Student: ____________________	**Instructor:** Sheila Tabanli	**Assignment:** Midterm#4
Date: ____________________	**Course:** Math 136	

1. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

Use the Ratio Test to determine if the following series converges absolutely or diverges.

$$\sum_{n=1}^{\infty} \frac{n^6}{(-6)^n}$$

Since the limit resulting from the Ratio Test is ________, which is (1) ________, (2) ________
(Simplify your answer.)

(1)
- ○ < 0
- ○ > 0
- ○ < 1
- ○ > 1

(2)
- ○ the series diverges.
- ○ the Ratio Test is inconclusive.
- ○ the series converges absolutely.

2. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

The Taylor polynomial of order 2 generated by a twice-differentiable function f(x) at x = a is called the quadratic approximation of f at x = a. Find
(a) linearization (Taylor polynomial of order 1)
(b) the quadratic approximation of f at x = 0.

$f(x) = \ln(\cos x)$

(a) Find the linearization approximation of f at x = 0.

____

(b) Find the quadratic approximation of f at x = 0.

____

3. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

Consider the series $\sum_{n=0}^{\infty} \frac{(x-9)^n}{4^n}$

(a) Does this series converge?
(b) Which convergence test did you use to check for convergence/divergence?
(c) Find the series' radius.
(d) For what values of x does the series converge absolutely?

(a) Select the correct choice below.

- **A.** The series converges.
- **B.** The series diverges.
- **C.** It can not be determined based on the given information.

(b) Select the correct choice below.

- **A.** I used the ratio test and found $r = 1$.
- **B.** I used the root test and found $\rho \geq 1$.
- **C.** I used the root test and found $\rho < 1$.
- **D.** I used the ratio test and found $r > 1$.
- **E.** I used the divergence test to show the series converges.

(c) The radius of convergence is __________.

(d) The series converges absolutely on the interval __________. (Write your answer in interval notation after checking the end points to determine open or closed intervals.)

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4. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

Find the domain and range and describe the level curves for the function f(x,y).

$f(x,y) = \sqrt{9 - x^2 - y^2}$

- **A.** Domain: all points in the xy-plane satisfying $x^2 + y^2 = 9$
  Range: real numbers $0 \leq z \leq 3$
- **B.** Domain: all points in the xy-plane $x^2 + y^2 \leq 9$
  Range: integer numbers $0 < z \leq 3$
- **C.** Domain: all points in the xy-plane
  Range: all real numbers
- **D.** Domain: all points in the xy-plane satisfying $x^2 + y^2 \leq 9$
  Range: real numbers $0 \leq z \leq 3$

5. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

You must show work and submit your scrap paper for this question.
Any discrepancy between the submitted work below and the scrap paper will result in deduction of points.

For the function f(x) = **ln** (1 + 8x), find the Taylor polynomials of orders 0, 1, 2, and 3 generated by *f* at *a* = 0.

$P_0(x) =$ ____________

$P_1(x) =$ ____________

$P_2(x) =$ ____________

$P_3(x) =$ ____________

Show your work below.

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6. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Find all the local maxima, local minima, and saddle points of the function.

$$f(x,y) = x^3 - 9xy + y^3$$

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** A local maximum occurs at __________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
  The local maximum value(s) is/are __________.
  (Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There are no local maxima.

Select the correct choice below and, if necessary, fill in the answer boxes to complete your choice.

- **A.** A local minimum occurs at __________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
  The local minimum value(s) is/are __________.
  (Type an exact answer. Use a comma to separate answers as needed.)
- **B.** There are no local minima.

Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- **A.** A saddle point occurs at __________.
  (Type an ordered pair. Use a comma to separate answers as needed.)
- **B.** There are no saddle points.

Show your work below.

7. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

Use power series operations to find the Taylor series at x = 0 for the following function.
Hint: Use the summary table for the commonly used Taylor series.

$14x e^{x}$

The Taylor series for e^{x} is a commonly known series. What is the Taylor series at x = 0 for e^{x}?

$\sum_{n=0}^{\infty}$ ____________ (Type an exact answer.)

Use power series operations and the Taylor series at x = 0 for e^{x} to find the Taylor series at x = 0 for the given function.

$\sum_{n=0}^{\infty}$ ____________ (Type an exact answer.)

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8. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Find the critical points of the following function.

$f(x,y) = 5x^{2} + 7xy^{2} - 6x + 1$

What are the critical points?

____
(Type an ordered pair. Use a comma to separate answers as needed.)

Show your work below.

9. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

Use the Root Test to determine if the following series converges absolutely or diverges.

$$\sum_{n=1}^{\infty} \frac{5}{(6n+7)^n}$$

Since the limit resulting from the Root Test is ________, which is (1) ________ (2) ________
(Type an exact answer.)

(1) ○ greater than 0, ○ less than 1, ○ equal to 1, ○ equal to 0, ○ greater than 1, ○ less than 0,

(2) ○ the series converges absolutely. ○ the series diverges. ○ the Root Test is inconclusive.

10. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

You must show work and submit your scrap work to earn credit for this question. Any discrepancy between the submitted work below and the scrap work will result in deduction of points.

Find all the second order partial derivatives of the given function.

$f(x,y) = x^2 + y - e^{x+y}$

○ **A.** $\frac{\partial^2 f}{\partial x^2} = 1 - e^{x+y}$; $\frac{\partial^2 f}{\partial y^2} = -e^{x+y}$; $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -e^{x+y}$

○ **B.** $\frac{\partial^2 f}{\partial x^2} = 2 - y^2 e^{x+y}$; $\frac{\partial^2 f}{\partial y^2} = -x^2 e^{x+y}$; $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -y^2 e^{x+y}$

○ **C.** $\frac{\partial^2 f}{\partial x^2} = 2 + e^{x+y}$; $\frac{\partial^2 f}{\partial y^2} = e^{x+y}$; $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = e^{x+y}$

○ **D.** $\frac{\partial^2 f}{\partial x^2} = 2 - e^{x+y}$; $\frac{\partial^2 f}{\partial y^2} = -e^{x+y}$; $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -e^{x+y}$

Show your work below.

11. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

Which order of differentiation will calculate f_{xy} faster, x first or y first?

$f(x,y) = y\ln(x) - 6\sin(x)$

- x first
- y first
- Neither will be faster

Explain your reasoning.

- **A.** When differentiating the function with respect to x first, I treated x as a constant which eliminiated the first term.
- **B.** When differentiating the function with respect to x first, I treated y as a constant which eliminiated the first term.
- **C.** When differentiating the function with respect to y first, I treated x as a constant which eliminiated the first term.
- **D.** When differentiating the function with respect to y first, I treated x as a constant which eliminiated the second term.
- **E.** The order does not matter, in each case I get to eliminate one term to differentiate.

Computing f_{xy} and f_{yx} gives the same answer.

- True
- False

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12. **RUTGERS UNIVERSITY MIDTERM#4 COPYRIGHT 2020**

If $f(0) = 0$, $f'(0) = 6$, $f''(0) = 0$, and $f'''(0) = 2$, then which of the following is the third-order Taylor polynomial generated by f(x) at x = 0?

Choose the correct answer below.

- **A.** $\frac{1}{3}x^3 + 6x$
- **B.** $\frac{1}{3}x^3 + \frac{1}{6}x$
- **C.** $2x^3 + x$
- **D.** $2x^3 - x$
- **E.** $\frac{2}{3}x^3 + \frac{1}{2}x$

1. $\frac{1}{6}$

(1) < 1

(2) the series converges absolutely.

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2. 0

$-\frac{x^2}{2}$

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3. A. The series converges.

C. I used the root test and found $\rho < 1$.

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(5,13)

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4. D. Domain: all points in the xy-plane satisfying $x^2 + y^2 \le 9$Range: real numbers $0 \le z \le 3$

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5. 0

$8x$

$8x - 32x^2$

$8x - 32x^2 + \frac{512}{3}x^3$

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6. B. There are no local maxima.

A. A local minimum occurs at **(3,3)** .(Type an ordered pair. Use a comma to separate answers as needed.)
The local minimum value(s) is/are **−27** .
(Type an exact answer. Use a comma to separate answers as needed.)

A. A saddle point occurs at **(0,0)** .(Type an ordered pair. Use a comma to separate answers as needed.)

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7. $\frac{x^n}{n!}$

$\frac{14x^{n+1}}{n!}$

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8. $\left(\frac{3}{5},0\right)$, $\left(0,\sqrt{\frac{6}{7}}\right)$, $\left(0,-\sqrt{\frac{6}{7}}\right)$

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9. 0

   (1) less than 1,

   (2) the series converges absolutely.

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10. D. $\frac{\partial^2 f}{\partial x^2} = 2 - e^{x+y}$; $\frac{\partial^2 f}{\partial y^2} = -e^{x+y}$; $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} = -e^{x+y}$

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11. y first

    D. When differentiating the function with respect to y first, I treated x as a constant which eliminiated the second term.

    True

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12. A. $\frac{1}{3}x^3 + 6x$
