Monday, January 18, 2021 2:56 PM

Agenda: Review of Algebra & Precalculus

State the domain of each function.

Fall 2019 Midterm #1 Q

State the domain of $f(x)=\dfrac{5x-10}{x+4} > 0$ in interval notation.

Recall: (Domain, Range) $\Rightarrow$ (x, y)

Find zeros of num., denom.

$5x-10=0$ | $x+4=0$

$x=2$ | $x=-4$

Construct a sign chart

test P. -10 0 10

$-\infty$ -4 2 $+\infty$

$f(x)$: $+$ $-$ $+$

>0 $+$

$f(x)=\dfrac{5x-10}{x+4}$

test P.

$f(-10)=\dfrac{5(-10)-10}{-10+4} \to \dfrac{-}{-} \to +$

$f(0)=\dfrac{-10}{4}<0 \to -$

Optimization in Calc.

Domain of $f(x)$ is $\underbrace{(-\infty,-4)\cup(2,\infty)}_{\text{interval notation}}$

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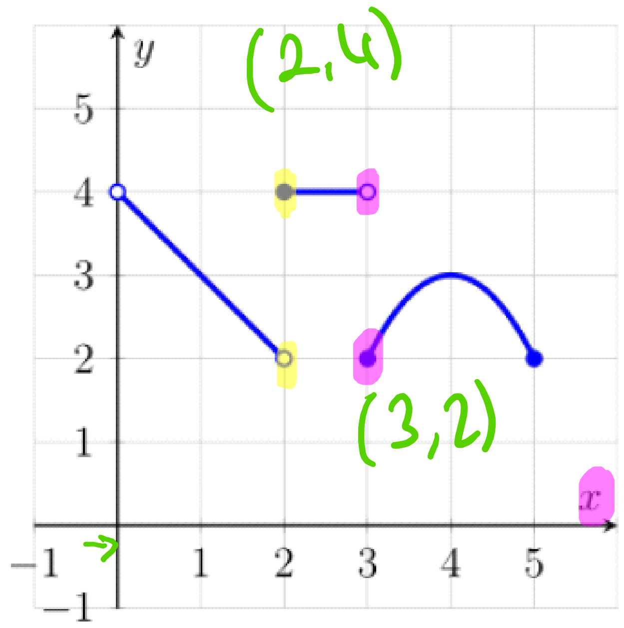
5. The graph of a function $y = g(x)$ is given below.



Find the domain of $g(x)$. Write your answer in interval notation.

o → excluded

$\boxed{(0,5]}$

0 is excluded, (open circle)

5 is included [closed circle]

• → include

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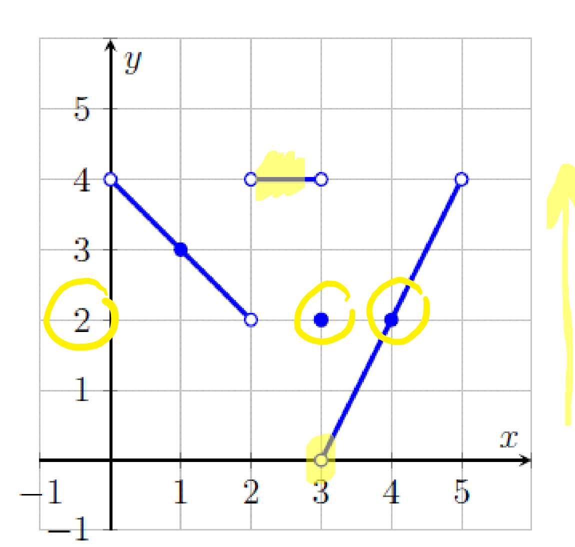
8. The graph of a function $y = g(x)$ is given below.



included

Is (2.5, 4) on the graph?

(x, y)

Yes!

Find the range of $g(x)$. Write your answer in interval notation.

y

range is : (0, 4]

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(,) o exclude

[,] • include

You try it!

State the domain of $2x^2-5x \leq 3$ in interval notation.

