# Ch 4.9 PART2

Wednesday, November 4, 2020 7:31 PM

## Introduction to Differential Equations

An equation involving an unknown function and its derivatives is called a **differential equation**. Here is an example to get us started.

Suppose you know that the derivative of a function $f$ satisfies the equation

$$f'(x) = 2x + 10.$$

To find a function $f$ that satisfies this equation, we note that the solutions are antiderivatives of $2x + 10$, which are $x^2 + 10x + C$, where $C$ is an arbitrary constant. So we have found an infinite number of solutions, all of the form $f(x) = x^2 + 10x + C$.

Now consider a more general differential equation of the form $f'(x) = g(x)$, where $g$ is given and $f$ is unknown. The solution $f$ consists of the antiderivatives of $g$, which involve an arbitrary constant. In most practical cases, the differential equation is accompanied by an **initial condition** that allows us to determine the arbitrary constant. Therefore, we consider problems of the form

$f'(x) = g(x)$, where $g$ is given, and — Differential equation

$f(a) = b$, where $a$ and $b$ are given. — Initial condition

A differential equation coupled with an initial condition is called an **initial value problem**.

**EXAMPLE 7** **Initial value problems** Solve the following initial value problems.

**a.** $f'(x) = x^2 - 2x;\ f(1) = \frac{1}{3}$ **b.** $f'(t) = 1 - \frac{1}{t^2};\ f(1) = 0$

$f(x) = ?$ $f(t) = ?$

1) find $f(x)$ w/ $C$ 2) use the initial value } $f(x)$

$f'(x) \to f(x)$

$$\int f'(x)\cdot dx = \int (x^2 - 2x)\cdot dx = \boxed{f(x)}$$

$$f(x) = \frac{x^3}{3} - x^2 + C \ ?$$

$$f(1) = \frac{1}{3} \Rightarrow f(1) = \cancel{\frac{1^3}{3}} - 1^2 + C = \cancel{\frac{1}{3}}$$

$$-1 + C = 0$$

$$\boxed{C = 1}$$

$$\boxed{f(x) = \frac{x^3}{3} - x^2 + 1}$$

# Application of Integrals in Physics

## Particle motion:

$s(t) \to$ position function

$v(t) \to$ velocity function

$a(t) \to$ acceleration function

$t \to$ time

$$\boxed{\frac{ds}{dt} = v(t)}$$

$$\frac{dv}{dt} = a(t) \text{ (Differentiation)}$$

(Integral)

$$ds = v(t) \cdot dt \qquad dv = a(t) \cdot dt$$

$$\int ds = \int v(t) \cdot dt \qquad \int dv = \int a(t) \cdot dt$$

$$s(t) = \int v(t) \cdot dt \qquad \boxed{v(t) = \int a(t) \cdot dt} \leftarrow$$

$$\cancel{dt} \cdot \frac{ds}{\cancel{dt}} = v(t) \cdot dt$$

$$\int ds = \int v(t) \cdot dt$$

$$\boxed{s(t) = \int v(t) \cdot dt}$$

Exp) A particle moves on a coordinate line w/ acceleration $\frac{d^2s}{dt^2} = 45\sqrt{t} - \frac{24}{\sqrt{t}}$, subject to the conditions that $\underbrace{\frac{ds}{dt}}_{v(t)} = 8$ and $s = 17$ when $t = 1$. Find the velocity and position in terms of $t$.

Given

$a(t) = 45 \cdot t^{1/2} - 24 \cdot t^{-1/2}$

$v(t) = 8,\ s(t) = 17$ @ $t = 1$

$v(1) = 8,\ s(1) = 17$

Asked

$v(t) = ?,\ s(t) = ?$

$$v(t) = \int a(t) \cdot dt = \int \left(45 \cdot t^{1/2} - 24 \cdot t^{-1/2}\right) dt$$

$$= \cancel{45}^{15} \cdot \frac{t^{3/2}}{{}_{1}\cancel{3}/2} - \overset{48}{24 \cdot \frac{t^{1/2}}{1/2}} + C_1$$

$$= 30t^{3/2} - 48t^{1/2} + C_1$$

Given: $v(1) = 8 \Rightarrow v(1) = 30 \cdot 1^{3/2} - 48 \cdot 1^{1/2} + C_1 = 8$

