Spring 2018 Midterm#1 Question

3. For each limit, calculate the value or show that it does not exist. Show all work.

(c) $\lim_{x \to 1} \left(\frac{5 - \sqrt{32 - 7x}}{x - 1} \right)$

try DSP $\Rightarrow \frac{5 - \sqrt{32 - 7 \cdot 1}}{1 - 1} = \frac{5 - \sqrt{25}}{0} = \text{"}\frac{0}{0}\text{"}$ indeterminate form

conjugate of $5 - \sqrt{32-7x}$ is $5 + \sqrt{32-7x}$

$(a-b)(a+b) = a^2 - b^2$

$$\lim_{x \to 1} \left(\frac{5 - \sqrt{32-7x}}{x-1} \right) \cdot \left(\frac{5 + \sqrt{32-7x}}{5 + \sqrt{32-7x}} \right)$$

$$\lim_{x \to 1} \left(\frac{5^2 - (\sqrt{32-7x})^2}{(x-1)(5+\sqrt{32-7x})} \right) = \lim_{x \to 1} \left(\frac{25 - (32-7x)}{(x-1)(5+\sqrt{32-7x})} \right)$$

$-7 + 7x = 7x - 7 = 7(x-1)$

$$\lim_{x \to 1} \left(\frac{25 - 32 + 7x}{(x-1)(5+\sqrt{32-7x})} \right) = \lim_{x \to 1} \left(\frac{7\cancel{(x-1)}}{\cancel{(x-1)}(5+\sqrt{32-7x})} \right)$$

$$\lim_{x \to 1} \left(\frac{7}{5+\sqrt{32-7x}} \right) \overset{\text{"DSP"}}{=} \frac{7}{5+\sqrt{32-7}} = \frac{7}{5+\sqrt{25}} = \frac{7}{10}$$

2.4: Infinite Limits

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An infinite limit occurs when function values increase or decrease without bound near a point.

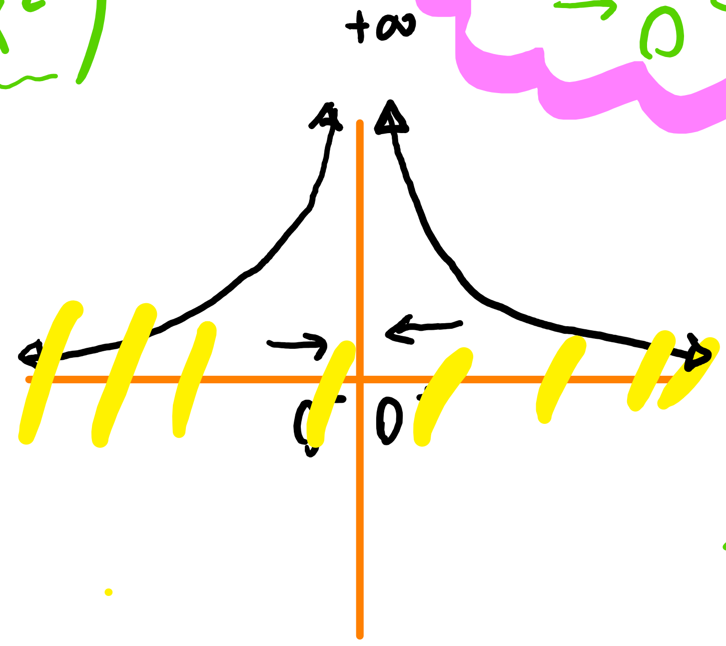
0^- 0^+

$\rightarrow 0 \leftarrow$

Example: $\lim_{x \to 0} \left(\frac{1}{x^2}\right)$

x	$f(x)$
± 0.1	100
± 0.01	$10{,}000$
± 0.001	$1{,}000{,}000$
$\downarrow$	$\downarrow$
0	$+\infty$

Numerically



Graphically

LL
RL

$$\frac{1}{(-0.1)^2} = \frac{1}{(+0.1)^2}$$

Calculus / limit way: try DSP

$$\lim_{x \to 0} \left(\frac{1}{x^2}\right)$$

$$\frac{1}{x^2} = \frac{\text{non-zero pos. \#}}{\text{pos. \# app. } 0} = \infty$$

(app. 0)

