Student: _____ **Date:** _____	**Instructor:** Sheila Tabanli **Course:** Math 136	**Assignment:** Midterm#1

1. Evaluate the following integral using integration by parts.

$$\int_0^{\pi} 2x \cos x \, dx$$

Use the integration by parts formula so that the new integral is simpler than the original one. Choose the correct answer below.

- **A.** $(2x)\big|_0^{\pi} - \int_0^{\pi} (-2\cos x)\, dx$
- **B.** $(2\cos x)\big|_0^{\pi} - \int_0^{\pi} (2x)\, dx$
- **C.** $(-2\cos x)\big|_0^{\pi} - \int_0^{\pi} (\sin x)\, dx$
- **D.** $(2x \sin x)\big|_0^{\pi} - \int_0^{\pi} (2\sin x)\, dx$

Evaluate the integral.

$\int_0^{\pi} 2x \cos x \, dx =$ _____ (Type an exact answer, using π as needed.)

*2. Find the area of the region enclosed by the curves $y = x^2 - 2$ and $y = 7$ between their intersections.

Set up the integral (or integrals) needed to compute this area. Use the smallest possible number of integrals. Give your limits of integration in increasing order. Select the correct choice below and fill in the answer boxes to complete your choice.

- **A.** $\int_{\_\_\_}^{\_\_\_} [\_\_\_\_\_] dx$
- **B.** $\int_{\_\_\_}^{\_\_\_} [\_\_\_\_\_] dx + \int_{\_\_\_}^{\_\_\_} [\_\_\_\_\_] dx$

The area of the region is _____ square unit(s).
(Type an exact answer in simplified form.)

3. The region R in the first quadrant bounded by the parabola $y = 49 - x^2$ and coordinate axes is revolved about the y-axis to produce a dome-shaped solid. Find the volume of the solid in the following ways.
a. Apply the disk method and integrate with respect to y.
b. Apply the shell method and integrate with respect to x.

a. Set up the integral using the disk method. Select the correct choice below and fill in the answer boxes to complete your choice.

- **A.** $\int_{\square}^{\square} (\quad) \, dx$
- **B.** $\int_{\square}^{\square} (\quad) \, dy$

b. Set up the integral using the shell method. Select the correct choice below and fill in the answer boxes to complete your choice.

- **A.** $\int_{\square}^{\square} (\quad) \, dx$
- **B.** $\int_{\square}^{\square} (\quad) \, dy$

The volume is _____.
(Type an exact answer.)

4. Find the volume of the solid generated by revolving the region bounded by $y = 4\sqrt{x}$ and the lines $y = 4\sqrt{11}$ and $x = 0$ about
a. the y-axis. **b.** the line $x = 11$.

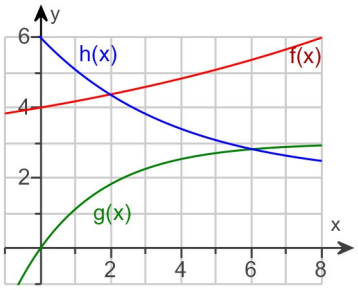
a. The volume of the solid generated by revolving the region bounded by $y = 4\sqrt{x}$ and the lines $y = 4\sqrt{11}$ and $x = 0$ about the y-axis is _____ cubic units.
(Type an exact answer, using π as needed.)

b. The volume of the solid generated by revolving the region bounded by $y = 4\sqrt{x}$ and the lines $y = 4\sqrt{11}$ and $x = 0$ about the line $x = 11$ is _____ cubic units.
(Type an exact answer, using π as needed.)

*5.

Shade the portion of the figure whose area is given by the following integral.

$$\int_0^2 [h(x) - g(x)]\,dx + \int_2^6 [f(x) - g(x)]\,dx$$



Choose the graph with the correct shading.

○ **A.** ○ **B.** ○ **C.** ○ **D.**

*6. The functions f and g are integrable, $\int_2^4 f(x)dx = 7$, $\int_2^7 f(x)dx = 4$, and $\int_2^7 g(x)dx = -5$. Find the values of the following definite integrals.

$\int_4^4 f(x)dx =$ _____
(Simplify your answer.)

$\int_7^2 g(x)dx =$ _____
(Simplify your answer.)

$\int_2^7 8g(x)dx =$ _____
(Simplify your answer.)

$\int_4^7 f(x)dx =$ _____
(Simplify your answer.)

$\int_2^7 [g(x) - f(x)]dx =$ _____
(Simplify your answer.)