A) $(-\frac{1}{2}, 3)$ B) $(\frac{1}{2}, 3)$ C) $[-\frac{1}{2}, 3]$ D) None

$$2x^2-5x \leq 3 \Rightarrow 2x^2-5x-3 \leq 0$$

temp. eq.

$$2x^2-5x-3 = 0$$

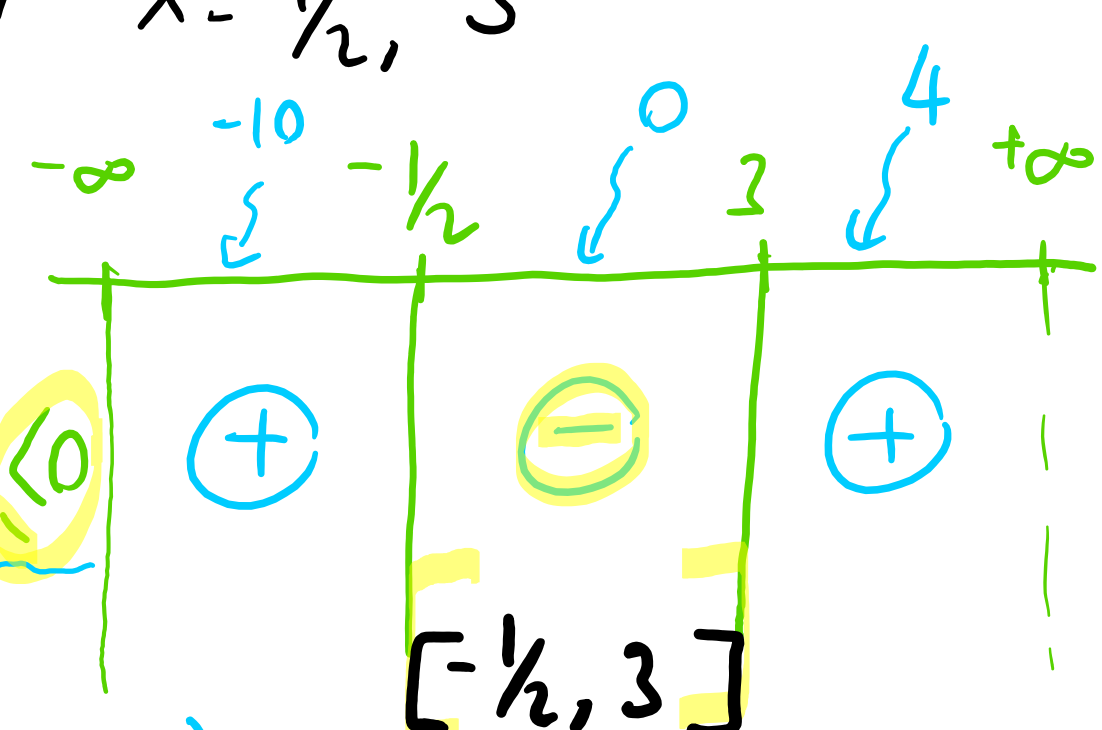
$$\begin{array}{ccc} 2x & & 1 \\ x & & -3 \end{array} \quad -6x+x = -5x \checkmark$$

cm

$$\underbrace{(2x+1)}_{0}\cdot\underbrace{(x-3)}_{0} = 0 \Rightarrow x = -\frac{1}{2}, 3$$

sign chart

$(2x+1)(x-3) \leq 0$



Subs. test P. in $(2x+1)(x-3)$

$x=-10$ $(-20+1)(-10-3) \Rightarrow \ominus \cdot \ominus = \oplus$

$x=0$ $1 \cdot -3 \Rightarrow \ominus$

Domain: $[-\frac{1}{2}, 3]$

Function Composition

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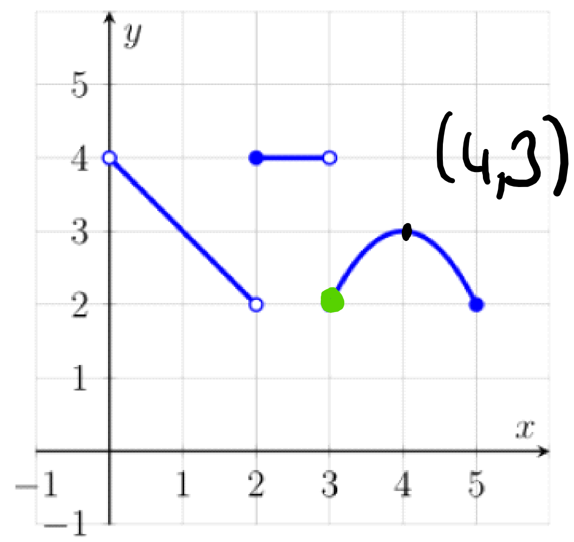
6. The graph of a function $y = g(x)$ is given below.