$x \to 0 \quad x = 10^{-20}$

$$f(x) = \frac{1}{x^2} \qquad f(10^{-20}) = \frac{1}{(10^{-20})^2} = \frac{1}{10^{-40}} = 10^{40}$$

$$\downarrow$$

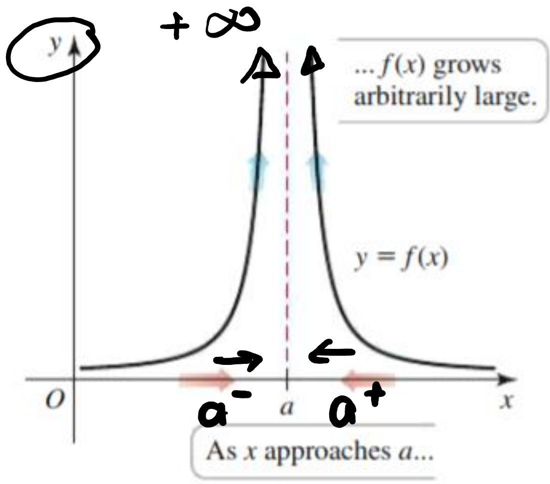
$$\infty$$

DEFINITION Infinite Limits

Suppose f is defined for all x near a. If $f(x)$ grows arbitrarily large for all x sufficiently close (but not equal) to a (Figure 2.24a), we write

$$\lim_{x \to a} f(x) = \infty$$

and say the limit of $f(x)$ as x approaches a is infinity.



(a)

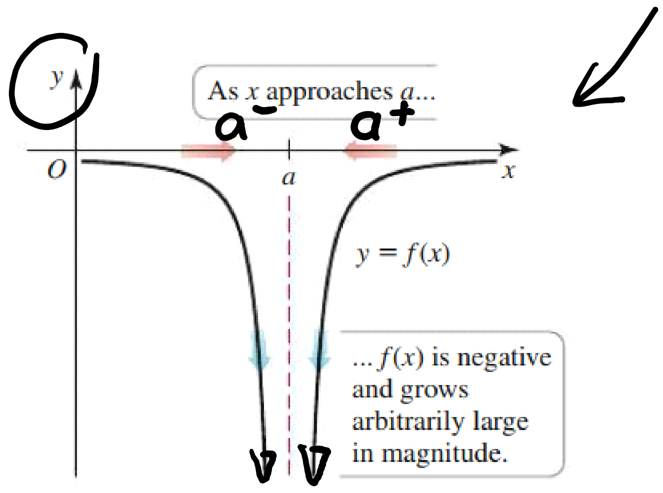
LL RL

DEFINITION Infinite Limits

If $f(x)$ is negative and grows arbitrarily large in magnitude for all x sufficiently close (but not equal) to a (Figure 2.24b), we write

$$\lim_{x \to a} f(x) = -\infty$$

and say the limit of $f(x)$ as x approaches a is negative infinity. *In both cases, the limit does not exist.*



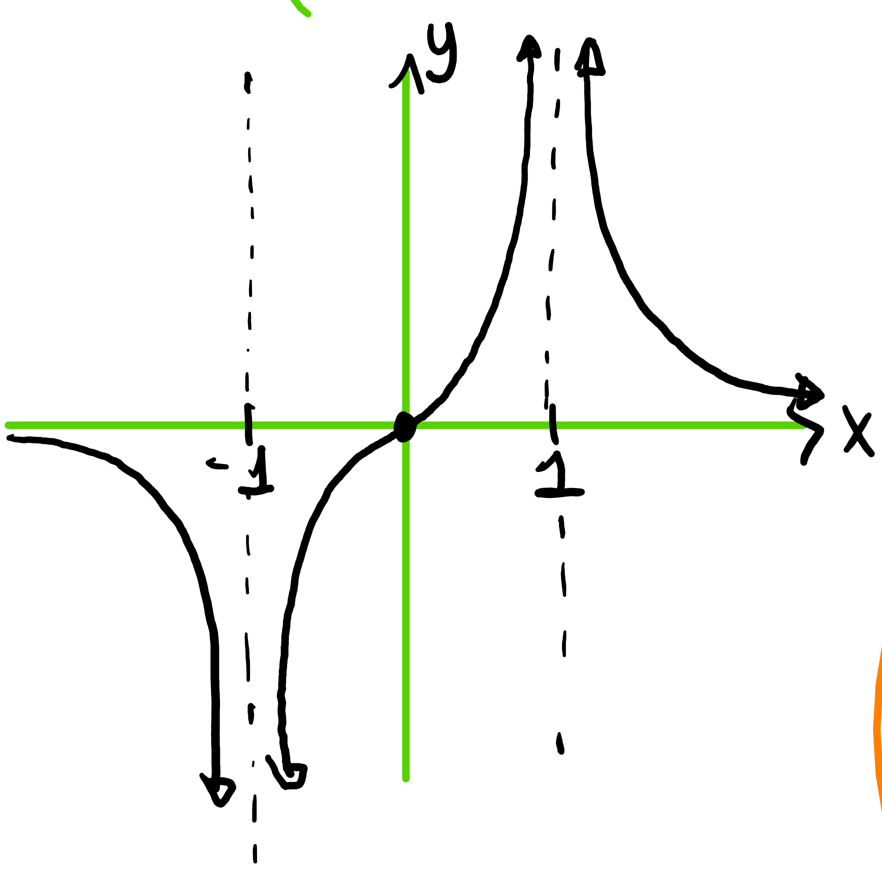
(b)

