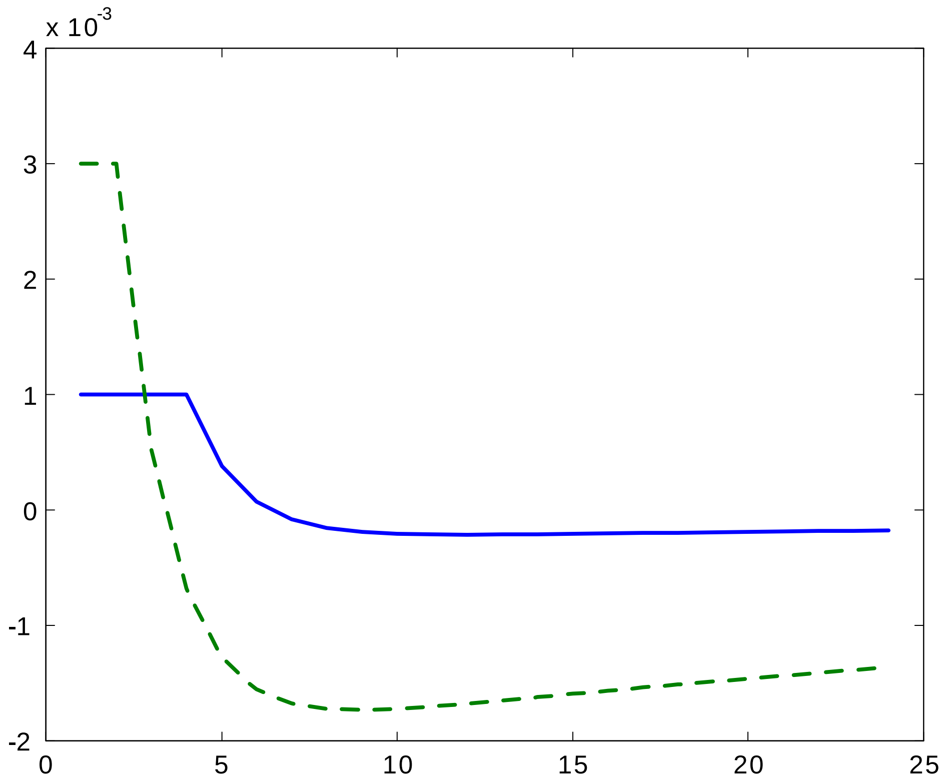

External Debt
x 10-3
Consumption
x 10-3
Exchange Rate
x 10-3
Interest Rate on Loans
x 10-4