$$= -18 + C_1 = 8 \Rightarrow \boxed{C_1 = 26}$$

$$v(t) = 30t^{3/2} - 48t^{1/2} + 26$$

$$\boxed{v(t)=30t^{3/2}-48t^{1/2}+26}$$

Given

$a(t)=45\cdot t^{1/2}-24\cdot t^{-1/2}$

$v(t)=8,\ s(t)=17$ @ $t=1$

$(1)=8,\ s(1)=17$

$s(t)=?$

$$s(t)=\int v(t)\cdot dt \Rightarrow s(t)=\int(30t^{3/2}-48t^{1/2}+26)\cdot dt$$

$$s(t)=\cancel{30}^{\,6}\cdot\frac{t^{5/2}}{\cancel{5}/2}-\cancel{48}^{\,16}\cdot\frac{t^{3/2}}{\cancel{3}/2}+26t+C_2$$

$$=12t^{5/2}-32t^{3/2}+26t+C_2$$

$$s(1)=17 \Rightarrow \quad s(1)=12\underbrace{-32}_{-20}+26+C_2=17$$

$$6+C_2=17\Rightarrow \boxed{C_2=11}$$

$$\boxed{s(t)=12t^{5/2}-32t^{3/2}+26t+11}$$

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# Application of Integrals in Business

**Review**

$C(x)$, $R(x)$, $P(x)$, $\boxed{A(x)=\frac{C(x)}{x}}$

$\downarrow$ $\downarrow$ $\downarrow$

$C'(x)$, $R'(x)$, $P'(x)$, $A'(x)$

$x \to$ level of production (# of items produced)

$C(x) \to C'(x)$ $(MC(x))$

Exp) A manufacturer estimates that the marginal revenue of a certain item is $R'(x)=240+0.1x$ when $x$ units are produced. Find the demand function $p(x)$.

$p(x) \to$ demand f. (price per unit)

Recall: $R(x)=p(x)\cdot x \Rightarrow p(x)=\frac{R(x)}{x}$

Given: $R'(x)=240+0.1x$

Asked: $p(x)=?$

$$\int R'(x)\cdot dx = R(x) \Rightarrow \int (240+0.1x)\,dx = R(x)$$

$$240x+0.1\frac{x^2}{2}+C=R(x)$$

when $x=0$ $R(x)=0$ $[R(0)=0]$

$$R(x)=240x+0.05x^2+C$$

$$R(0)=0 \Rightarrow R(0)=0+0+C=0 \Rightarrow \boxed{C=0}$$

$$p(x)=\frac{R(x)}{x}=\frac{240x+0.05x^2}{x}=\underbrace{240+0.05x}$$

# Ch 4 -

Thursday, November 5, 2020 11:02 AM

# Indefinite Integral

$$\int f(x) \cdot dx = F(x) + C$$

Find the antiderivative of $f$ (ALL antider.)

$\int$ integral sign

$f(x) \rightarrow$ integrand

$dx \rightarrow$ differential ($x$ is the indep. var, $x$ is the variable of integration)

$F(x) \rightarrow$ complete family of antiderivatives

$C \rightarrow$ constant of integration

**THEOREM 4.17** **Constant Multiple and Sum Rules**

**Constant Multiple Rule:** $\int cf(x)\,dx = c\int f(x)\,dx$, for real numbers $c$

**Sum Rule:** $\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

The following example shows how Theorems 4.16 and 4.17 are used.

**EXAMPLE 2** **Indefinite integrals** Determine the following indefinite integrals.

**a.** $\int (3x^5 + 2 - 5\sqrt{x})\,dx$ **b.** $\int \left(\frac{4x^{19} - 5x^{-8}}{x^2}\right) dx$ **c.** $\int (z^2 + 1)(2z - 5)\,dz$

No such thing as anti-quotient or anti-product rule, use algebra!

b. $$\int \left(\frac{4x^{19}}{x^2} - \frac{5x^{-8}}{x^2}\right) dx = \int \left(4x^{17} - 5x^{-10}\right) dx$$

$$= 4\cdot\frac{x^{18}}{18} + 5\cdot\frac{x^{-9}}{+9} + C = \frac{2x^{18}}{9} + \frac{5}{9x^9} + \underline{\underline{C}}$$

c. $$\int (z^2+1)(2z-5)\cdot dz \underset{\text{FOIL}}{=} \int (2z^3 - 5z^2 + 2z - 5)\,dz$$

$$= 2\frac{z^4}{4} - 5\frac{z^3}{3} + \cancel{2}\frac{z^2}{\cancel{2}} - 5\frac{z}{1} + C$$

$$= \frac{z^4}{2} - \frac{5}{3}z^3 + z^2 - 5z + C$$