

Supplementary Problem - 4.5.

Q12 Page 280 - "B Level"

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x \cdot \tan x} \stackrel{\text{"DSP"}}{=} \frac{0 - \sin 0}{0 \cdot \tan 0} = \frac{0}{0} = \frac{0}{0}$$

Use L.R.

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{1 \cdot \tan x + x \cdot \sec^2 x} \stackrel{\text{"DSP"}}{=} \frac{1 - \cos 0}{\tan 0 + 0 \cdot \sec^2 0} = \frac{0}{0}$$

(product rule for $x \cdot \tan x$)

Use L.R. again

$$\lim_{x \rightarrow 0} \frac{\sin x}{\sec^2 x + 1 \cdot \sec^2 x + x \cdot 2 \cdot \sec x \cdot \tan x \cdot \sec x} \stackrel{\text{"DSP"}}{=} \frac{0}{1 + 1 + 0} = 0$$

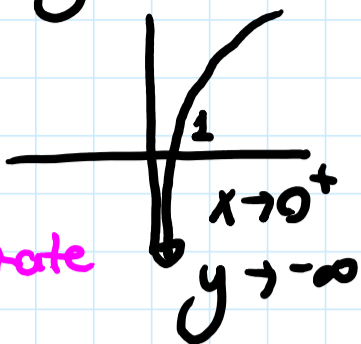
(product rule for $x \cdot \sec^2 x$)

Q37 - Page 281

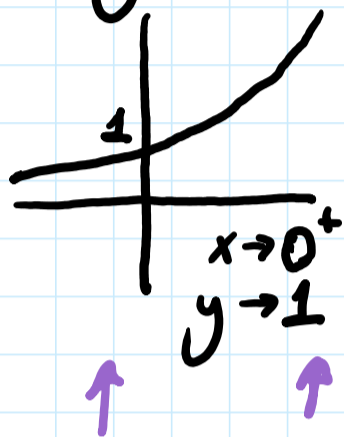
"A Level"

↓ Recall ↓

Graph of $y = \ln x$



Graph of $y = e^x$



$$\lim_{x \rightarrow 0^+} (e^x - 1)^{\frac{1}{\ln x}} \stackrel{\text{"OSP"}}{=} (e^0 - 1)^{\frac{1}{\ln 0^+}} \stackrel{\text{"OSP"}}{=} (1 - 1)^{\frac{1}{-\infty}} = 0^0$$

"Other" Indeterminate form

Use Algebra to re-write:

Let $L = \lim_{x \rightarrow 0^+} (e^x - 1)^{\frac{1}{\ln x}}$

$$\ln L = \ln \left(\lim_{x \rightarrow 0^+} (e^x - 1)^{\frac{1}{\ln x}} \right)$$

$$\ln L = \lim_{x \rightarrow 0^+} \left(\ln (e^x - 1)^{\frac{1}{\ln x}} \right)$$

$$\ln L = \lim_{x \rightarrow 0^+} \left(\frac{1}{\ln x} \cdot \ln (e^x - 1) \right) = \lim_{x \rightarrow 0^+} \left(\frac{\ln (e^x - 1)}{\ln x} \right)$$

"Use OSP"

$$= \frac{\ln (e^{0^+} - 1)}{\ln 0^+} = \frac{\ln (1^+ - 1)}{\ln 0^+} = \frac{\ln 0^+}{-\infty} = \frac{-\infty}{-\infty}$$

"Use LR"

$$= \lim_{x \rightarrow 0^+} \frac{\frac{e^x}{e^x - 1}}{\frac{1}{x}} = \lim_{x \rightarrow 0^+} \frac{e^x \cdot x}{e^x - 1} \stackrel{\text{"OSP"}}{=} \frac{0}{0}$$

"Use LR again"

$$= \lim_{x \rightarrow 0^+} \frac{e^x \cdot (1 + e^x \cdot x)}{e^x} = \lim_{x \rightarrow 0^+} \frac{e^x (1+x)}{e^x} = 1$$

Final Answer

$\ln L = 1$
 $L = e$