

### 3.1 Group Activity Problems



**DEFINITION Rate of Change and the Slope of the Tangent Line**

The **average rate of change** in  $f$  on the interval  $[a, x]$  is the slope of the corresponding secant line:

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}.$$

The **instantaneous rate of change** in  $f$  at  $a$  is

$$m_{\text{tan}} = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}, \tag{1}$$

which is also the **slope of the tangent line** at  $(a, f(a))$ , provided this limit exists. The **tangent line** is the unique line through  $(a, f(a))$  with slope  $m_{\text{tan}}$ . Its equation is

$$y - f(a) = m_{\text{tan}}(x - a).$$

**ALTERNATIVE DEFINITION Rate of Change and the Slope of the Tangent Line**

The **average rate of change** in  $f$  on the interval  $[a, a + h]$  is the slope of the corresponding secant line:

$$m_{\text{sec}} = \frac{f(a + h) - f(a)}{h}.$$

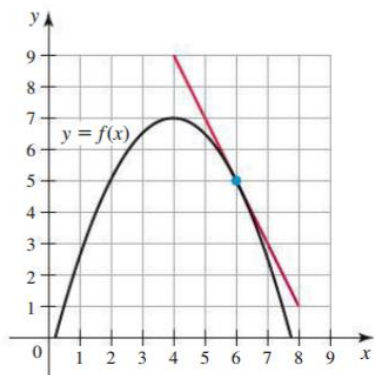
The **instantaneous rate of change** in  $f$  at  $a$  is

$$m_{\text{tan}} = \lim_{h \rightarrow 0} \frac{f(a + h) - f(a)}{h}, \tag{2}$$

which is also the **slope of the tangent line** at  $(a, f(a))$ , provided this limit exists.

4. Explain the relationships among the slope of a tangent line, the instantaneous rate of change, and the value of the derivative at a point.

6. The following figure shows the graph of  $f$  and a line tangent to the graph of  $f$  at  $x = 6$ . Find  $f(6)$  and  $f'(6)$ .



8. An equation of the line tangent to the graph of  $g$  at  $x = 3$  is  $y = 5x + 4$ . Find  $g(3)$  and  $g'(3)$ .
9. If  $h(1) = 2$  and  $h'(1) = 3$ , find an equation of the line tangent to the graph of  $h$  at  $x = 1$ .

Use the definition (1) and definition (2) to:

- Find the slope of the line tangent to the graph of  $f$  at  $P$ .
- Determine an equation of the tangent line at  $P$ .
- Plot the graph of  $f$  and the tangent line at  $P$ .

**Model Question – Q18 (Refer to the problem-solving session)**

18.  $f(x) = \frac{4}{x^2}; P(-1, 4)$

**You Try It!**

15.  $f(x) = x^2 - 5; P(3, 4)$

Determine which definition (1 or 2) is more practical to use for the following question and then solve the problem.

- a) Find the slope of the line tangent to the graph of  $f$  at  $P$ .
- b) Determine an equation of the tangent line at  $P$ .

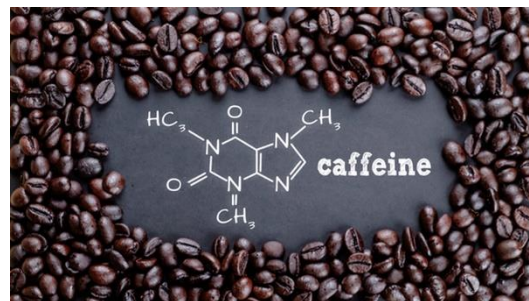
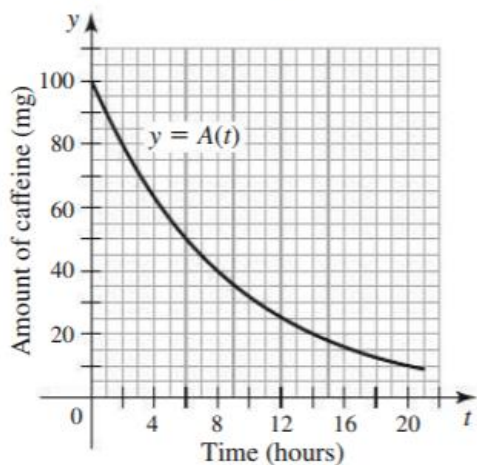
30.  $f(x) = \sqrt{x - 1}; P(2, 1)$

### 33–42. Derivatives and tangent lines

- a. For the following functions and values of  $a$ , find  $f'(a)$ .
- b. Determine an equation of the line tangent to the graph of  $f$  at the point  $(a, f(a))$  for the given value of  $a$ .

