Lecture#2 : Review of Precalculus

Monday, January 25, 2021 10:23 AM

Exp) Let $\log_k p = 5$, $\log_k q = -2$; find $\log_k(p^3 \cdot q^2)$; $k>0$

Given

?

$$\log_k(p^3 \cdot q^2) = \log_k p^3 + \log_k q^2 = 3 \cdot \log_k p + 2 \cdot \log_k q$$

Expand

$$= 3 \cdot 5 + 2 \cdot (-2)$$

$$= 15 - 4 = 11$$

We used the prop. of log. f. to expand & subs. give info.

We will use these for LOGARITHMIC DIFFERENTIATION.

Exp) Find the EXACT SOLUTION: $\ln(3x) + \ln\left(x+\frac{2}{3}\right) = 0$ ($3x>0$, $x+\frac{2}{3}>0$)

A) $x=\frac{4}{3}$ B) $\frac{1}{3}$ C) 1 D) -1 E) Nne

$x=\frac{1}{3}$

$x=-1$

$$\ln(3x) + \ln\left(x+\frac{2}{3}\right) = \ln\left(3x \cdot \left(x+\frac{2}{3}\right)\right) = \ln\left(3x^2 + 3x \cdot \frac{2}{3}\right)$$

Condense

$\ln$ just

$$\ln(3x^2+2x) = 0 \Rightarrow 3x^2+2x = e^0 = 1 \Rightarrow 3x^2+2x-1=0$$

e

$3x \cdot x$, $-1 \cdot 1$

$3x - x = 2x$ ✓

$(3x-1)(x+1)=0$; $x=\frac{1}{3}$, $x=-1$ extraneous solution

Exp) Solve the eq. $3^{4-x} = 27^{x}$

$27 = 3^3$

$3^{4-x} = (3^3)^x = 3^{3x}$

$4 - x = 3x$

$4 = 4x$

$x = 1$

Verify: $3^{4-1} \stackrel{?}{=} 27^1$

$3^3 = 27$ ✓

Exp) Solve the eq: $5^{x^2-4x+5} = 25$; $x = ?$

A) -3 B) 3 C) -1 D) 1 E) None

$5^{x^2-4x+5} = 5^2 \Rightarrow x^2 - 4x + 5 = 2$

$x^2 - 4x + 3 = 0 \Rightarrow (x-1)(x-3) = 0$

-3 -1

$x = 1, \quad x = 3$

Exp) Solve: $x \cdot \left(\ln\left(\frac{1}{6}\right)\right) = \ln 6$ $x = ?$

$\frac{1}{6} = 6^{-1}$

$x \cdot \left(\ln\left(6^{-1}\right)\right) = \ln 6$

$\frac{x \cdot (-1) \cdot \ln 6}{\ln 6} = \frac{\ln 6}{\ln 6} \Rightarrow x \cdot (-1) = 1 \Rightarrow x = -1$

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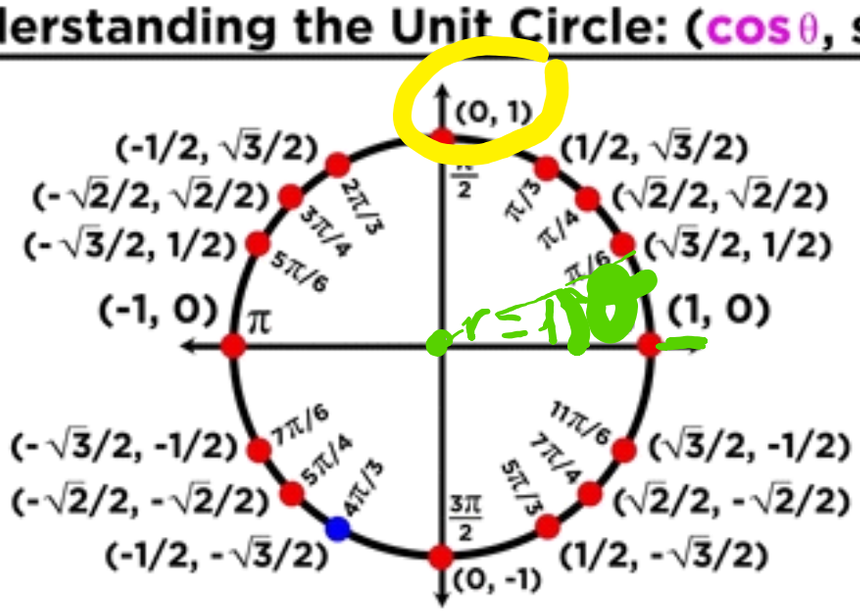
$[0, 2\pi]$ ↺ counter-clockwise (+) Angle

↻ clockwise (−)

Understanding the Unit Circle: ($\cos\theta$, $\sin\theta$)



$(x, y) \rightarrow (\cos\theta, \sin\theta)$

SOH CAH TOA

$\sin\theta = \frac{Opp}{H}$ $\cos\theta = \frac{Adj}{H}$ $\tan\theta = \frac{Opp}{Adj}$