Find $g(g(4))$.

$$\underbrace{g(g(4))}_{\text{always start from inside!}}$$

$$g(4)=3 \Rightarrow g(g(4))$$

$$g(3) = \boxed{2}$$

Difference Quotient : $\dfrac{f(x+h)-f(x)}{h}$ (Limits, Derivatives)

Simplify the difference quotient for $f(x)=\sqrt{x+2}$

$f(x)=\sqrt{x+2}$

$f(x+h)=\sqrt{x+h+2}$

$$\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\cdot\frac{(\sqrt{x+h+2}+\sqrt{x+2})}{(\sqrt{x+h+2}+\sqrt{x+2})}$$

Factoring Diff. of Squares: $(a-b)(a+b)=a^2-b^2$

$$\frac{(\sqrt{x+h+2})^2-(\sqrt{x+2})^2}{h(\sqrt{x+h+2}+\sqrt{x+2})}=\frac{\cancel{x}+h+\cancel{2}-(\cancel{x+2})}{h(\sqrt{x+h+2}+\sqrt{x+2})}=\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+2}+\sqrt{x+2})}$$

$$=\frac{1}{\sqrt{x+h+2}+\sqrt{x+2}}$$

$f(x)=x+\sqrt{2}$

$f(x+h)=x+h+\sqrt{2}$

$f(x)=\sqrt{2-x}$

$f(x+h)=\sqrt{2-(x+h)}$

Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ for $f(x)=\frac{2}{x}$

A) $\frac{3}{x(x+h)}$ B) $\frac{2}{x(x+h)}$ C) $\frac{-2}{x(x+h)}$ D) None

$$f(x+h)=\frac{2}{x+h}$$

$$f(x)=\frac{2}{x}$$

$$\frac{\frac{2}{x+h}\cdot\frac{(x)}{(x)}-\frac{2}{x}\cdot\frac{(x+h)}{(x+h)}}{h}=\frac{\frac{2x}{x(x+h)}-\frac{2(x+h)}{x(x+h)}}{h}$$

$$=\frac{\frac{\cancel{2x}-\cancel{2x}-2h}{x(x+h)}}{h}=\frac{\frac{-2\cancel{h}}{x(x+h)}}{\cancel{h}}=\frac{-2}{x(x+h)}$$

C) $\frac{-2}{x(x+h)}$

Evaluate Functions

Recall: $e^0 = 1$, $\sin 0 = 0$

Let $g(x) = 3e^x - 7$ Evaluate $g(0)$, $h(0)$

$h(x) = -4 + \sin(x)$ Are they equal. (Yes / No)

$$g(0) = 3e^0 - 7 = 3 - 7 = -4$$
$$h(0) = -4 + \sin(0) = -4$$

Yes

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9. True or false? "For the function $f(x)$ below, $\lim_{x \to 0} f(x)$ exists."

$$f(x) = \begin{cases} 3e^x - 7 & , \quad x < 0 \\ -4 + \sin(x) & , \quad x \geq 0 \end{cases}$$

f(-2) =?

f(0) =?

Justify your response.

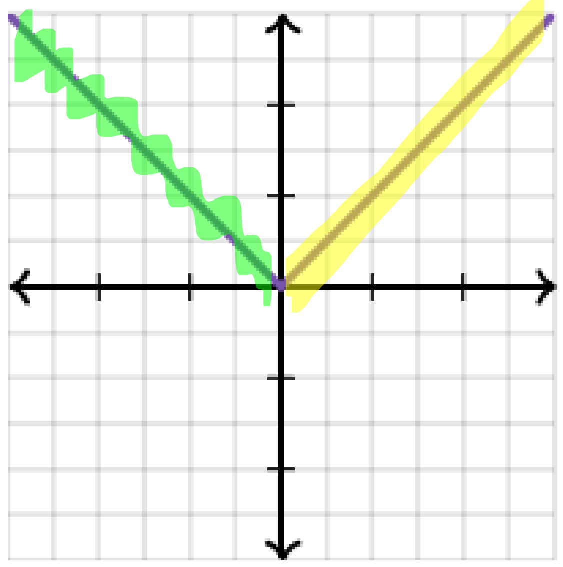
$$f(-2) = 3 \cdot e^{-2} - 7 = \frac{3}{e^2} - 7$$

