CTh 4.5 Optimization Problems

Monday, November 16, 2020 8:44 AM

The theme of this section is *optimization*, a topic arising in many disciplines that rely on mathematics. A structural engineer may seek the dimensions of a beam that maximize strength for a specified cost. A packaging designer may seek the dimensions of a container that maximize the volume of the container for a given surface area. Airline strategists need to find the best allocation of airliners among several hubs to minimize fuel costs and maximize passenger miles. In all these examples, the challenge is to find an *efficient* way to carry out a task, where "efficient" could mean least expensive, most profitable, least time consuming, or, as you will see, many other measures.

2vars

Guidelines for Optimization Problems

1. Read the problem carefully, identify the variables, and organize the given information with a picture. 1var.
2. Identify the objective function (the function to be optimized). Write it in terms of the variables of the problem. Obj. F.
3. Identify the constraint(s). Write them in terms of the variables of the problem. Constraint eq.
4. Use the constraint(s) to eliminate all but one independent variable of the objective function. variables
5. With the objective function expressed in terms of a single variable, find the interval of interest for that variable. → endpoints
6. Use methods of calculus to find the absolute maximum or minimum value of the objective function on the interval of interest. If necessary, check the endpoints.

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Exp) The difference of two numbers is 8. Find the smallest possible product.

$\underbrace{x - y = 8}_{\text{Constraint eq.}}$

$P = x \cdot y$ MIN. (Obj. F.)

Goal: To min. product

$P(x, y) = x \cdot y$ Obj. F.

$x - y = 8$ Constraint eq.

$x - 8 = y$

$P(x) = x \cdot (x - 8) = x^2 - 8x$ — $f(x)$

Interval: $(-\infty, +\infty)$

Critical P.

$P'(x) = (x^2 - 8x)' = 2x - 8 = 0$ or ~~DNE~~

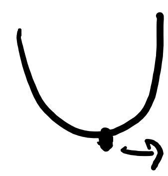
$\boxed{x = 4}$

Sign chart

	$-\infty$	4	$+\infty$
P'	$-$		$+$
P		local MIN	

local MIN at $x = 4$

1) $P''(x) = 2 > 0$



local min / global min

2) If there's ONLY 1 crit. P & local min/max THEN it's global min/max

$P(x) = x(x - 8)$

$x = 4$ MIN $P(x)$

$P(4) = 4 \cdot -4 = \boxed{-16}$

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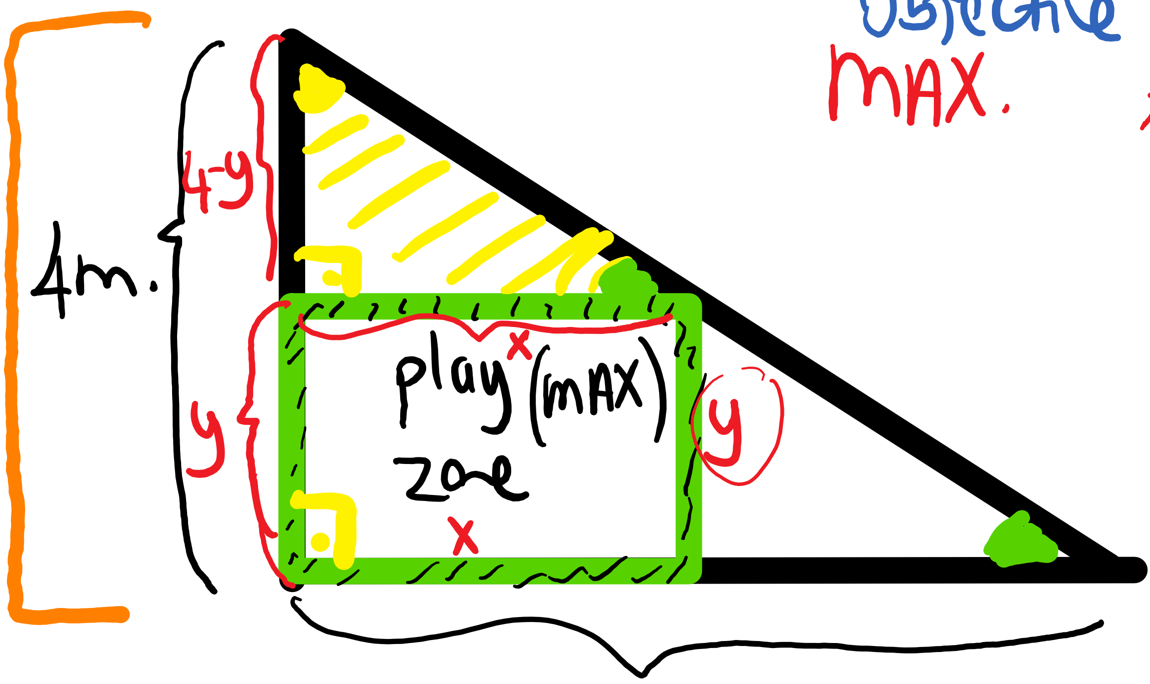
Wednesday, November 18, 2020 10:34 PM

Exp) You need to build a rectangular fence to enclose a play zone for children. What's the maximum area for this play zone if it is to fit into a right-triangular plot with sides measuring 4 m. and 12m. ?



12m.

Objective F.

MAX. $A(x,y) = x \cdot y$ ↑

$\hookrightarrow A(x),\ A(y)$?

Constraint eq. ?

Interval: y $(0,4)$

min for x, y 0

yellow shaded Δ is my smaller Δ

right-triangular plot is my big Δ

Similarity statement between Δs

$$\frac{base_s}{base_l} \rightarrow \frac{height_s}{height_l} \Rightarrow \frac{x}{12} = \frac{4-y}{4}$$

$$\cancel{12}^{3}(4-y) = \cancel{4}^{1} \cdot x$$

Constraint eq. $3(4-y) = x$ Constraint eq.

$A(x,y) \longrightarrow A(y)$

$$A(x,y) = x \cdot y \rightarrow A(y) = 3 \cdot (4-y) \cdot y = 12y - 3y^2$$

↑ max

$A'(y) = 12 - 6y = 0$ or DNE

$\boxed{y = 2}$ critical P.

$\boxed{y=2}$ critical p.

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constraint eq. $\boxed{3(4-y) = x}$ constraint eq.

$A(x,y) \xrightarrow{\text{constraint eq.}} A(y)$ ↑ max

$$A(x,y) = x \cdot y \rightarrow A(y) = 3\cdot(4-y)\cdot y = 12y - 3y^2$$

$A'(y) = 12 - 6y = 0$ or DNE

$\boxed{y=2}$ critical P.

sign chart

	0	1	2	3	4
$A'(y)$		+	0	−	
$A(y)$		↗	local MAX	↘	

global max at $x=2$

$(0,4) \rightarrow [0,4]$

$$\underbrace{\left.\begin{aligned} A(0) &= 0 \\ A(4) &= 0 \\ A(2) &= 6\cdot 2 = 12 \end{aligned}\right\}}_{\text{endpoints}}$$

$A(y) = 3y(4-y)$

$A''(y) = (12-6y)' = -6$ → global/local max

MAX. area for the playground is $12m$.