Trigonometric Identities

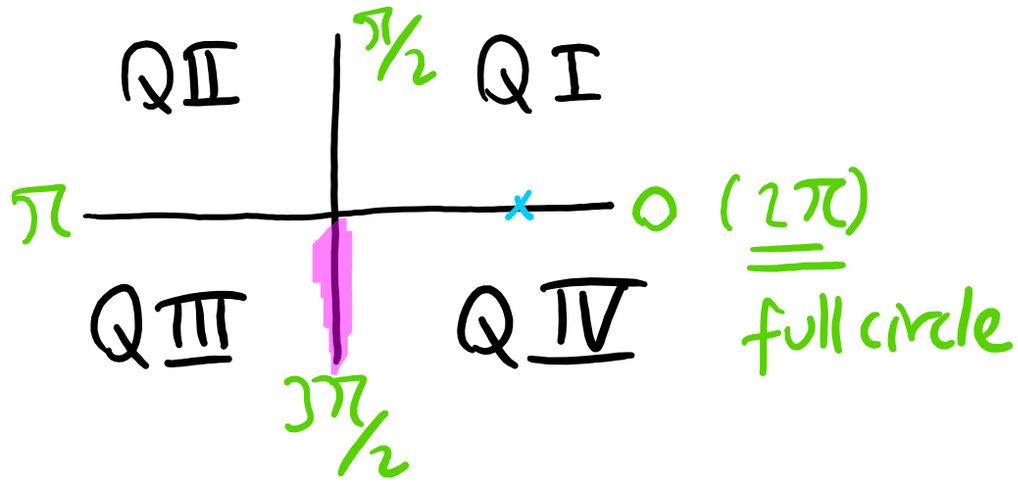
Reciprocal Identities	Quotient Identities
$\cot\theta = \frac{1}{\tan\theta}$ ←	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ ←
$\csc\theta = \frac{1}{\sin\theta}$	$\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\sec\theta = \frac{1}{\cos\theta}$	

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$



related rates prb.

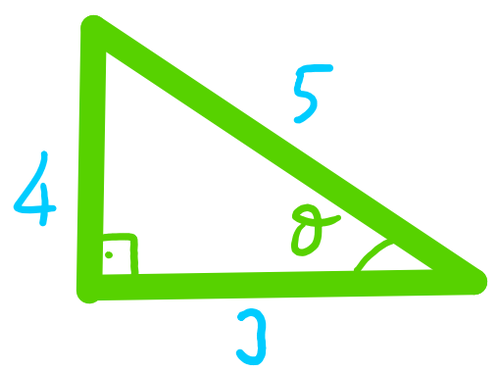
Exp) Let $\sec\theta = \frac{5}{3}$, find $\tan\theta$, $\sin\theta$ in $\frac{3\pi}{2} < \theta < 2\pi$

$(x, y) \rightarrow (\cos\theta, \sin\theta)$

$\tan\theta = -\left(\frac{\sin\theta}{\cos\theta}\right)$, $\sin\theta$ −

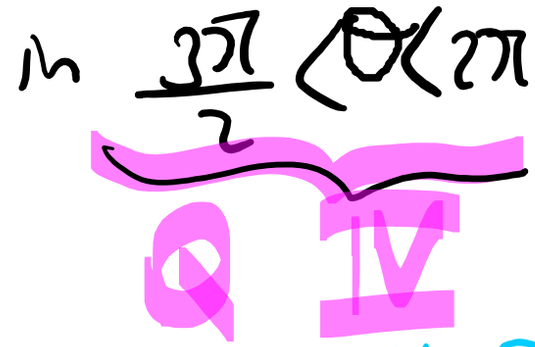
$\sec\theta = \frac{5}{3} \Rightarrow \cos\theta = \frac{3}{5}$

pos. in QIV



$\sin\theta = \frac{Opp}{H} = \frac{-4}{5}$

$\tan\theta = \frac{Opp}{Adj} = \frac{-4}{3}$



$\sec\theta = \frac{1}{\cos\theta} = \frac{H}{A} = \frac{5}{3}$

Exp) Solve $\ln(\log x) = 0$ $x = ?$

$\ln(\log_{10} x) = 0 \Rightarrow \log_{10} x = e^0 = 1 \Rightarrow \log_{10} x = 1$

$x = 10^1 = 10$

e.g: $\log_2 x = 3 \Rightarrow x = 2^3$

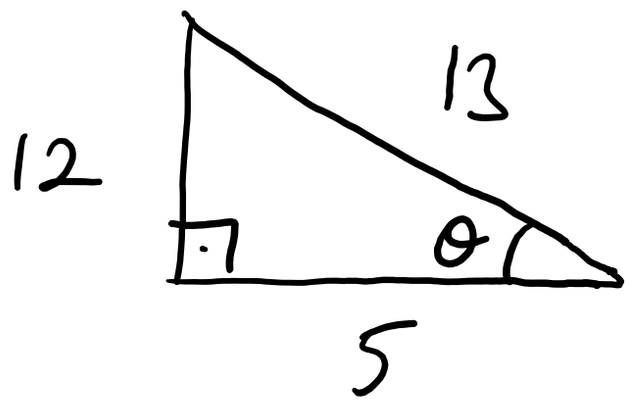
QI

Exp) Eval. $\cot\theta$, $\sin\theta$ when $\csc\theta = \frac{13}{12}$, $0 < \theta < \pi/2$

