Student: ____________________ **Date:** ____________________	**Instructor:** Sheila Tabanli **Course:** Math 136	**Assignment:** Midterm#2-June17

1. I acknowledge that I have read and understood the Math 136 assessment policies and procedures in its entirety and agree to abide by them.

   - True
   - False

2. Evaluate the following indefinite integral.

   $$\int 7\,\sin^3 x\,\cos^2 x\,dx$$

   The first step is to use the half-angle formulas to transform the integrand into a polynomial in cos2x.

   - True
   - False

   HINT: Do not forget to include the constant of integration, C.

   $\int 7\,\sin^3 x\,\cos^2 x\,dx =$ ____________

3. Determine whether the following statements are true or false and give an explanation or counterexample.

Answer parts **a**–**d** below.

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a. Determine whether the following statement is true or false. If $x = 10 \tan u$, then $\csc u = \frac{10}{x}$. Choose the correct answer below.

- **A.** False, because $\csc u \neq \tan u$.
- **B.** True, because $\csc u = \frac{1}{\sin u}$.
- **C.** True, because $\csc u = \frac{1}{\tan u}$.
- **D.** False, because $\csc u \neq \frac{1}{\tan u}$.

b. Determine whether the following statement is true or false. The integral $\int_7^{14} \sqrt{49 - x^2}\, dx$ does not have a finite real value.

Choose the correct answer below.

- **A.** True, because $\int \sqrt{49 - x^2}\, dx$ is impossible to integrate.
- **B.** True, because $\sqrt{49 - x^2}$ does not have real values throughout the interval [7,14].
- **C.** False, because $\sqrt{49 - x^2}$ has real values throughout the interval [7,14].
- **D.** False, because $\sqrt{49 - x^2}$ has real values throughout the interval [0,7].

c. Determine whether the following statement is true or false. The integral $\int_2^{4} \sqrt{x^2 - 4}\, dx$ does not have a finite real value.

Choose the correct answer below.

- **A.** True, because $\int \sqrt{x^2 - 4}\, dx$ is impossible to integrate.
- **B.** False, because $\sqrt{x^2 - 4}$ has real values throughout the interval [0,2].
- **C.** True, because $\sqrt{x^2 - 4}$ does not have real values throughout the interval [2,4].
- **D.** False, because $\sqrt{x^2 - 4}$ has real values throughout the interval [2,4].

d. Determine whether the following statement is true or false. The integral $\int \frac{dx}{x^2 + 3x + 8}$ cannot be evaluated using a trigonometric substitution. Choose the correct answer below.

- **A.** False, because $x^2 + 3x + 8$ can be solved using the quadratic equation.
- **B.** True, because there is no trigonometric substitution for the form $x^2 + bx + c$.
- **C.** False, because $x^2 + 3x + 8$ can be written in the form $u^2 + a^2$ or $u^2 - a^2$ by completing the square.

4. Evaluate the following integral.

$$\int \frac{dx}{x^2 + 8x + 25}$$

Rewrite the integrand by completing the square.

$$\int \frac{dx}{x^2 + 8x + 25} = \int \left(\underline{\qquad\qquad} \right) dx$$

(Simplify your answer.)

Use a u-substitution to rewrite this integral in a form that better suggests a trigonometric substitution. What should be used for u?

$u = \underline{\qquad\qquad}$

(Type an expression using x as the variable.)

After the substitution in the previous step is used, what trigonometric substitution will be the most helpful for evaluating this integral?

- **A.** $u = 3 \sin \theta$
- **B.** $u = 3 \sec \theta$
- **C.** $u = 3 \tan \theta$

Find du.

$du = \left(\underline{\qquad\qquad} \right) d\theta$

Rewrite the given integral using this substitution.

$$\int \frac{dx}{x^2 + 8x + 25} = \int \left(\underline{\qquad\qquad} \right) d\theta$$

(Simplify your answer. Type an exact answer.)

Evaluate the indefinite integral.

$$\int \frac{dx}{x^2 + 8x + 25} = \underline{\qquad\qquad}$$

(Type an exact answer.)

5. Determine whether the following statements are true and give an explanation or counterexample.

Answer parts **a**.–**d**. below.

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a. Is the following statement true? To evaluate $\int \frac{4x^6}{x^4 + 7x^2} dx$, the first step is to find the partial fraction decomposition of the integrand. Choose the correct answer below.

- **A.** Yes, because the denominator can be factored into simple linear factors, including a repeated linear factor.
- **B.** Yes, because the degree of the numerator is greater than the degree of the denominator.
- **C.** No, because the denominator is not yet in factored form.
- **D.** No, because the degree of the numerator is greater than the degree of the denominator.

b. Is the following statement true? The easiest way to evaluate $\int \frac{10x + 2}{5x^2 + 2x} dx$ is with a partial fraction decomposition of the integrand. Choose the correct answer below.