$-\infty$

EXAMPLE 1 Infinite limits Analyze $\lim_{x \to 1} \frac{x}{(x^2 - 1)^2}$ and $\lim_{x \to -1} \frac{x}{(x^2 - 1)^2}$ using the graph of the function.

$$\lim_{x \to 1} \frac{x}{((x-1)(x+1))^2}$$

V.A → $x = \mp 1$



"DSP"

$$\lim_{x \to 1} \frac{x}{((x-1)(x+1))^2} = \frac{1}{0^+}$$

$0, 0^+$ $\quad 2$

$x \to 1^+, \ x \to 1^-$

1^- $\quad 1$ $\quad 1^+$

0.999 $\quad$ 1.0001

"pos. #"

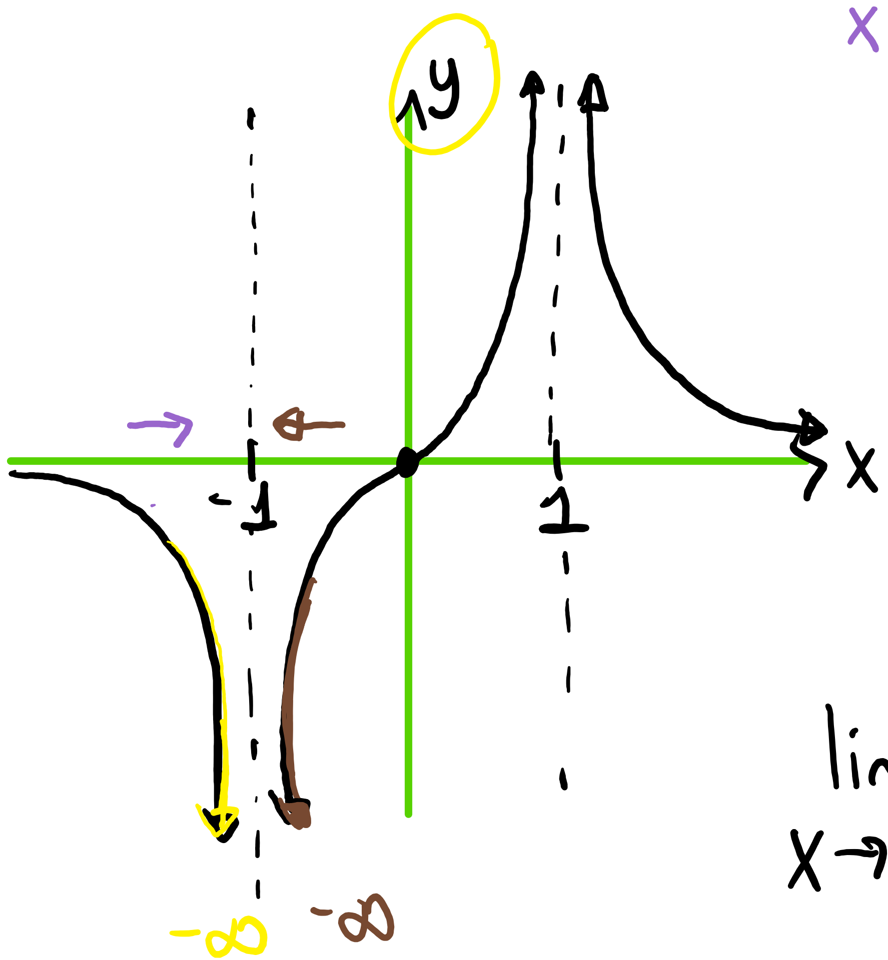
$$\frac{1}{0^+} = \infty$$

"pos. # approaches 0"

