4.3 - Part2 Poll Questions

Note Title 4/2/2020

(True/False) All second-order critical numbers are points of inflection.

(True/False) The second-order derivative test also helps to identify the local min/max.

(True/False) The x-coordinates of vertical asymptotes are also candidates for PoI (point of inflection).

Poll Q- Use the second-derivative test to determine where the function is concave up/down.

$$f(x) = \frac{1}{3}x^3 - 9x + 2$$

Solution:

$$f(x) = \frac{1}{3}x^3 - 9x + 2$$

$$f'(x) = \frac{3}{3}x^2 - 9 = x^2 - 9 = (x-3)(x+3)$$

First-order critical #s: $f'(x) = 0$ or $\underbrace{\text{DNE}}_{\text{none}}$

(where c is defined) $\quad (x-3)(x+3) = 0 \quad$ f is poly.

$\boxed{x = 3, -3}$

$$f''(x) = (x^2 - 9)' = 2x$$

Second-order critical #s: $\underbrace{f''(x) = 0 \text{ or DNE}}_{\text{where c is defined}}$

$$2x = 0 \Rightarrow x = 0$$

Sign-chart for $f''(x)$

$f''(x) = 2x$

		0	
Sign for $f''(x)$	$-$		$+$
concave up/down	$\cap$		$\cup$

f is concave down on $(-\infty, 0)$

f is concave up on $(0, \infty)$

There's a PoI at $x = 0$

(concavity changes at $x = 0$)

Poll Q- Use the second-derivative test to classify each critical number as a local (relative) min, local max, or neither.

$$f(x) = \frac{x^2 - x + 5}{x+4}$$

Find the error: Kyle found the first-order critical #s as: $x = 1, x = {}^-9, x = 4$.

a) Kyle is right!

b) He factored it wrong, the answer is $x = {}^-1, 9, 4$.

c) He included the x-coord. of V.A, the answer is $x = -1, 9$.

d) He included the x-coord. of V.A and did the factoring wrong; the answer is $x = 1, {}^-9$.

Solution: $f(x)=\dfrac{x^2-x+5}{x+4}$

Use quotient rule to find $f'(x)$:

$$f'(x)=\frac{(x^2-x+5)'\cdot(x+4)-(x^2-x+5)\cdot(x+4)'}{(x+4)^2}$$

$$f'(x)=\frac{(2x-1)(x+4)-(x^2-x+5)(1)}{(x+4)^2}$$

$9 \quad -1$

$$=\frac{2x^2+8x-x-4-x^2+x-5}{(x+4)^2}=\frac{x^2+8x-9}{(x+4)^2}$$

$$=\frac{(x+9)(x-1)}{(x+4)^2}$$

First-order critical numbers: $\underbrace{f'(x)=0 \text{ or DNE}}_{f(c) \text{ is defined}}$

$$f'(x)=\frac{(x+9)(x-1)}{(x+4)^2}=0 \text{ or DNE} \left\} \begin{array}{l} x=-9,\ 1 \\ x=-4 \text{ is } x\text{-coord. of V.A.} \end{array} \right.$$

f is undef at $x=-4$!

Second-order critical numbers: $\underbrace{f''(x)=0 \text{ or DNE}}_{f(c) \text{ is defined}}$

$$f''(x)=\left(\frac{x^2+8x-9}{(x+4)^2}\right)'$$

use quotient rule again

$$f''(x)=\frac{(2x+8)(x+4)^2-(x^2+8x-9)\cdot\overbrace{2(x+4)\cdot 1}^{\text{chain rule}}}{(x+4)^4}$$

$$=\frac{2(x+4)\left[(x+4)^2-(x^2+8x-9)\right]}{(x+4)^4}$$

$$=\frac{2(x+4)\left[x^2+8x+16-x^2-8x+9\right]}{(x+4)^4}$$

$$=\frac{2(x+4)\cdot 25}{(x+4)^4}=\frac{50}{(x+4)^3} \rightarrow$$

use $x=-4$ in sign chart

sign chart for $f''(x) = \frac{50}{(x+4)^3}$

$x = -9, 1$ (first-order critical #s)

	-9	-4	1
sign of $f''(x)$	$-$		$+$
concave up/down ($f(x)$)	$\cap$		$\cup$

since $f''(-9) < 0$ local max at $x = -9$

$f''(1) > 0$ local min at $x = 1$

$x = {}^-4$ is the x-coordinate of V.A.

$x = -4$ is undefined, therefore, there's

NO PoI at $x = {}^-4$