- **A.** No, because the integrand is an improper fraction and, therefore, cannot be put into a partial fraction decomposition.
- **B.** No, because it is easier to evaluate with a u-substitution with $u = 5x^2 + 2x$.
- **C.** Yes, because partial fraction decomposition takes fewer steps than trigonometric substitution or u-substitution.
- **D.** Yes, because it is a proper fraction, and the denominator can be factored into simple linear factors.

c. Is the following statement true? The rational function $f(x) = \frac{1}{x^2 - 14x + 45}$ has an irreducible quadratic denominator. Choose the correct answer below.

- **A.** Yes, because the trinomial $x^2 - 14x + 45$ is a quadratic.
- **B.** Yes, because the trinomial $x^2 - 14x + 45$ cannot be factored.
- **C.** No, because the trinomial $x^2 - 14x + 45$ can be factored.
- **D.** No, because the trinomial $x^2 - 14x + 45$ is not a quadratic.

d. Is the following statement true? The rational function $f(x) = \frac{1}{x^2 - 14x + 50}$ has an irreducible quadratic denominator. Choose the correct answer below.

- **A.** Yes, because the trinomial $x^2 - 14x + 50$ cannot be factored.
- **B.** Yes, because the trinomial $x^2 - 14x + 50$ is a quadratic.
- **C.** No, because the trinomial $x^2 - 14x + 50$ is not a quadratic.
- **D.** No, because the trinomial $x^2 - 14x + 50$ can be factored.

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6. Does the function $y(t) = 9t$ satisfy the differential equation $y'''(t) + y'(t) = -9$?

Choose the correct answer below.

- A. Yes, because when y' and y''' are substituted into the equation, the result is a true statement.
- B. Yes, because there is no given initial condition, so the function satisfies the differential equation for all values of t.
- C. No, because there is no given initial condition, so there could be values of t which do not satisfy the differential equation.
- D. No, because when y' and y''' are substituted into the equation, the result is not a true statement.

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7. Solve the following initial value problem.

$$y'(t) - 4y = 16, \quad y(0) = 0$$

$y(t) =$ ____________ (Type an exact answer in terms of e.)

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8. Find the general solution of the differential equation. Use C, C_1, C_2, . . . to denote arbitrary constants. If used, do not simplify hyperbolic functions.

$y''(t) = 35e^{7t} + \sin 5t$

- A. $y = \frac{5e^{7t}}{7} + \frac{\sin 5t}{5} + C_1 t + C_2$
- B. $y = \frac{5e^{7t}}{7} - \frac{\sin 5t}{25} + C_1 t + C_2$
- C. $y = \frac{5e^{7t}}{7} + \frac{\sin 5t}{25} + C_1 t + C_2$
- D. $y = \frac{5e^{7t}}{7} - \frac{\sin 5t}{25} + C_1 t$

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*9. Find the equilibrium price and quantity for $p = D(x) = 71 - \frac{1}{10}x$ and $p = S(x) = 35 + \frac{1}{20}x$.

- A. $p = 47$
  $q = 288$
- B. $p = 47$
  $q = 240$
- C. $p = 47$
  $q = 180$
- D. $p = 50$
  $q = 240$

10. Find the consumers' surplus and the producers' surplus at the equlibrium level for the given price-demand and price-supply equations. Include a graph that identifies the consumers' surplus and the producers' surplus. Round all values to the nearest integer.

$p = D(x) = 42.5 - 0.09x;\ \ p = S(x) = 10 + 0.04x$

The value of x at equilibrium is __________.

The value of p at equilibrium is $__________.

The consumers' surplus at equilibrium is $__________.

The producers' surplus at equilibrium is $__________.

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11. Re-write the following integral using trigonometric substitution.

$$\int_{\frac{21}{2}}^{21} \sqrt{441 - x^2}\, dx$$

What substitution will be the most helpful for evaluating this integral?

- A. $x = 21\,\mathbf{sin}\,\theta$
- B. $x = 21\,\mathbf{tan}\,\theta$
- C. $x = 21\,\mathbf{sec}\,\theta$

Rewrite the given integral using this substitution.

$$\int_{\frac{21}{2}}^{21} \sqrt{441 - x^2}\, dx = \int_{\frac{\pi}{6}}^{\_\_\_\_\_} (\_\_\_\_\_\_\_\_\_\_)\, d\theta$$

(Simplify your answers. Type exact answers.)

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12. Evaluate the following definite integral.

$$\int_{-1}^{3} \frac{6y}{y^2 - 2y - 8}\, dy$$

Find the partial fraction decomposition of the integrand.

$$\int_{-1}^{3} \frac{6y}{y^2 - 2y - 8}\, dy = \int_{-1}^{3} (\_\_\_\_\_\_\_\_\_\_)\, dy$$

Evaluate the definite integral.

$$\int_{-1}^{3} \frac{6y}{y^2 - 2y - 8}\, dy = \_\_\_\_\_\_\_\_\_\_$$

(Type an exact answer by using ln. Do **not** type it as a decimal)

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13. Verify that the given function y(t) is a solution of the initial value problem that follows it.