EXAMPLE 1 **Infinite limits** Analyze $\lim_{x\to 1} \frac{x}{(x^2-1)^2}$ and $\lim_{x\to -1} \frac{x}{(x^2-1)^2}$ using the graph of the function.

$\infty \neq -\infty$

$x \to -1^-$, $x \to -1^+$



$$\lim_{x\to -1} \frac{x}{(x^2-1)^2} = -\infty$$

-1^- -1^+ x

-1

try DSP

$x \to -1$ (neg. #)

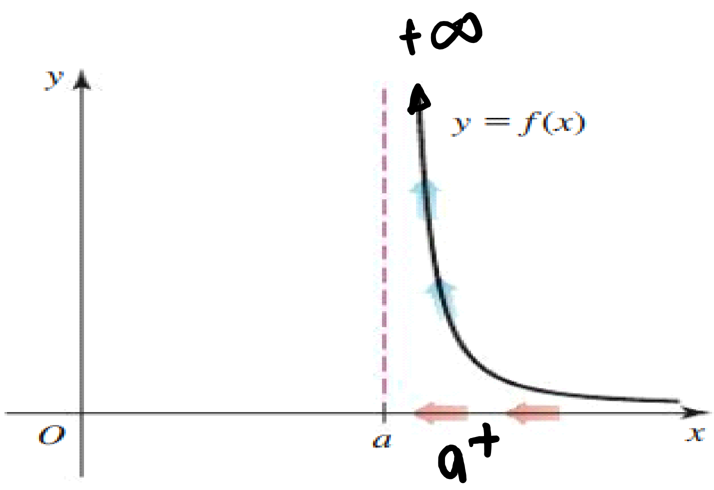
$$\lim_{x\to -1} \frac{x}{\underbrace{(x^2-1)^2}} = -\infty$$

pos. #
app. 0

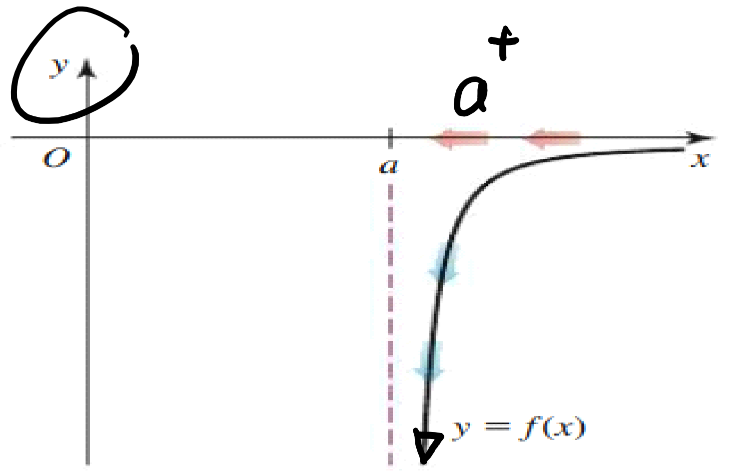
DEFINITION One-Sided Infinite Limits

Suppose f is defined for all x near a with $x > a$. If $f(x)$ becomes arbitrarily large for all x sufficiently close to a with $x > a$, we write $\lim_{x \to a^+} f(x) = \infty$ (Figure 2.26a).