$(-2 < 0)$

$$f(0) = -4 + \sin 0 = -4$$

$(0 = 0)$

Absolute Value Function (A Famous Piece-wise Function)

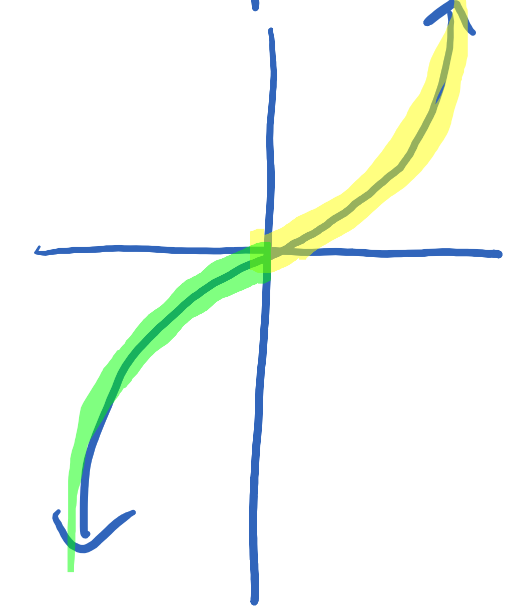


$$f(x) = |x| \quad \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

E.g: $\sqrt[2]{x^6} = |x^3| \quad \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$