$y(t) = 14e^{3t} - 23$

$y'(t) - 3y(t) = 69, \quad y(0) = -9$

Differentiate the given function, y(t). What is the result?

(1) ____________ = ____________

What is the next step?

- **A.** Substitute y into the differential equation and verify that a true statement results.
- **B.** Substitute $y'(0)$ and $y(0)$ into the differential equation and verify that a true statement results..
- **C.** Substitute y' and $y(0)$ into the differential equation and verify that a true statement results.
- **D.** Substitute y and y' into the differential equation and verify that a true statement results.

Substitute and simplify. What is the result?

- **A.** $14e^{3t} - 23 = 69$
- **B.** $3 = 69$
- **C.** $69 = 69$
- **D.** $3y(t) = -9$

What is the best next step?

- **A.** Solve for y(t) in the differential equation and substitute the result into the given function and verify that a true statement results.
- **B.** Substitute 0 for t in y(t) to verify the initial conditions.
- **C.** Substitute y(t) in the differential equation and verify that a true statement results.
- **D.** Solve for $y'(t)$ in the differential equation and substitute the result into the given function and verify that a true statement results.

Does a true statement result from the operation performed?

- No
- Yes

How has the solution been verified?

- **A.** It has been shown that substituting the solution and its derivative into the differential equation results in a true statement, $y'(0) = 0$, and $y(0) = -9$.
- **B.** It has been shown that substituting the solution and its derivative into the differential equation results in a true statement and $y(0) = -9$.
- **C.** It has been shown that substituting $y'(t)$ into the given solution results in a true statement and $y(0) = -9$.
- **D.** It has been shown that substituting the given solution into the differential equation results in a statement that is equal to the derivative to the solution.

(1) ○ $y(t)$
○ $y'(t)$

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14. Consider a loan repayment plan described by the following initial value problem, where the amount borrowed is $B(0) = \$80,000$, the monthly payments are \$400, and $B(t)$ is the unpaid balance of the loan. Use the initial value problem to answer parts **a** through **b**.

$$B'(t) = 0.02B - 400,\ B(0) = 80,000$$

a) Find the solution of the initial value problem and explain why B is an increasing solution.

$B(t) =$ ____________

Why is B an increasing function?

- ○ **A.** The function is increasing because it is an exponential function with a positive exponent.
- ○ **B.** The function is increasing because it is an exponential function with a positive coefficient and a positive exponent.
- ○ **C.** The function is increasing because it is an exponential function with a positive coefficient and a negative exponent.
- ○ **D.** The function is increasing because it is an exponential function with a positive coefficient.

b) What is the most that you can borrow under the terms of this loan without going further into debt each month?

The most you could borrow is \$____________.

1. True

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2. False

$\frac{7}{5}\cos^5 x - \frac{7}{3}\cos^3 x + C$

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3. D. False, because $\csc u \neq \frac{1}{\tan u}$.

   B. True, because $\sqrt{49 - x^2}$ does not have real values throughout the interval [7,14].

   D. False, because $\sqrt{x^2 - 4}$ has real values throughout the interval [2,4].

   C. False, because $x^2 + 3x + 8$ can be written in the form $u^2 + a^2$ or $u^2 - a^2$ by completing the square.

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4. $\frac{1}{(x+4)^2 + 9}$

   $x + 4$

   C. $u = 3\tan\theta$

   $3\sec^2\theta$

   $\frac{1}{3}$

   $\frac{1}{3}\tan^{-1}\frac{x+4}{3} + C$

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5. D. No, because the degree of the numerator is greater than the degree of the denominator.

   B. No, because it is easier to evaluate with a u-substitution with $u = 5x^2 + 2x$.

   C. No, because the trinomial $x^2 - 14x + 45$ can be factored.

   A. Yes, because the trinomial $x^2 - 14x + 50$ cannot be factored.

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6. D. No, because when y' and y''' are substituted into the equation, the result is not a true statement.

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7. $4e^{4t} - 4$

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8. B. $y = \frac{5e^{7t}}{7} - \frac{\sin 5t}{25} + C_1 t + C_2$

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9. B. $p = 47q = 240$

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10. 250

    20

    2813

    1250

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11. A. $x = 21 \sin \theta$

    $\frac{\pi}{2}$

    $441 \cos^2 \theta$

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12. $\frac{4}{y-4} + \frac{2}{y+2}$

    $-2 \ln 5$

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13. (1) $y'(t)$

    $42 e^{3t}$

    D. Substitute y and y' into the differential equation and verify that a true statement results.

    C. $69 = 69$

    B. Substitute 0 for t in y(t) to verify the initial conditions.

    Yes

    B.
    It has been shown that substituting the solution and its derivative into the differential equation results in a true statement and $y(0) = -9$.

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14. $60{,}000 e^{0.02t} + 20{,}000$

    B. The function is increasing because it is an exponential function with a positive coefficient and a positive exponent.

    20,000