37.  $f(x) = \frac{1}{\sqrt{x}}; a = \frac{1}{4}$

54. **Caffeine levels** Let  $A(t)$  be the amount of caffeine (in mg) in the bloodstream  $t$  hours after a cup of coffee has been consumed (see figure). Estimate the values of  $A'(7)$  and  $A'(15)$ , rounding answers to the nearest whole number. Include units in your answers and interpret the physical meaning of these values.



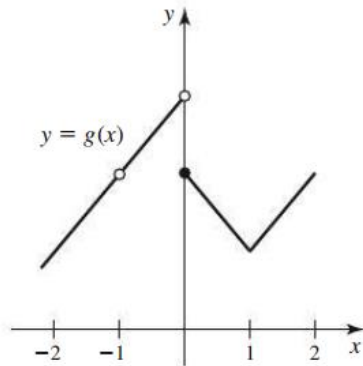
### 3.2 Group Activity Problems

6. Sketch a graph of a function  $f$ , where  $f(x) > 0$  and  $f'(x) < 0$  for all  $x$  in  $(0, 2)$ .

9. Describe the graph of  $f$  if  $f(0) = 1$  and  $f'(x) = 3$ , for  $-\infty < x < \infty$ .

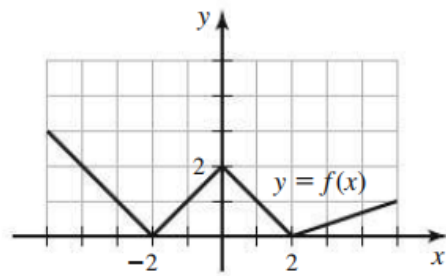
20. Use the graph of  $g$  in the figure to do the following.

- Find the values of  $x$  in  $(-2, 2)$  at which  $g$  is not continuous.
- Find the values of  $x$  in  $(-2, 2)$  at which  $g$  is not differentiable.



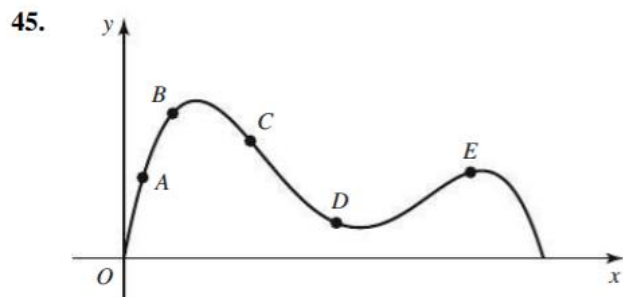
17–18. **Sketching derivatives** Reproduce the graph of  $f$  and then sketch a graph of  $f'$  on the same axes.

17.



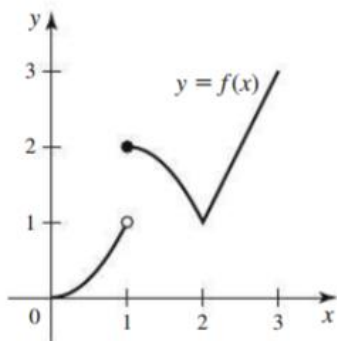
**45–46. Analyzing slopes** Use the points  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  in the following graphs to answer these questions.

- At which points is the slope of the curve negative?
- At which points is the slope of the curve positive?
- Using  $A$ – $E$ , list the slopes in decreasing order.



**53. Where is the function continuous? Differentiable?** Use the graph of  $f$  in the figure to do the following.

- Find the values of  $x$  in  $(0, 3)$  at which  $f$  is not continuous.
- Find the values of  $x$  in  $(0, 3)$  at which  $f$  is not differentiable.
- Sketch a graph of  $f'$ .



## 21–30. Derivatives

- a. Use limits to find the derivative function  $f'$  for the following functions  $f$ .
- b. Evaluate  $f'(a)$  for the given values of  $a$ .

(You may pick one to solve during the recitation, however, complete both problems after the recitation)

26.  $f(x) = \frac{x}{x+2}; a = -1, 0$

28.  $f(w) = \sqrt{4w-3}; a = 1, 3$