even index #

$f(x) = x^3$



Solve the inequality: $\overbrace{|3x-4|}^{} > 8$

-10

$|-10| > 8$

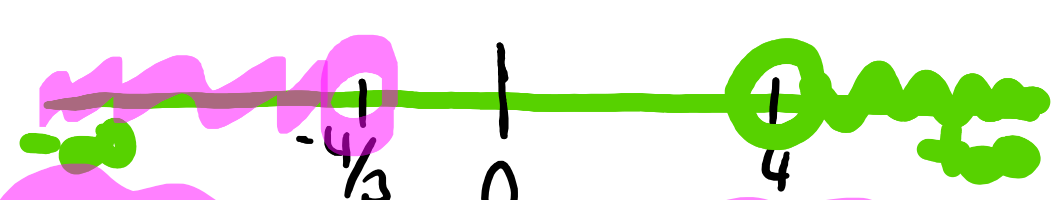
$\checkmark \; 10 > 8$

$3x - 4 > 8$ or $3x - 4 < -8$

$+4 \quad +4$ $\qquad$ $+4 \quad +4$

$\frac{3x}{3} > \frac{12}{3}$ or $\frac{3x}{3} < \frac{-4}{3}$

$x > 4$ or $x < -4/3$



$(-\infty, -4/3), (4, \infty)$

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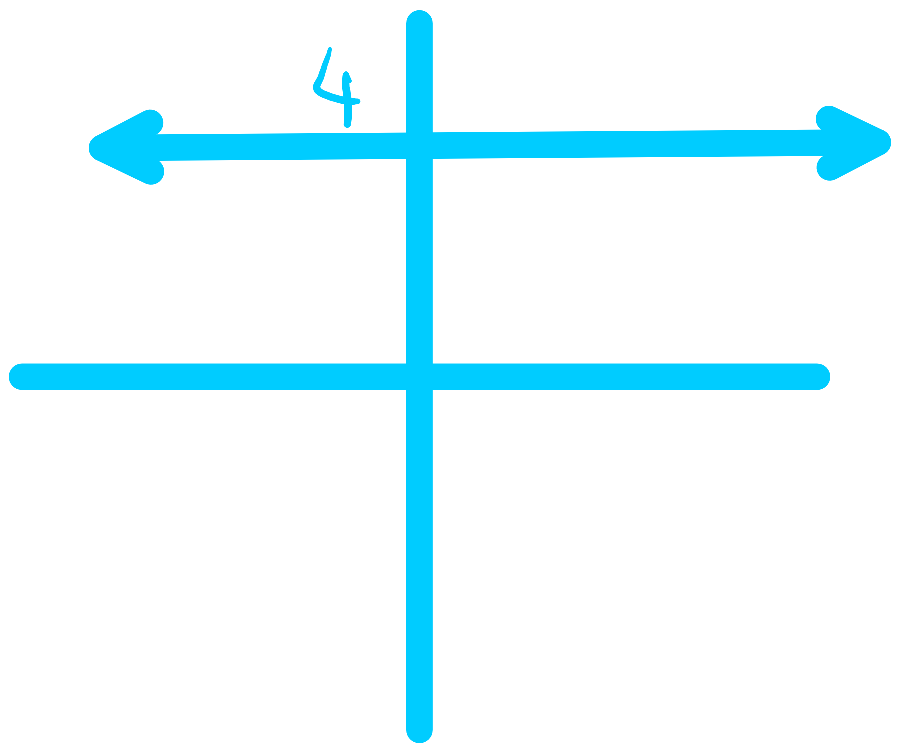
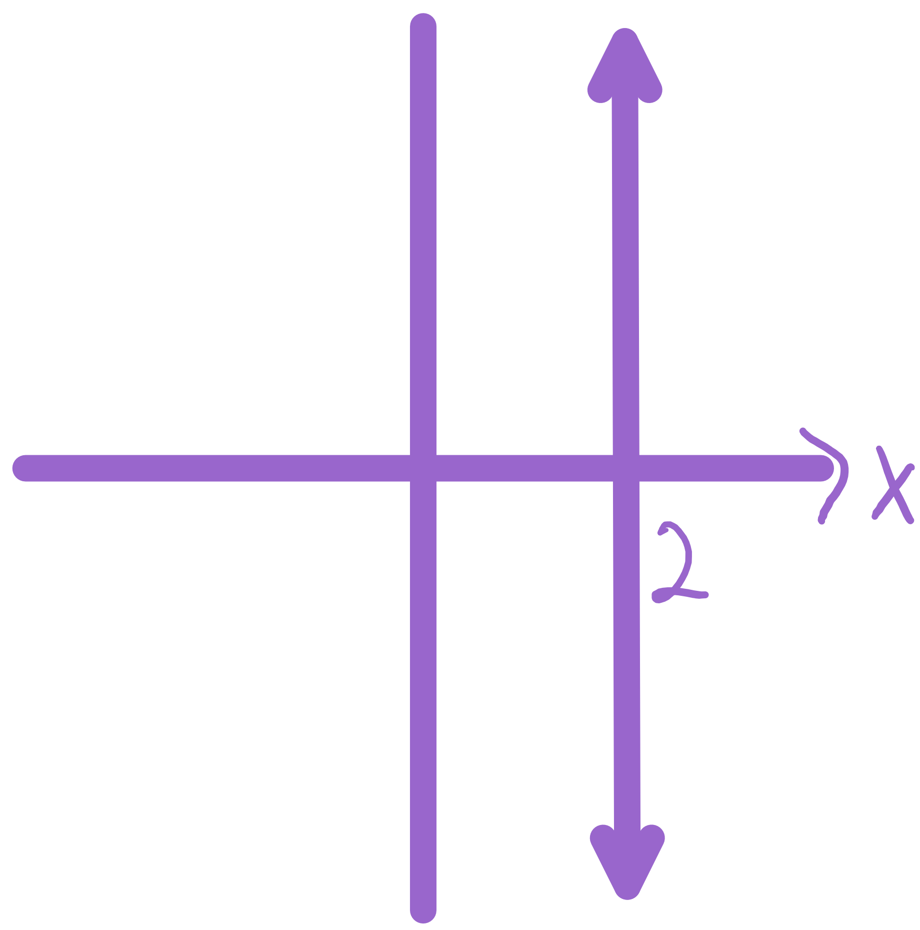
Linear Equations and Slopes



$y = 4$ zero slope

rate of chage



$x = 2$, $y = 2$?