x 10-3
4
3
2
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10
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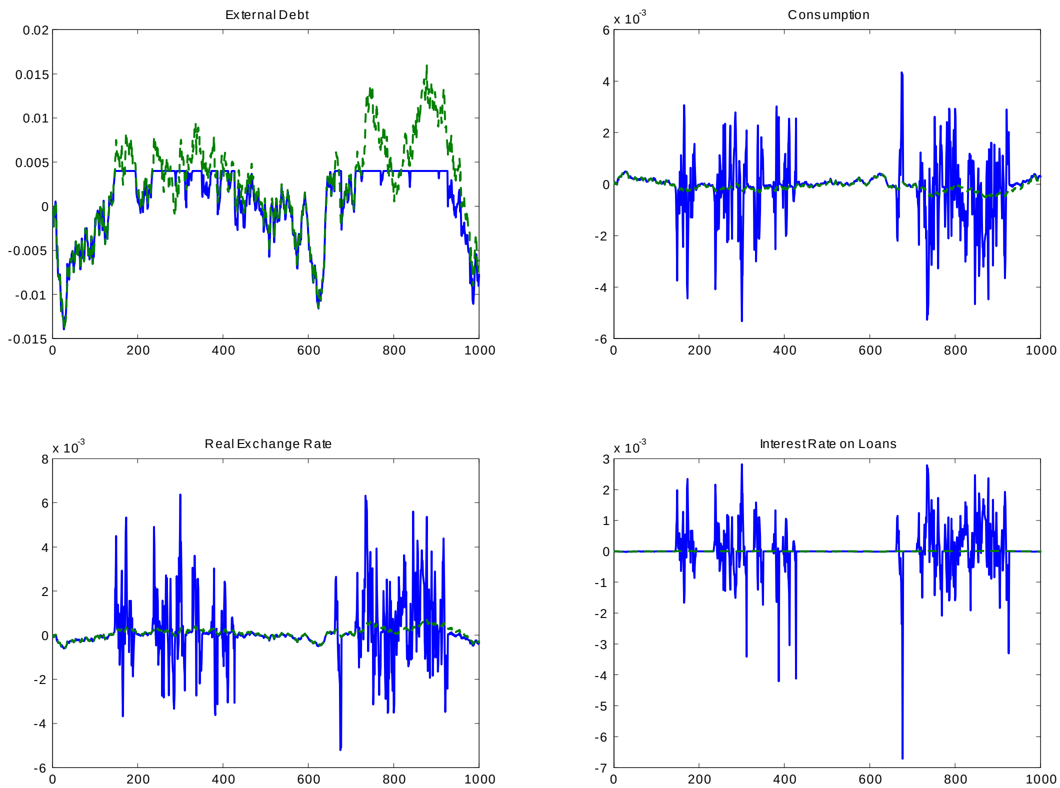
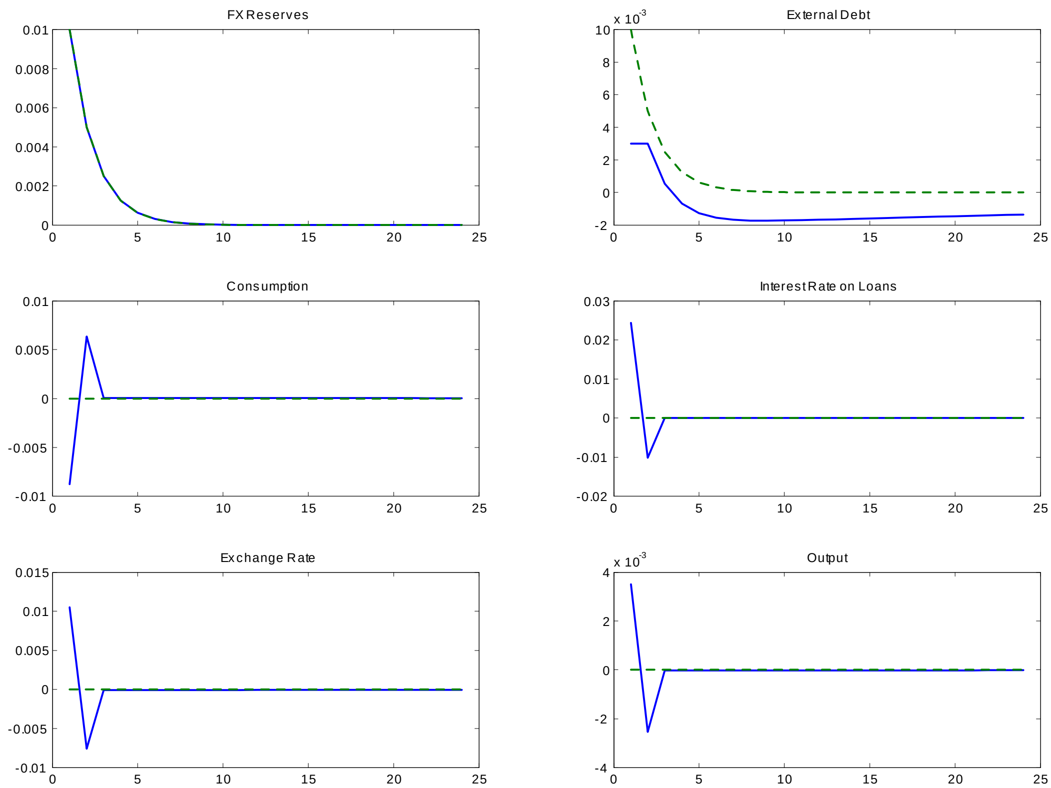
