Ch 4.4 - Graphing Functions

Monday, November 16, 2020 7:44 AM

Graphing Guidelines for $y = f(x)$

1. **Identify the domain or interval of interest.** On what interval(s) should the function be graphed? It may be the domain of the function or some subset of the domain.

$f(x)=x^2$ y-axis $f(x)=x^3$ origin

2. **Exploit symmetry.** Take advantage of symmetry. For example, is the function *even* ($f(-x) = f(x)$), *odd* ($f(-x) = -f(x)$), or neither?
3. **Find the first and second derivatives.** They are needed to determine extreme values, concavity, inflection points, and intervals of increase and decrease. Computing derivatives—particularly second derivatives—may not be practical, so some functions may need to be graphed without complete derivative information.
4. **Find critical points and possible inflection points.** Determine points at which $f'(x) = 0$ or f' is undefined. Determine points at which $f''(x) = 0$ or f'' is undefined.
5. **Find intervals on which the function is increasing/decreasing and concave up/down.** The first derivative determines the intervals of increase and decrease. The second derivative determines the intervals on which the function is concave up or concave down.

sign charts → include V.A.

6. **Identify extreme values and inflection points.** Use either the First or Second Derivative Test to classify the critical points. Both x- and y-coordinates of maxima, minima, and inflection points are needed for graphing. $\pm\infty$
7. **Locate all asymptotes and determine end behavior.** Vertical asymptotes often occur at zeros of denominators. Horizontal asymptotes require examining limits as $x \to \pm\infty$; these limits determine end behavior. Slant asymptotes occur with rational functions in which the degree of the numerator is one more than the degree of the denominator.
8. **Find the intercepts.** The y-intercept of the graph is found by setting $x = 0$. The x-intercepts are found by solving $f(x) = 0$; they are the real zeros (or roots) of f.
9. **Choose an appropriate graphing window and plot a graph.** Use the results of the previous steps to graph the function. If you use graphing software, check for consistency with your analytical work. Is your graph *complete*—that is, does it show all the essential details of the function?

Ch 4 - Warm up

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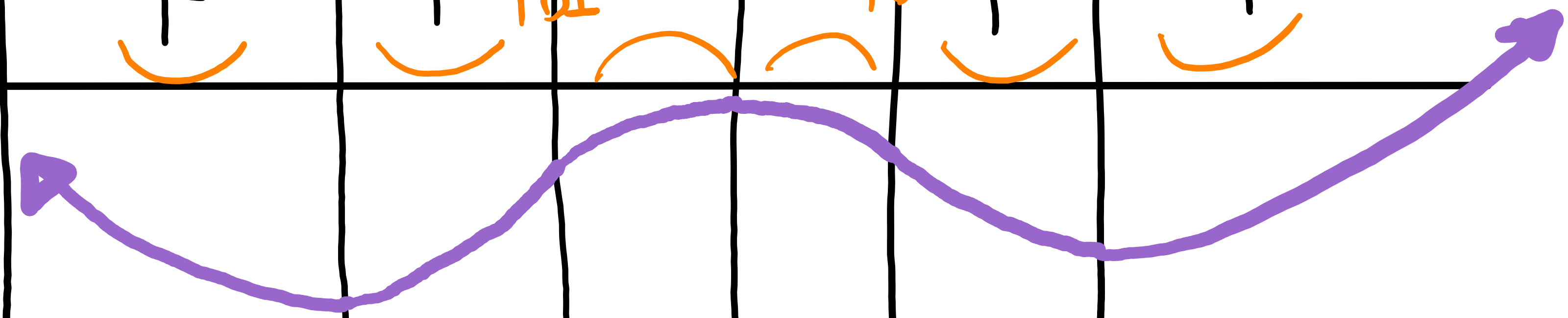
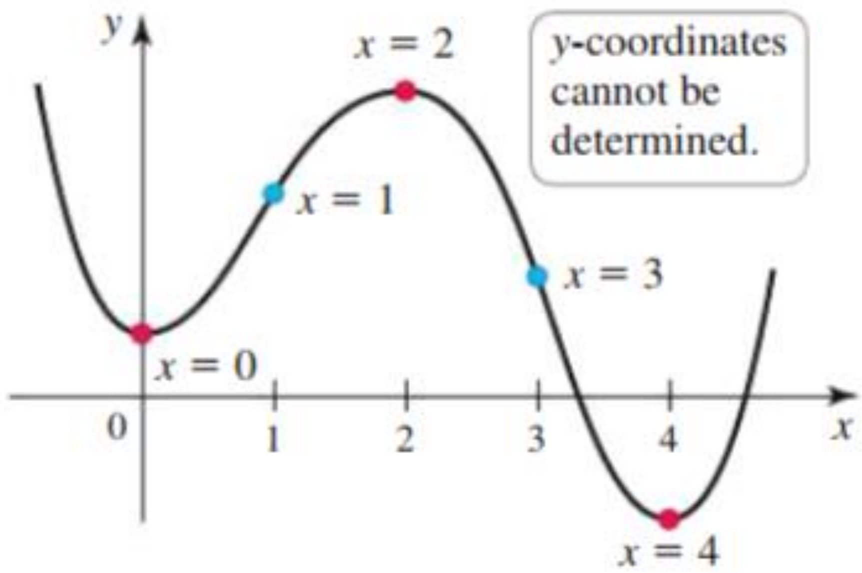
EXAMPLE 1 **A warm-up** Given the following information about the first and second derivatives of a function f that is continuous on $(-\infty, \infty)$, summarize the information using a sign graph, and then sketch a possible graph of f.