slope → undefined

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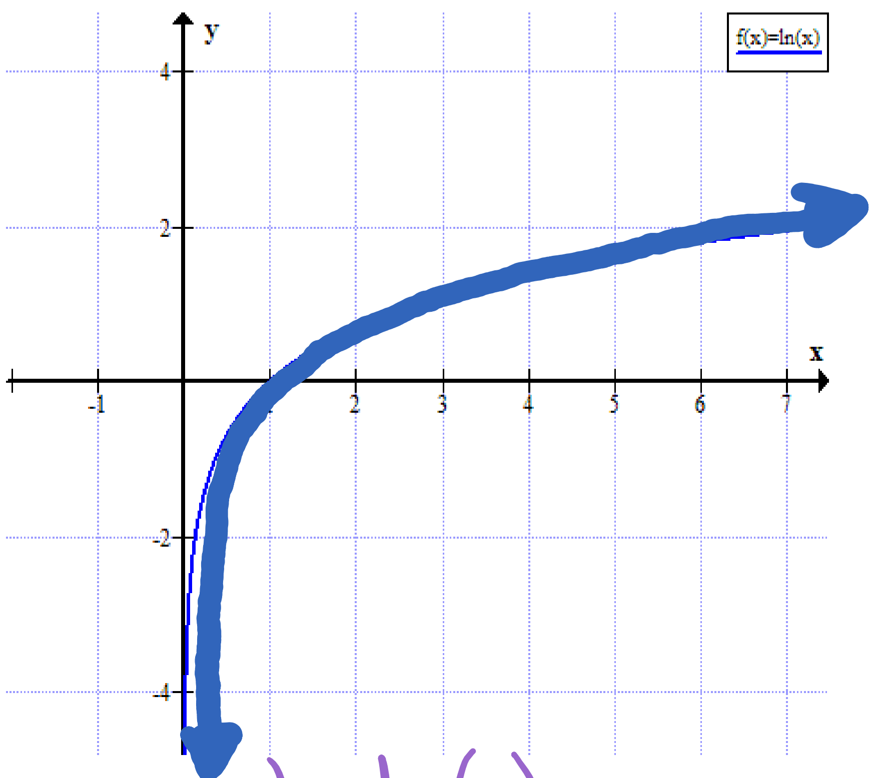
Logarithmic (and Exponential) Functions

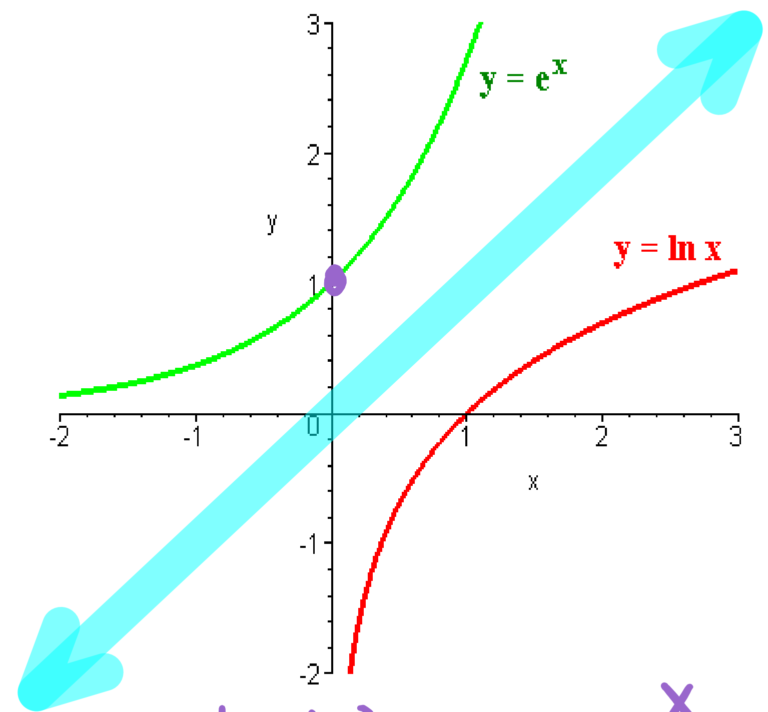
$e^0 = 1 \quad (0,1)$



$f(x) = \ln(x)$

$x > 0$

$D: (0, \infty)$



$\ln(x)$ vs. e^x

e^x & $\ln x$ are inverse f.

Review: properties of ln

1) $\ln(ab) = \ln a + \ln b$
2) $\ln\frac{a}{b} = \ln a - \ln b$
3) $\ln a^k = k \ln a$
4) $\ln e = 1$
5) $\ln 1 = 0$

E.g: Solve exponential eq. Express the solution set in terms of natural logs.

$e^{2x-1}=7$

A) $\ln 4$ B) $\frac{\ln 8}{2}$ C) $\frac{\ln(7)+1}{2}$ D) $\frac{\ln(7)-1}{2}$ E) None

$e^{2x-1}=7$

$\ln e^{2x-1}=\ln 7$

$(2x-1)\cdot\underbrace{\ln e}_{1}=\ln 7$

$2x-1=\ln 7$

$2x=\ln(7)+1$

$x=\frac{\ln(7)+1}{2}$