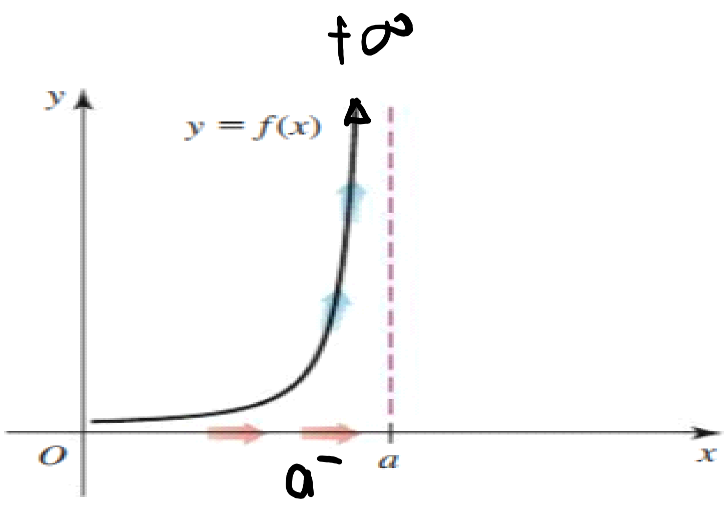
The one-sided infinite limits $\lim_{x \to a^+} f(x) = -\infty$ (Figure 2.26b), $\lim_{x \to a^-} f(x) = \infty$ (Figure 2.26c), and $\lim_{x \to a^-} f(x) = -\infty$ (Figure 2.26d) are defined analogously.



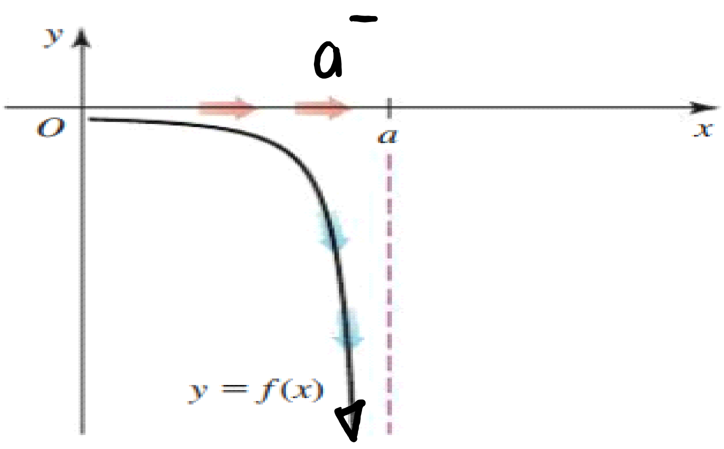
$$\lim_{x \to a^+} f(x) = \infty$$



$$\lim_{x \to a^+} f(x) = -\infty$$



$$\lim_{x \to a^-} f(x) = \infty$$



$$\lim_{x \to a^-} f(x) = -\infty$$

Vertical Asymptote

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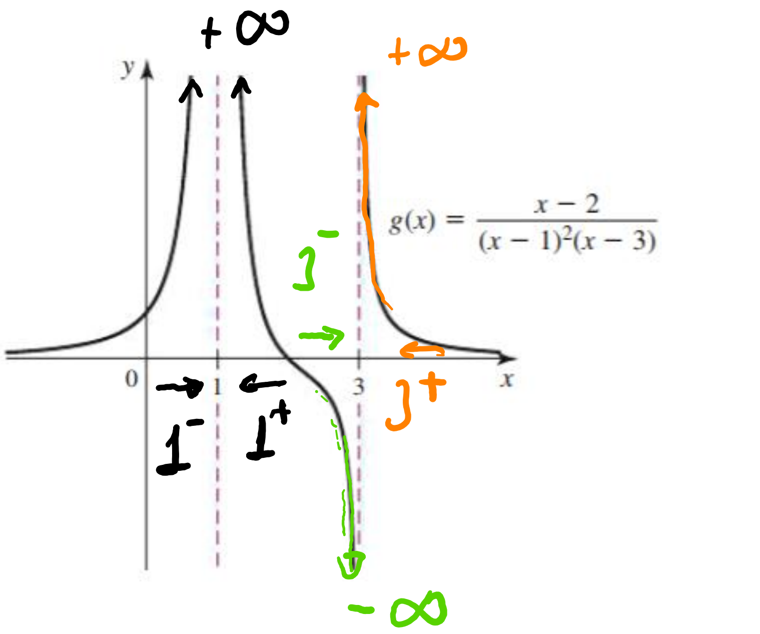
DEFINITION Vertical Asymptote

If $\lim_{x \to a} f(x) = \pm\infty$, $\lim_{x \to a^+} f(x) = \pm\infty$, or $\lim_{x \to a^-} f(x) = \pm\infty$, the line $x = a$ is called a **vertical asymptote** of f.

