Note Title 4.3 - Part 1 Poll Questions 3/26/2020

True/False Every local min/max is a critical #.

TRUE.

True/False Every critical # is a local min/max.

FALSE.

Counter-example: $f(x)=x^3 \Rightarrow f'(x)=3x^2 \Rightarrow x=0$ critical #
however, $x=0$ is NOT a local min/max!

True/False Local min/max may occur at endpoints.

FALSE.

local min/max occur in an open interval;

for example: GLOBAL min/max may occur at endpoints; however, local min/max do not occur at endpoints.

MC What does the first-derivative test tell us?

a) where the graph of $f(x)$ is concave up/down

b) where critical #s of $f(x)$ occur

c) where local min/max of $f(x)$ occur

d) in which intervals $f'(x)$ is increasing or decreasing

C)

Think/Tank Prb. Sketch a graph of $f(x)$ that satisfies the following:

$f'(x) > 0$ when $x < -5$ and $x > 1$

$f'(x) < 0$ when $-5 < x < 1$

$f(-5) = 4$ and $f(1) = -1$

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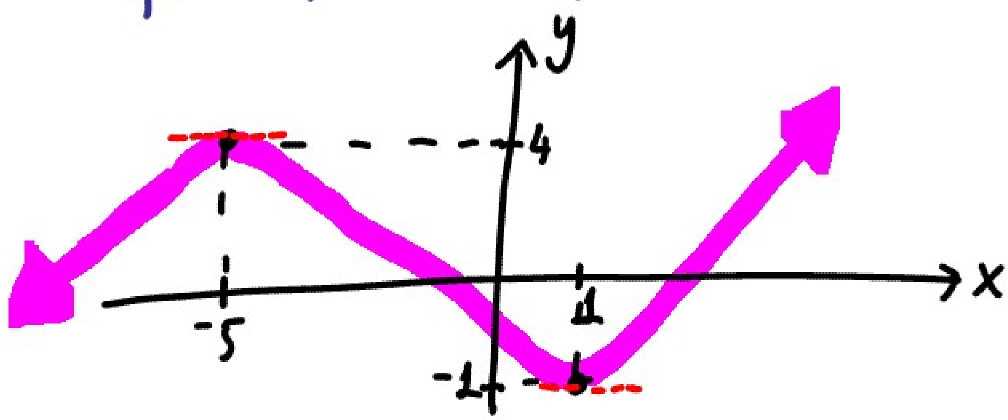
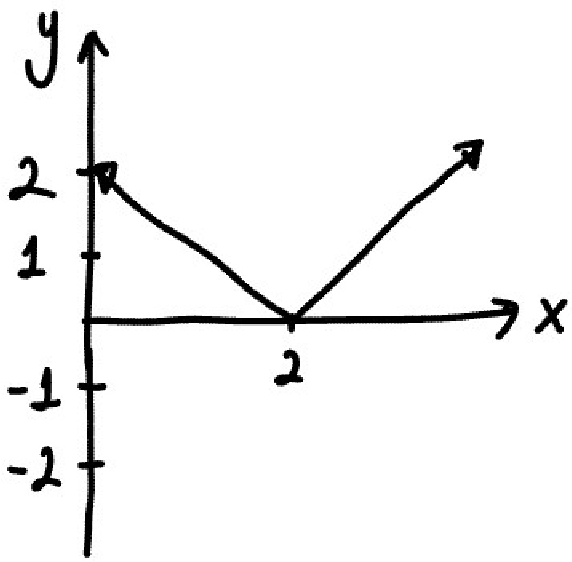


Sketch Draw a curve that represents the derivative of function f.



You try it! Use first-order derivative test to classify each critical number as local min/max or neither.

$f(x) = (x^3 - 3x + 1)^7$ at $x = 1, -1$

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$$f(x) = (x^3 - 3x + 1)^7 \text{ at } x = 1, -1$$

Solution

$$f'(x) = 7(x^3 - 3x + 1)^6 \cdot \underbrace{\underbrace{(3x^2 - 3)}_{3(x^2-1)}}_{3(x-1)(x+1)}$$

identify critical #: $x = 1, -1$

$$f'(x) = 7(x^3 - 3x + 1)^6 \cdot 3(x-1)(x+1)$$

Construct a sign chart:

$$f'(x) = \overbrace{21(x^3 - 3x + 1)^6}^{\text{always positive}}(x-1)(x+1)$$

Cut p: $x = 1, -1$

test p: $x = -10, 0, 10$

test the signs of $\underbrace{f'(-10)}_{+}, \underbrace{f'(0)}_{-}, \underbrace{f'(10)}_{+}$

	-10	-1	0	1	+10
sign of $f'(x)$	+		−		+
$f(x)$ incr./decr.		local max		local min	

Local min. at $x=1$, local max. at $x=-1$.