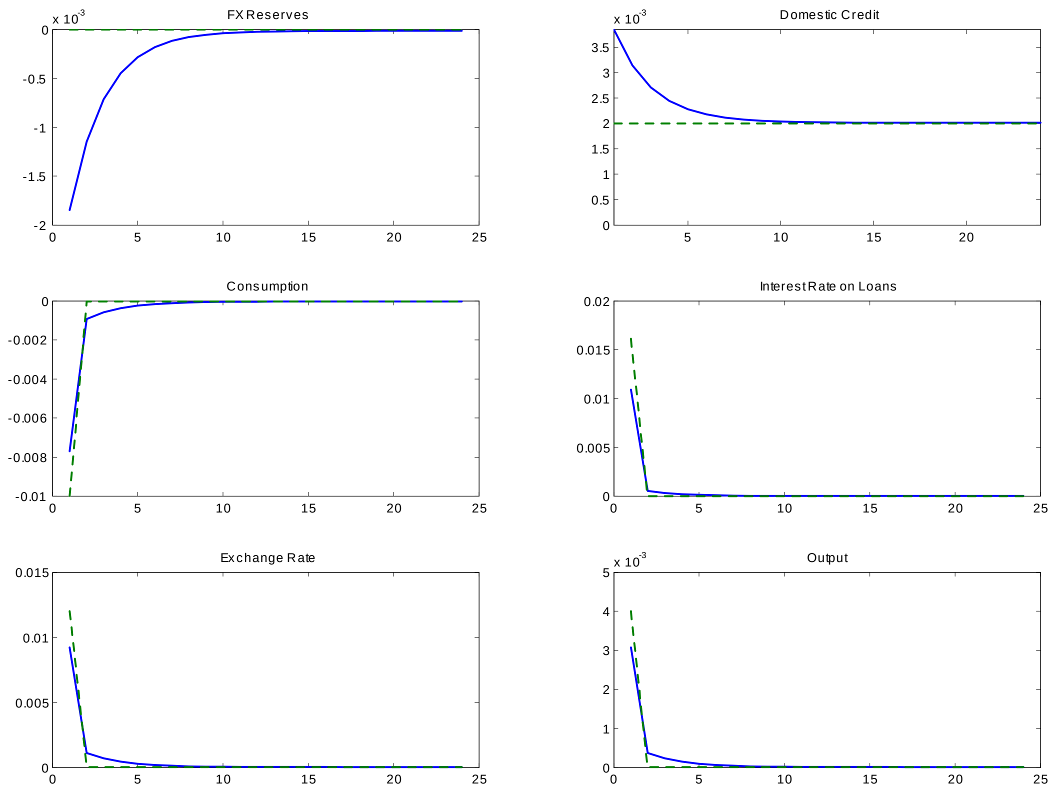
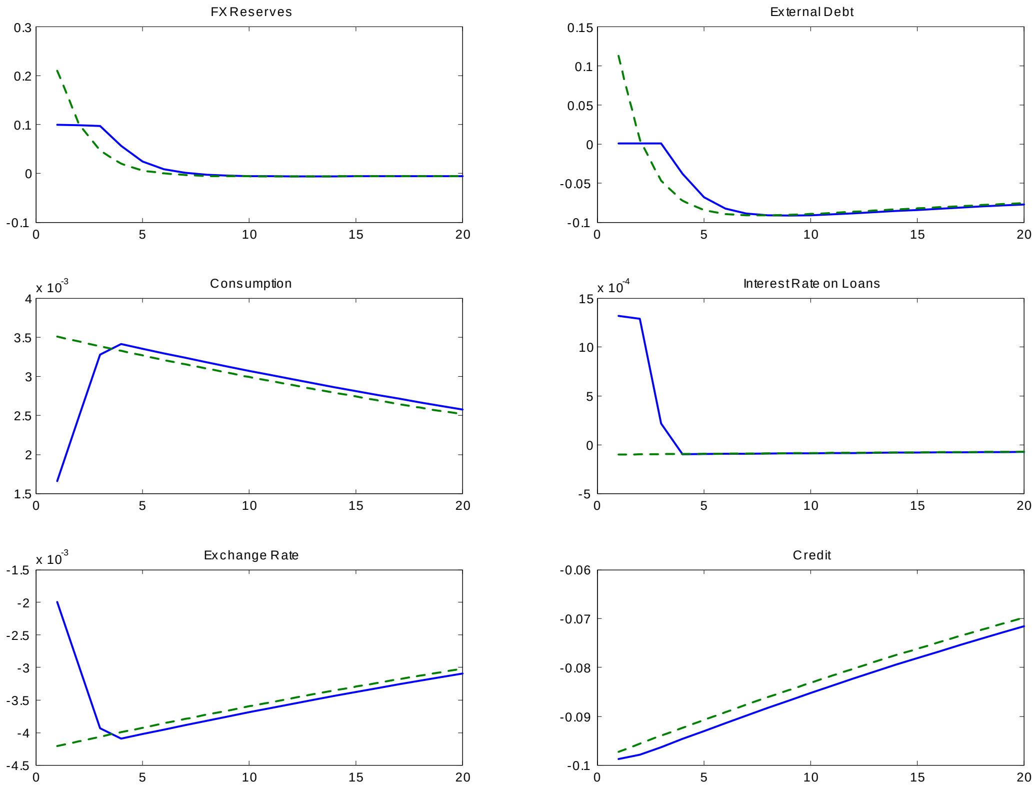
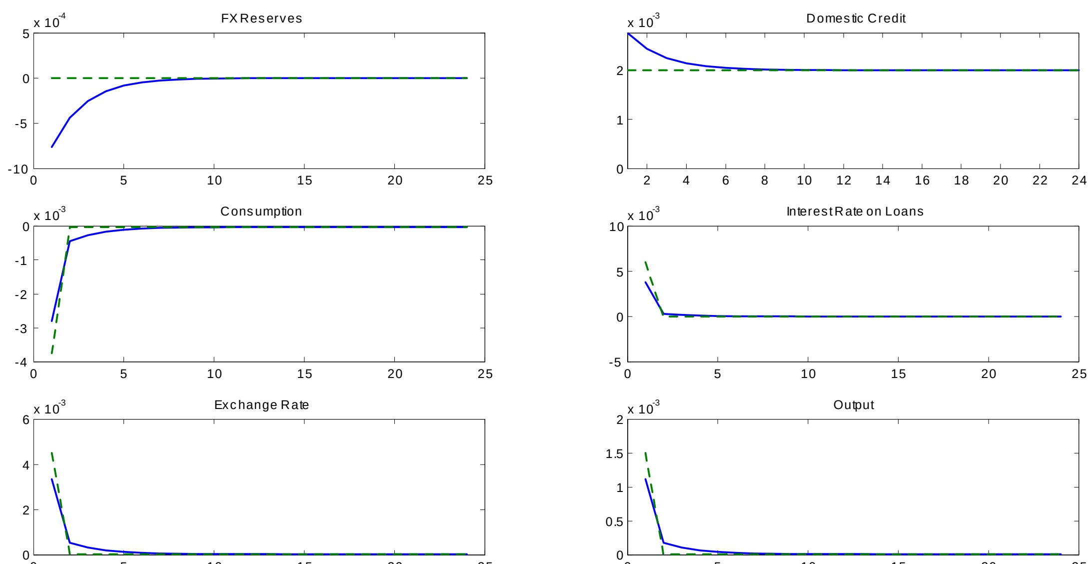
20
25

