Student: ____________________	**Instructor:** Sheila Tabanli	**Assignment:** Final Exam-Part 2
Date: ____________________	**Course:** Math 136	

1. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Find all the first order partial derivatives for the following function.

$$f(x,y) = \ln\left(\frac{y^8}{x^3}\right)$$

- **A.** $\frac{\partial f}{\partial x} = -\ln\left(\frac{3y^8}{x^4}\right); \frac{\partial f}{\partial y} = \ln\left(\frac{8y^7}{x^3}\right)$
- **B.** $\frac{\partial f}{\partial x} = \frac{8}{y}; \frac{\partial f}{\partial y} = \frac{3}{x}$
- **C.** $\frac{\partial f}{\partial x} = -\ln\left(\frac{3}{x}\right); \frac{\partial f}{\partial y} = \ln\left(\frac{8}{y}\right)$
- **D.** $\frac{\partial f}{\partial x} = -\frac{3}{x}; \frac{\partial f}{\partial y} = \frac{8}{y}$

Show your work below.

2. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Find the quadratic approximation of f at x = 0.

$f(x) = \ln(1 + \sin 10x)$

- **A.** $Q(x) = 10x + 50x^2$
- **B.** $Q(x) = 10x - 50x^2$
- **C.** $Q(x) = 1 + 10x + 50x^2$
- **D.** $Q(x) = 1 - 10x + 50x^2$

Show your work below.

3. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Find all the local maxima, local minima, and saddle points of the function.

$$f(x,y) = x^3 + y^3 + 3x^2 - 9y^2 - 8$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** A local maximum occurs at __________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
  The local maximum value(s) is/are __________.
  (Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There are no local maxima.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** A local minimum occurs at __________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
  The local minimum value(s) is/are __________.
  (Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There are no local minima.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** A saddle point occurs at __________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** There are no saddle points.

Show your work below.

4. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Evaluate the double integral over the given region R.

$$\iint_R e^{x-y}\,dx\,dy \quad R: 0 \le x \le \ln(6),\ 0 \le y \le \ln(5)$$

$\iint_R e^{x-y}\,dx\,dy =$ ____________ (Type an integer or a simplified fraction.)

Show your work below.

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5. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Given the function

$f(x,y) = \dfrac{1}{\sqrt{9-25x^2-25y^2}}$, answer the following questions.

a. Find the function's domain.

b. Find the function's range.

a. Choose the correct domain.

- **A.** All points in the xy-plane.
- **B.** The set of all points in the xy-plane that satisfy $x^2+y^2 \le \frac{9}{25}$.
- **C.** All points in the xy-plane except those that lie on the circle $x^2+y^2 = \frac{9}{25}$.
- **D.** The set of all points in the xy-plane that satisfy $x^2+y^2 < \frac{9}{25}$.

b. Choose the correct range.

- **A.** $0 < z \le \frac{1}{3}$
- **B.** $0 < z \le 3$
- **C.** $z \ge \frac{1}{3}$
- **D.** $z \ge 3$

6. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Use the Limit Comparison Test to determine if the following series converges or diverges.

$\left(\text{Hint: Limit Comparison with } \sum_{n=1}^{\infty} \frac{1}{n^2}\right)$

$$\sum_{n=1}^{\infty} \frac{n-2}{n^3-n^2+6}$$

Apply the Limit Comparison Test with $\sum a_n = \sum_{n=1}^{\infty} \frac{n-2}{n^3-n^2+6}$ and $\sum b_n = \sum_{n=1}^{\infty} \frac{1}{n^2}$. Complete the sentence below.

The series $\sum a_n$ (1) ______ because $\lim_{n\to\infty} \frac{a_n}{b_n} =$ ______, a_n, b_n (2) ______ for all $n > 2$, and $\sum b_n$ (3) ______

(1) ○ diverges ○ converges

(2) ○ > 0 ○ $= 0$ ○ ≤ 0

(3) ○ converges. ○ diverges.

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7. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Determine whether the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{n}{21}\right)^n$ converges or diverges.

Let $a_n > 0$ represent the *n*th term of the given series. Identify and describe a_n. Select the correct choice below and fill in any answer box in your choice.

- **A.** a_n = ______________ nonincreasing in magnitude for n greater than some index N.
- **B.** a_n = ______________ is nondecreasing in magnitude for n greater than some index N.
- **C.** a_n = ______________ for any index N, there are some values of $n > N$ for which $a_{n+1} \geq a_n$ and some values of $n > N$ for which $a_{n+1} \leq a_n$.

Evaluate $\lim_{n\to\infty} a_n$.

$\lim_{n\to\infty} a_n$ = ______________

Does the series converge? Choose the correct answer below and, if necessary, fill in the answer box to complete your choice.

- **A.** The series does not satisfy the conditions of the Alternating Series Test but converges because it is a p-series with p = ______________.
- **B.** The series does not satisfy the conditions of the Alternating Series Test but diverges because it is a p-series with p = ______________.
- **C.** The series does not satisfy the conditions of the Alternating Series Test but converges because it is a geometric series with r = ______________.
- **D.** The series does not satisfy the conditions of the Alternating Series Test but diverges by the Root Test because the limit used is infinite.
- **E.** The series converges by the Alternating Series Test.

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8. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

If $\sum a_n$ converges and $\sum b_n$ diverges, can anything be said about their term-by-term sum $\sum (a_n + b_n)$? Give reasons for your answer.

