## 2.6 Group Activity Problems


**1.** Which of the following functions are continuous for all values in their domain? Justify your answers.

**a.** $a(t) =$ altitude of a skydiver $t$ seconds after jumping from a plane

**b.** $n(t) =$ number of quarters needed to park legally in a metered parking space for $t$ minutes

**2.6.1**

a. $a(t)$ is a continuous function during the time period from when she jumps from the plane and when she touches down on the ground, because her position is changing continuously with time.

b. $n(t)$ is not a continuous function of time. The function "jumps" at the times when a quarter must be added.

**Determine whether the following functions are continuous at *a*.**

**19.** $f(x) = \sqrt{x - 2};\ a = 1$

**2.6.19** $f$ is discontinuous at 1, because 1 is not in the domain of $f$; $f(1)$ is not defined.

**74.** $$g(x) = \begin{cases} \dfrac{x^3 - 5x^2 + 6x}{x - 2} & \text{if } x \neq 2 \\ -2 & \text{if } x = 2 \end{cases};\ a = 2$$

**74** Observe that $g(2) = -2$ and $\lim_{x\to 2} g(x) = \lim_{x\to 2} \frac{x^3 - 5x^2 + 6x}{x - 2} = \lim_{x\to 2} \frac{x(x-2)(x-3)}{x-2} = \lim_{x\to 2} x(x-3) = -2.$ Therefore $g$ is continuous at $x = 2$.

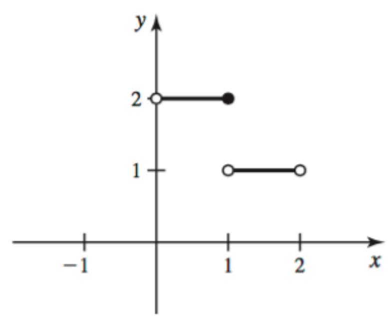
**24.** $f(x) = \begin{cases} \dfrac{x^2 + x}{x + 1} & \text{if } x \neq -1 \\ 2 & \text{if } x = -1 \end{cases}; a = -1$

**2.6.24** $f$ is discontinuous at $-1$ because $\lim_{x\to-1} f(x) = \lim_{x\to-1} \frac{x(x+1)}{x+1} = \lim_{x\to-1} x = -1 \neq f(-1) = 2.$

**81.** Sketch the graph of a function that is continuous on $(0, 1]$ and on $(1, 2)$ but is not continuous on $(0, 2)$.

**81**

One such possible graph is pictured to the right.



**87. An unknown constant** Let

$$g(x) = \begin{cases} x^2 + x & \text{if } x < 1 \\ a & \text{if } x = 1 \\ 3x + 5 & \text{if } x > 1. \end{cases}$$

**a.** Determine the value of $a$ for which $g$ is continuous from the left at 1.

**b.** Determine the value of $a$ for which $g$ is continuous from the right at 1.

**c.** Is there a value of $a$ for which $g$ is continuous at 1? Explain.

**2.6.87**

a. In order for $g$ to be continuous from the left at $x = 1$, we must have $\lim_{x\to 1^-} g(x) = g(1) = a$. We have $\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-} (x^2 + x) = 2$. So we must have $a = 2$.

b. In order for $g$ to be continuous from the right at $x = 1$, we must have $\lim_{x\to 1^+} g(x) = g(1) = a$. We have $\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+} (3x + 5) = 8$. So we must have $a = 8$.

c. Because the limit from the left and the limit from the right at $x = 1$ don't agree, there is no value of $a$ which will make the function continuous at $x = 1$.

**79. Determining unknown constants** Let

$$g(x) = \begin{cases} 5x - 2 & \text{if } x < 1 \\ a & \text{if } x = 1 \\ ax^2 + bx & \text{if } x > 1. \end{cases}$$

Determine values of the constants $a$ and $b$, if possible, for which $g$ is continuous at $x = 1$.

**79** In order for $g$ to be left continuous at 1, it is necessary that $\lim_{x\to 1^-} g(x) = g(1)$, which means that $a = 3$. In order for $g$ to be right continuous at 1, it is necessary that $\lim_{x\to 1^+} g(x) = g(1)$, which means that $a + b = 3 + b = 3$, so $b = 0$.