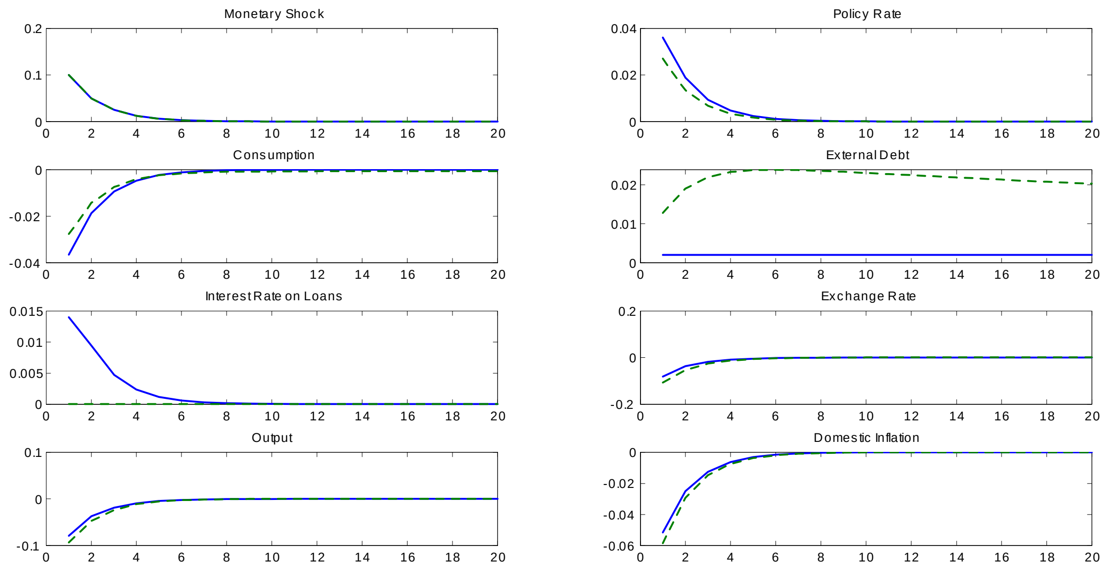

External Debt
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
0
200
400
600
800
1000
Consumption
x 10-3
6
4
2
0
-2
-4
-6
0
200
400
600
800
1000
Real Exchange Rate
x 10-3
8
6
4
2
0
-2
-4
-6
0
200
400
600
800
1000
Interest Rate on Loans
x 10-3
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
0
200
400
600
800
1000

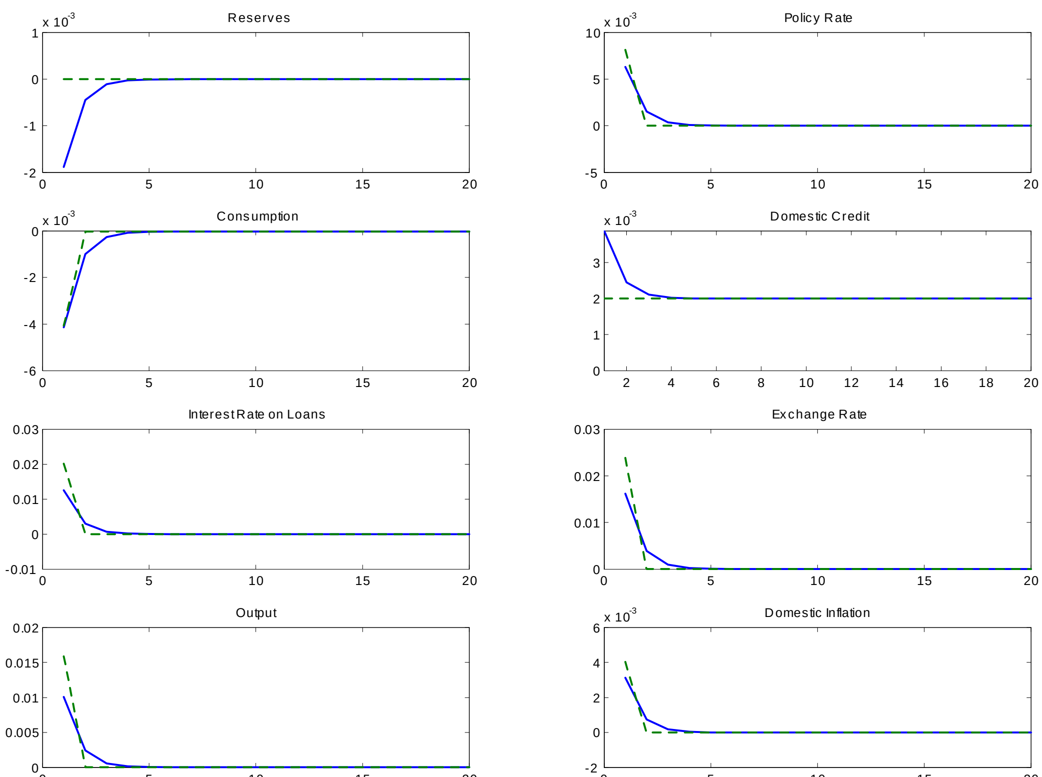

FX Reserves
External Debt
x 10-3
Consumption
Interest Rate on Loans
Exchange Rate
Output
x 10-3