Choose the correct answer below.

- **A.** Yes; $\sum (a_n + b_n)$ diverges because every nonzero constant multiple of a divergent series diverges.
- **B.** Yes; $\sum (a_n + b_n)$ converges because $\sum a_n$ converges and $\sum b_n$ diverges and the sum of a convergent and divergent series always converges.
- **C.** Yes; $\sum (a_n + b_n)$ converges because every nonzero constant multiple of a convergent series converges.
- **D.** Yes; $\sum (a_n + b_n)$ diverges because $\sum a_n$ converges and $\sum b_n$ diverges and the sum of a convergent and divergent series always diverges.
- **E.** No; there is not enough information to say anything about $\sum (a_n + b_n)$.

9. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

(a) Find the series' radius and interval of convergence. Find the values of x for which the series converges **(b)** absolutely and **(c)** conditionally.

$$\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2+7}}$$

(a) The radius of convergence is __________.
(Simplify your answer.)

Determine the interval of convergence. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

HINT: Check the endpoints to determine if the interval includes or excludes the endpoint(s).

- **A.** The interval of convergence is __________.
  (Type your answer in interval notation.)
- **B.** The series converges only at x = __________.
  (Type an integer or a simplified fraction.)
- **C.** The series converges for all values of x.

(b) Based on the convergence test, for what values of x does the series converge absolutely? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** The series converges absolutely for __________.
  (Type your answer in interval notation.)
- **B.** The series converges absolutely at x = __________.
  (Type an integer or a simplified fraction.)
- **C.** The series converges absolutely for all values of x.

(c) For what values of x does the series converge conditionally? Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

HINT: Compare your answers from part (a) and part (b) to respond to this part.

- **A.** The series converges conditionally at x = __________.
  (Type an integer or a simplified fraction. Use a comma to separate answers as needed.)
- **B.** The series converges conditionally for __________.
  (Type your answer in interval notation.)
- **C.** There are no values of x for which the series converges conditionally.

10. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Find the first three nonzero terms of the Maclaurin series for the function.

$$f(x) = -\cos x - \left(\frac{3}{1-x}\right)$$

The first three nonzero terms are ____________.
(Use a comma to separate answers as needed.)

Show your work below.

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11. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

Write out the first few terms of the geometric series, $\sum_{n=0}^{\infty} 3\left(\frac{x-5}{3}\right)^n$, to find a and r, and find the sum of the series. Then, express the inequality, $|r| < 1$, in terms of x and find the values of x for which the inequality holds and the series converges.

The general geometric series is $a + ar + ar^2 + ar^3 + \cdots + ar^n + \cdots$. What is the first term of the given series?

____

What is the second term of the given series?

____

What is a?

____

What is r?

____

If the series is convergent, what is the series' sum?

____

Substitute the expression in x for r in the inequality $|r| < 1$ to obtain $-1 <$ ____________ < 1.

Isolate x. For what values of x does the series converge?

____________ $< x <$ ____________

12. **RUTGERS UNIVERSITY FINAL EXAM COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Find the maximum and minimum values of $3x^2 + 3y^2$ subject to the constraint $x^2 - 6x + y^2 - 8y = 0$.

The minimum is ____________.

The maximum is ____________.

Show your work below.

1. D. $\frac{\partial f}{\partial x} = -\frac{3}{x}; \frac{\partial f}{\partial y} = \frac{8}{y}$

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2. B. $Q(x) = 10x - 50x^2$

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3. A. A local maximum occurs at **(− 2,0)** .(Type an ordered pair. Use a comma to separate answers as needed.)
   The local maximum value(s) is/are **− 4** .
   (Type an exact answer. Use a comma to separate answers as needed.)

   A. A local minimum occurs at **(0,6)** .(Type an ordered pair. Use a comma to separate answers as needed.)
   The local minimum value(s) is/are **− 116** .
   (Type an exact answer. Use a comma to separate answers as needed.)

   A. A saddle point occurs at **(− 2,6),(0,0)** .(Type an ordered pair. Use a comma to separate answers as needed.)

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4. 4

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5. D. The set of all points in the xy-plane that satisfy $x^2 + y^2 < \frac{9}{25}$.

   C. $z \geq \frac{1}{3}$

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6. (1) converges

   1

   (2) > 0

   (3) converges.

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7. B. $a_n = \left(\frac{\mathbf{n}}{\mathbf{21}}\right)^{\mathbf{n}}$ is nondecreasing in magnitude for n greater than some index N.

   ∞

   D.
   The series does not satisfy the conditions of the Alternating Series Test but diverges by the Root Test because the limit used is infinite.

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8. D.
   Yes; $\sum(a_n + b_n)$ diverges because $\sum a_n$ converges and $\sum b_n$ diverges and the sum of a convergent and divergent series always diverges.

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9. 1

A. The interval of convergence is **[− 1, 1)** . (Type your answer in interval notation.)

A. The series converges absolutely for **(− 1, 1)** .(Type your answer in interval notation.)

A. The series converges conditionally at x = **− 1** .
(Type an integer or a simplified fraction. Use a comma to separate answers as needed.)

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10. $-4, -3x, -\frac{5}{2}x^2$

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11. 3

$3\left(\frac{x-5}{3}\right)$

3

$\frac{x-5}{3}$

$\frac{9}{8-x}$

$\frac{x-5}{3}$

2

8

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12. 0

300