EXAMPLE 2 Determining limits graphically The vertical lines $x = 1$ and $x = 3$ are vertical asymptotes of the function $g(x) = \dfrac{x-2}{(x-1)^2(x-3)}$. Use Figure 2.27 to analyze the following limits.

a. $\lim_{x \to 1} g(x)$ **b.** $\lim_{x \to 3^-} g(x)$ **c.** $\lim_{x \to 3} g(x)$



a. $\lim_{x \to 1} g(x) = \infty = \lim_{x \to 1^-} g(x) = \lim_{x \to 1^+} g(x)$

b. $\lim_{x \to 3^-} g(x) = -\infty$

c. $\lim_{x \to 3} g(x)$ (check LL, RL)

$\lim_{x \to 3^+} g(x) = +\infty$

since $\lim_{x \to 3^-} g(x) \neq \lim_{x \to 3^+} g(x)$; $\lim_{x \to 3} g(x)$ DNE

2.999... 3.00001

$3^- \rightarrow \; 3 \; \leftarrow 3^+$

EXAMPLE 3 **Determining limits analytically** Analyze the following limits.

a. $\lim_{x\to 3^+} \frac{2-5x}{x-3}$ **b.** $\lim_{x\to 3^-} \frac{2-5x}{x-3}$

a. $\lim_{x\to 3^+}\left(\frac{2-5x}{x-3}\right) = \lim_{x\to 3^+}\left(\frac{\overbrace{2-5\cdot 3}^{\text{app. } -13}}{x-3}\right) = \text{"}\frac{\text{neg. non-zero \#}}{\text{pos. \# app. } 0}\text{"}$

$3^+ - 3 \rightarrow 0^+$

$= -\infty$

b. $\lim_{x\to 3^-}\left(\frac{2-5x}{x-3}\right) = \lim_{x\to 3^-}\left(\frac{\overbrace{2-5\cdot 3}^{\text{app. } -13}}{\underbrace{x-3}_{\text{neg. \# app. } 0}}\right) = \text{"}\frac{\text{neg. non-zero\#}}{\text{neg. \# app. } 0}\text{"}$

3^-

$2.999 \rightarrow 3$

$= +\infty$

c. $\lim_{x\to 3}\left(\frac{2-5x}{x-3}\right)$ DNE LL $\neq$ RL $(+\infty \neq -\infty)$

EXAMPLE 4 **Determining limits analytically** Analyze $\lim_{x\to -4^+} \dfrac{-x^3 + 5x^2 - 6x}{-x^3 - 4x^2}$.

$x=0$
$x+4=0$
$x \neq -4$

$$\lim_{x\to -4^+} \frac{\cancel{+x}(x^2-5x+6)}{+x^{\cancel{2}}(x+4)} = \lim_{x\to -4^+} \frac{(x-3)(x-2)}{x(x+4)}, \quad x\neq 0$$

-4^+
-3.9999

$-4 \quad -3 \quad -2 \quad -1 \quad 0$

approaches $(-4-3)(-4-2)=(-7)\cdot(-6)=42$ ($+$ #)

$$\lim_{x\to -4^+} \frac{(x-3)(x-2)}{x(x+4)} = -\infty$$

neg. # $\quad 0^+$ (pos. # app. 0)

sign matters

∞

$$0.00\frac{1}{10^{-20}} \quad -1 \quad -20$$

EXAMPLE 5 **Location of vertical asymptotes** Let $f(x) = \dfrac{x^2 - 4x + 3}{x^2 - 1}$. Determine the following limits and find the vertical asymptotes of f. Verify your work with a graphing utility.

a. $\lim_{x \to 1} f(x)$ **b.** $\lim_{x \to -1^-} f(x)$ **c.** $\lim_{x \to -1^+} f(x)$

"DSP"

a. $\lim_{x \to 1} \dfrac{(x-3)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}} = \dfrac{1-3}{1+1} = \dfrac{-2}{2} = -1$

$x = 1$

$f(1)$ is undefined ; $x = 1$ is NOT a V.A of f.

app. -4 (neg. #)

b. $\lim_{x \to -1^-} \dfrac{(x-3)}{(x+1)}$ $\Rightarrow +\infty$

neg. # app. 0

-1.0001 -0.999 -2 -1 0

app. -4 (neg. #)

c. $\lim_{x \to -1^+} \dfrac{(x-3)}{(x+1)} = -\infty$

pos. # app. 0