3.11 Related Rates Group Activity Problems - Solutions

Procedure for solving related rates prb.

1. Draw a figure, assign variables to quantities that vary (What's not changing (a constant) vs. what's changing (variable))
2. Find a formula or an equation that relates the variables
3. Differentiate the equation (usually implicitly w/ respect to time)
4. Substitute specific values and solve algebraically for any required rate (use correct units)

5. A rectangular swimming pool 10 ft wide by 20 ft long and of uniform depth is being filled with water.

a. If t is elapsed time, h is the height of the water, and V is the volume of the water, find equations relating V to h and dV/dt to dh/dt.

b. At what rate is the volume of the water increasing if the water level is rising at $\frac{1}{4}$ ft/min?

c. At what rate is the water level rising if the pool is filled at a rate of 10 ft^3/min?

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a $V = 200h$, so $\dfrac{dV}{dt} = 200\dfrac{dh}{dt}$.

b. $\dfrac{dV}{dt} = 200 \cdot \dfrac{1}{4} = 50\,\text{ft}^3/\text{min}.$

c. $\dfrac{dh}{dt} = \dfrac{dV/dt}{200} = \dfrac{10}{200} = \dfrac{1}{20}\,\text{ft}/\text{min}.$

8. At all times, the length of the long leg of a right triangle is 3 times the length x of the short leg of the triangle. If the area of the triangle changes with respect to time t, find equations relating the area A to x and dA/dt to dx/dt.

3.11.8 $A = \frac{3}{2}x^2$, so $\frac{dA}{dt} = 3x\frac{dx}{dt}$.

26. Bug on a parabola A bug is moving along the right side of the parabola $y = x^2$ at a rate such that its distance from the origin is increasing at 1 cm/min.

a. At what rate is the x-coordinate of the bug increasing at the point $(2, 4)$?

b. Use the equation $y = x^2$ to find an equation relating $\frac{dy}{dt}$ to $\frac{dx}{dt}$.

c. At what rate is the y-coordinate of the bug increasing at the point $(2, 4)$?

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a. Let s be the distance from the origin to the bug's position $P(x, x^2)$ on the parabola. By the Pythagorean Theorem, $s^2 = x^2 + x^4$. Differentiating with respect to t, we have

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 4x^3\frac{dx}{dt} = 2x(1 + 2x^2)\frac{dx}{dt}.$$

Dividing through by 2 and solving for $\frac{dx}{dt}$ gives

$$\frac{dx}{dt} = \frac{s(ds/dt)}{x(1 + 2x^2)}.$$

When the bug is at $(2, 4)$ we have $s = \sqrt{20} = 2\sqrt{5}$, so

$$\frac{dx}{dt} = \frac{2\sqrt{5}(1)}{2(1 + 2(4))} = \frac{\sqrt{5}}{9} \text{ cm/min}.$$

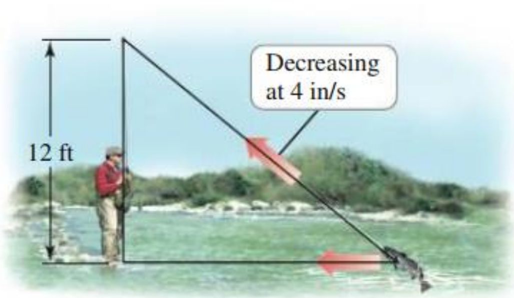
b. Differentiating both sides of $y = x^2$ with respect to t gives

$$\frac{dy}{dt} = 2x\frac{dx}{dt}.$$

c. Using the previous parts of this problem, we have

$$\frac{dy}{dt} = 2(2)\left(\frac{\sqrt{5}}{9}\right) = \frac{4\sqrt{5}}{9} \text{ cm/min}.$$

29. Fishing story An angler hooks a trout and reels in his line at 4 in/s. Assume the tip of the fishing rod is 12 ft above the water and directly above the angler, and the fish is pulled horizontally directly toward the angler (see figure). Find the horizontal speed of the fish when it is 20 ft from the angler.



3.11.29

Let x be the distance between the fish and the fisherman's feet, and let D be the distance between the fish and the tip of the pole. Then $D^2 = x^2 + 144$, so $2D(dD/dt) = 2x(dx/dt)$. Note that $dD/dt = -1/3\,\text{ft/sec}$, so when $x = 20\,\text{ft}$, we have $dx/dt = \sqrt{400+144}/20 \cdot (-1/3) \approx -0.3887\,\text{ft/sec} \approx -4.66\,\text{in/sec}$. The fish is moving toward the fisherman at about 4.66 in/sec.