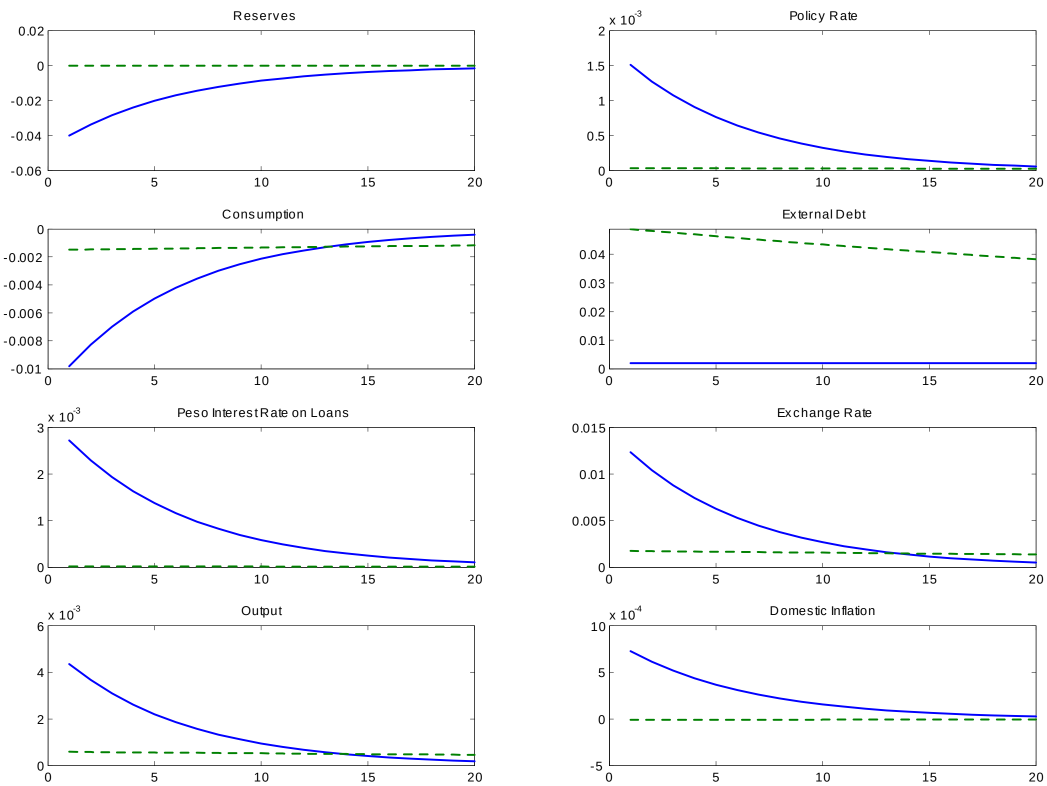

FX Reserves
x 10^-3
Domestic Credit
Consumption
Interest Rate on Loans
Exchange Rate
Output


FX Reserves
External Debt
x 10^-3
Consumption
x 10^-4
Interest Rate on Loans
x 10^-3
Exchange Rate
Credit


x 10^-4
FX Reserves
x 10^-3
Domestic Credit
x 10^-3
Consumption
x 10^-3
Interest Rate on Loans
x 10^-3
Exchange Rate
x 10^-3
Output


Monetary Shock
Policy Rate
Consumption
External Debt
Interest Rate on Loans
Exchange Rate
Output
Domestic Inflation


Reserves
Policy Rate
Consumption
Domestic Credit
Interest Rate on Loans
Exchange Rate
Output
Domestic Inflation


Reserves
Policy Rate
x 10^-3
Consumption
External Debt
x 10^-3
Peso Interest Rate on Loans
Exchange Rate
x 10^-3
Output
x 10^-4
Domestic Inflation