$\theta \in (0, \pi/2)$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{13}{12}$$

$$\sin\theta = \frac{12}{13} \checkmark$$



$$\cot\theta = \frac{\cos\theta}{\sin\theta} \rightarrow \frac{Adj}{Opp} = \frac{5}{12}$$

Exp) Find solution(s) in $[0, 2\pi]$ in radians:

$2\cdot\sin^2 x + \sin x - 3 = 0$

A) $\frac{3\pi}{2}$ B) $-\frac{3\pi}{2}$ C) $\frac{\pi}{2}$ D) None

$2\cdot\sin^2 x + \sin x - 3 = 0 \Rightarrow (2\sin x + 3)(\sin x - 1) = 0$

$2\sin x \quad 3$

$\sin x \quad -1$

$3\sin x - 2\sin x = \sin x \checkmark$

$\sin x = -\frac{3}{2}$, $\sin x = 1$

$x = \frac{\pi}{2}$

C) $\frac{\pi}{2}$

MyLab HW Problem Review

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Sales records indicate that if Blu-ray players are priced at $300, then a large store sells an average of 9 units per day. If they are priced at $200, then the store sells an average of 15 units per day. Find and graph the linear demand function for Blu-ray sales. For what prices is the demand function defined?

Given: If item is priced at \$300, then are 9 units/day is sold

" \$200, then are 15 units/day is sold

Asked: Linear demand f., graph, domain

Demand f. is a function of price.

$f(x) = y = mx + b$

$x \to$ price/unit

$f(x) \to$ demand f. (# units per day)

$(x, y) \to$ (price per unit, #units/day)

$(300, 9), (200, 15)$ $\quad m = \frac{9-15}{300-200}$

$m = \frac{-6}{100}$

Use $(200, 15)$ $\quad m = \frac{-6}{100}$

$y = mx + b \Rightarrow 15 = \frac{-6}{100} \cdot 200 + b \Rightarrow 15 = -12 + b$

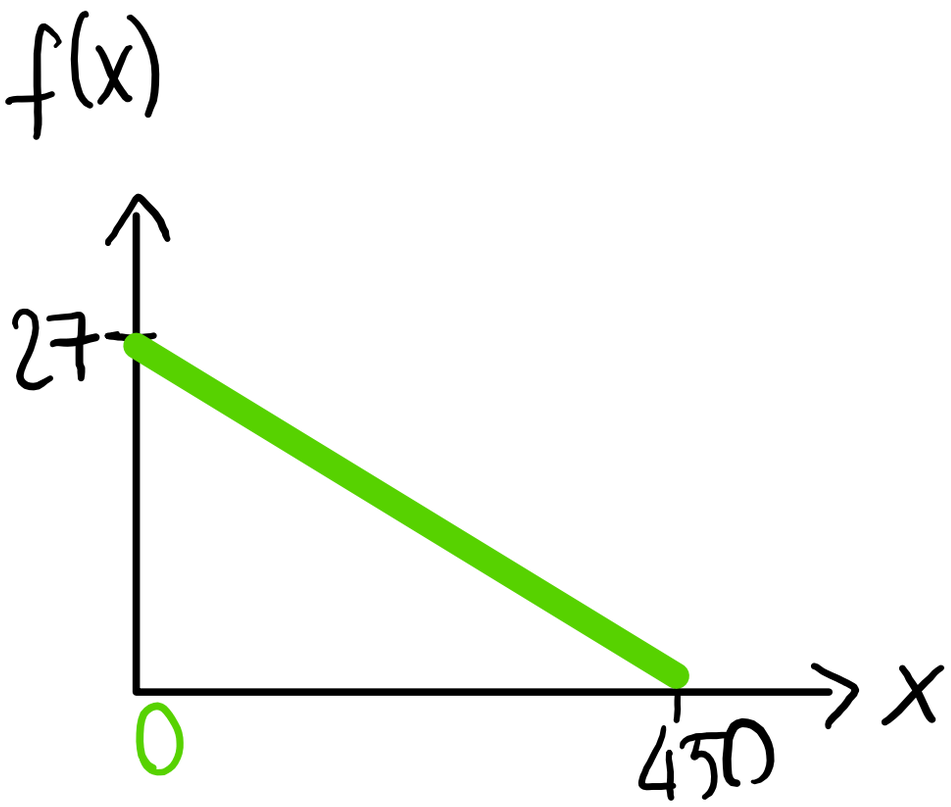
$b = 27$

$$\boxed{y = \frac{-6}{100}x + 27}$$ Demand f.

$$\boxed{y = \frac{-6}{100}x + 27}$$ Demand f.

$(x=0)$ $b = 27$ $(y\text{-int.})$ $\rightarrow$ $y = 27$

$y = 0$ $(x\text{-int.})$ $\rightarrow$ $x = 450$



$$0 = \frac{-6}{100}x + 27$$

$$-27 = \frac{-6}{100}x$$

$$x = 450$$

Domain: $[0, 450]$