$f' < 0, f'' > 0$ on $(-\infty, 0)$ $f' > 0, f'' > 0$ on $(0, 1)$ $f' > 0, f'' < 0$ on $(1, 2)$
$f' < 0, f'' < 0$ on $(2, 3)$ $f' < 0, f'' > 0$ on $(3, 4)$ $f' > 0, f'' > 0$ on $(4, \infty)$

} conditions

	$-\infty$		0		1		2		3		4		$+\infty$
f'		—	0 local min	+		+	0 local max	—		—	0 local min	+	
f''		+		+	PoI	—		—	PoI	+		+	
f													





EXAMPLE 2 **A deceptive polynomial** Use the graphing guidelines to graph $f(x) = \frac{x^3}{3} - 400x$ on its domain.

1) Domain $(-\infty, \infty)$

2) Symmetry $f(x) = \frac{x^{3 \to \text{odd}}}{3} - 400x^{1 \to \text{odd}}$

Odd

Sym. based on the origin

3) f', f'' $f'(x) = \frac{3x^2}{3} - 400 = \underbrace{x^2 - 400}$

$f''(x) = 2x$

4) First-order crit. P. $f'(x) = 0$ or ~~DNE~~ poly.

$f'(x) = x^2 - 400 = 0$

$x = \mp 20$

candidate for local min/max

second-order crit. P. $f''(x) = 0$ or ~~DNE~~

$\underbrace{f''(x) = 2x = 0}$ $x = 0$

candidate for PoI

5) Sign Chart:

local max at $x = -20$

local min at $x = 20$

concavity changes at $x = 0$ (PoI)

	$-\infty$ $\quad$ -30	-20		0		20	$+\infty$
f'	$+$	0	$-$		$-$	0	$+$
f''	$-$		$-$	0	$+$		$+$
f	incr.		decr.		decr.		incr.

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5) Sign Chart:

-30

	$-\infty$	-20		0		20	$+\infty$
f'	+	0	−		−	0	+
f''	−		−	0	+		+
f	incr.		decr.		decr.		incr.

6) local max at $x=-20$

local min at $x=20$

concavity changes at $x=0$ (P. of I.)

f is increasing on $(-\infty,-20)$, $(20,\infty)$

f is decreasing on $(-20,20)$

f is concave up on $(0,\infty)$

f is concave down on $(-\infty,0)$

7) Polynomials have no Asymptotes

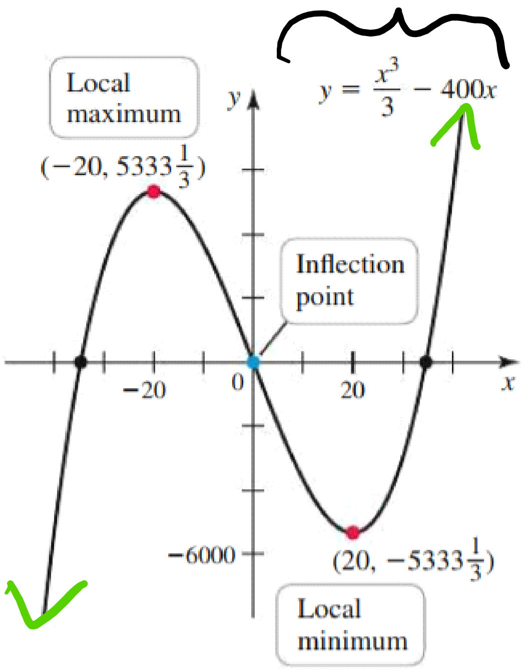

Figure 4.50

$$\lim_{x\to\infty}\left(\frac{x^3}{3}-400x\right)\to\infty$$

$$\lim_{x\to-\infty} f(x)\to -\infty$$

8) Intercepts: $x\left(\frac{x^2}{3}-400\right)$

X-int: $y=0=\frac{x^3}{3}-400x \Rightarrow x=\pm\sqrt{1200}$

y-int: $x=0 \Rightarrow y=0$ $(0,0)$