$\int_2^7 [6g(x) - f(x)]dx =$ _____
(Simplify your answer.)

7. For the region bounded by $y = x$, the x-axis, and $x = 2$, determine which of the following is greater: the volume of the solid generated when the region is revolved about the x-axis or about the y-axis.

When the region is revolved about the x-axis, the volume is _____.
(Type an exact answer, using π as needed.)

When the region is revolved about the y-axis, the volume is _____.
(Type an exact answer, using π as needed.)

Which volume is greater?

- **A.** The volume about the x-axis is greater.
- **B.** The volumes are equal.
- **C.** The volume about the y-axis is greater.

8. Find the area of the region bounded by the graph of $f(x) = x \sin x^2$ and the x-axis between $x = 0$ and $x = \sqrt{\pi}$.

Determine a change of variables from x to u. Choose the correct answer below.

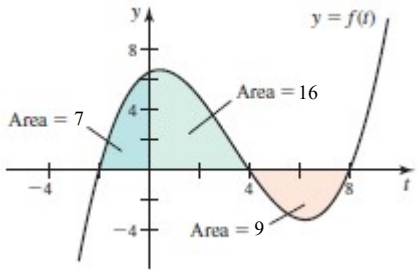
- **A.** $u = x$
- **B.** $u = x^2$
- **C.** $u = x \sin x^2$
- **D.** $u = \sin x^2$

Write the integral in terms of u.

$$\int_0^{\sqrt{\pi}} x \sin x^2 \, dx = \int_0^{\square} (\square) \, du$$

The area of the given region is _____.
(Simplify your answer.)

9. The graph of f is shown in the figure to the right. Let $A(x) = \int_{-2}^{x} f(t)\,dt$ and $F(x) = \int_{4}^{x} f(t)\,dt$ be two area functions for f. Evaluate the following area functions.



a. $A(-2)$ **b.** $F(8)$ **c.** $A(4)$ **d.** $F(4)$ **e.** $A(8)$

a. $A(-2) =$ _____ (Simplify your answer.)

b. $F(8) =$ _____ (Simplify your answer.)

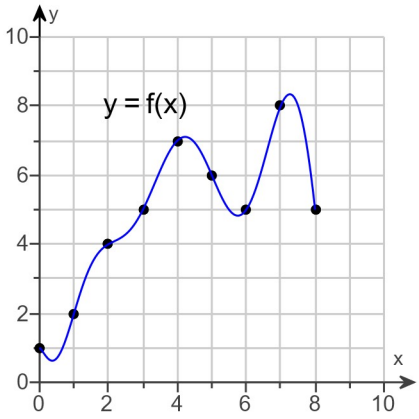
c. $A(4) =$ _____ (Simplify your answer.)

d. $F(4) =$ _____ (Simplify your answer.)

e. $A(8) =$ _____ (Simplify your answer.)

10. Compute the following estimate of $\int_0^8 f(x)\,dx$ using the graph in the figure.

S(4)



Using Simpson's Rule, S(4) = _____.
(Type an integer or a simplified fraction.)

11. Select the correct choices that complete the sentence below.

A region R is revolved about the y-axis. The volume of the resulting solid could (in principle) be found by using the disk/washer method and integrating with respect to (1) _____ or using the shell method and integrating with respect to (2) _____

(1) ○ x ○ y (2) ○ x. ○ y.

1.

D. $(2x \sin x)\Big|_0^{\pi} - \int_0^{\pi} (2 \sin x)\, dx$

-4

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2. A. $\int_{-3}^{3} \left[9 - x^2 \right] dx$

36

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3. B. $\int_0^{49} \left(\pi(49 - y) \right) dy$

A. $\int_0^{7} \left(2\pi x \left(49 - x^2 \right) \right) dx$

$\frac{2401\pi}{2}$

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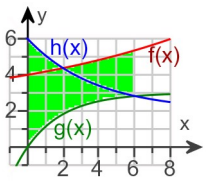
4. $\frac{484\pi\sqrt{11}}{5}$

$\frac{3388\pi\sqrt{11}}{15}$

5.

A.



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6. 0

5

-40

-3

-9

-34

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7. $\frac{8\pi}{3}$

$\frac{16\pi}{3}$

C. The volume about the y-axis is greater.

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8. B. $u = x^2$

π

$\frac{1}{2}$ **sin** u

1

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9. 0

-9

23

0

14

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10. $\frac{112}{3}$

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11. (1) y

(2) x.