$f(x)$ is increasing on $(-\infty,-1), (1,\infty)$

$f(x)$ is decreasing on $(-1,1)$

You try it! Use first-order derivative test to classify critical #s ($x=1, \frac{1}{2}$) as local min, local max or neither.

$$f(x)=\frac{e^{-x^2}}{3-2x}$$

Solution:

Step 1) Find critical #s ($f'(x)=0$ or DNE)

$$f'(x)=\frac{(e^{-x^2})'(3-2x)-(e^{-x^2})(3-2x)'}{(3-2x)^2}$$ quotient rule

$$=\frac{-2x\cdot e^{-x^2}\cdot(3-2x)-(e^{-x^2})\cdot(-2)}{(3-2x)^2}$$

$$= \frac{-2\cdot e^{-x^2}\left[x\cdot(3-2x)-1\right]}{(3-2x)^2}$$

$$= \frac{-2\cdot e^{-x^2}\left(3x-2x^2-1\right)}{(3-2x)^2}$$

$$= \frac{-2\cdot e^{-x^2}\left(-2x^2+3x-1\right)}{(3-2x)^2}$$

$$-2x^2+3x-1$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ -2x & \times & 1 \\ x & & -1 \end{array} \Big\} \ 3x \checkmark$$

$$2x + x = 3x$$

$$= \frac{-2\cdot e^{-x^2}(-2x+1)(x-1)}{(3-2x)^2}$$

$\underbrace{f'(x)=0}_{\text{numerator}}$ or $\underbrace{f'(x) \text{ DNE}}_{\text{denominator}}$

$$\underbrace{-2\cdot e^{-x^2}}_{\neq 0}\underbrace{(-2x+1)}_{x=1/2}\underbrace{(x-1)}_{x=1}=0$$

$$(3-2x)^2=0$$

$$x=3/2$$

critical #s are: $x = 1/2,\ 1,\ 3/2$ (cutpoints)

Step 2) construct a sign chart:

	$-\infty$	0	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$ $(5/4)$	$\frac{3}{2}$	2	$+\infty$
sign of $f'(x)$		$+$		$-$		$+$		$+$	
incr/decr. $f(x)$		↗	local max	↘	local min	↗		↗	

testpoints: (points in each sub-interval cut by critical #s)

I chose $x = 0, 3/4, 5/4, 2$

$$f'(x) = \frac{-2e^{-x^2}(-2x+1)(x-1)}{(3-2x)^2}$$

check sign for $-2(-2x+1)(x-1)$

$e^{-x^2} \to$ always pos.

$(3-2x)^2 \to$ always pos. except $x = 3/2$

only check the sign for: $-2(-2x+1)(x-1)$

$f'(0)$ gives $-2 \cdot 1 \cdot (-1) \to (+)$

$f'(3/4)$ gives $-2\left(\frac{-6}{4}+1\right)\left(3/4-1\right)$

$$\underbrace{\ominus \cdot \ominus \cdot \ominus}_{(-)}$$

$f'(5/4)$ gives $-2 \cdot \left(-10/4+1\right)\left(5/4-1\right)$

$$\underbrace{\ominus \cdot \ominus \cdot \oplus}_{\oplus}$$

$f'(2)$ gives $-2(-4+1)\cdot(2-1)$

$\underbrace{- \cdot - \cdot +}_{+}$

In summary: $f(x)$ is increasing on:

$(-\infty, \tfrac{1}{2}), (1, \underbrace{\tfrac{3}{2}), (\tfrac{3}{2}}_{\text{V.A: } x = 3/2}, \infty)$

$f(x)$ is decreasing on: $(\tfrac{1}{2}, 1)$

$x = \tfrac{1}{2}$ is the local max.

$x = 1$ is the local min.