3.3-3.5, 3.9 Group Activity Problems

DEFINITION Rate of Change and the Slope of the Tangent Line

The **average rate of change** in f on the interval $[a, x]$ is the slope of the corresponding secant line:

$$m_{\text{sec}} = \frac{f(x) - f(a)}{x - a}.$$

The **instantaneous rate of change** in f at a is

$$m_{\text{tan}} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}, \qquad (1)$$

which is also the **slope of the tangent line** at $(a, f(a))$, provided this limit exists. The **tangent line** is the unique line through $(a, f(a))$ with slope m_{tan}. Its equation is

$$y - f(a) = m_{\text{tan}}(x - a).$$

ALTERNATIVE DEFINITION Rate of Change and the Slope of the Tangent Line

The **average rate of change** in f on the interval $[a, a + h]$ is the slope of the corresponding secant line:

$$m_{\text{sec}} = \frac{f(a + h) - f(a)}{h}.$$

The **instantaneous rate of change** in f at a is

$$m_{\text{tan}} = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}, \qquad (2)$$

which is also the **slope of the tangent line** at $(a, f(a))$, provided this limit exists.

DIFFERENTIATION RULES

1. *Constant Rule:* If $f(x) = c$ (c constant), then $f'(x) = 0$.
2. *Power Rule:* If r is a real number, $\frac{d}{dx}x^r = rx^{r-1}$
3. *Constant Multiple Rule:* $\frac{d}{dx}(c \cdot f(x)) = c \cdot f'(x)$
4. *Sum Rule:* $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
5. *Product Rule:* $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$
6. *Quotient Rule:* $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

Dr. Tabanli's Spring 2020 Exam#1 Question

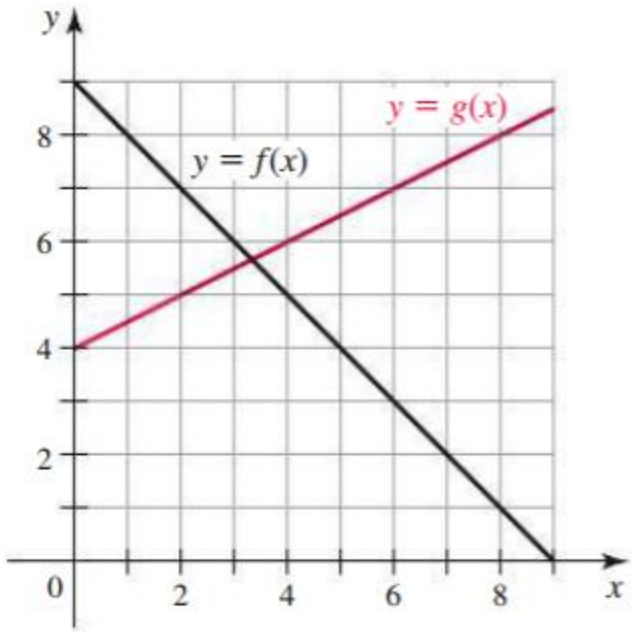
14. For both parts of this problem let $f(x) = 3x^2 - 4x + 2$.

(a) (2 points) Calculate $f'(4)$ by using derivative rules to receive full credit.

(b) (8 points) Calculate $f'(4)$ by using the limit definition of derivative and proper notation to receive full credit. If you simply quote a derivative rule without using the limit definition, you will receive no credit.

8. If $f'(0) = 6$ and $g(x) = f(x) + e^x + 1$, find $g'(0)$.

9–11. *Let* $F(x) = f(x) + g(x)$, $G(x) = f(x) - g(x)$, *and* $H(x) = 3f(x) + 2g(x)$, *where the graphs of f and g are shown in the figure. Find each of the following.*



9. $F'(2)$ **10.** $G'(6)$ **11.** $H'(2)$

18. The line tangent to the graph of f at $x = 3$ is $y = 4x - 2$ and the line tangent to the graph of g at $x = 3$ is $y = -5x + 1$. Find the values of $(f + g)(3)$ and $(f + g)'(3)$.

66. Finding slope locations Let $f(x) = 2e^x - 6x$.

a. Find all points on the graph of f at which the tangent line is horizontal.

b. Find all points on the graph of f at which the tangent line has slope 12.

77. Tangent line given Determine the constants b and c such that the line tangent to $f(x) = x^2 + bx + c$ at $x = 1$ is $y = 4x + 2$.

74. Tangent lines Suppose $f(2) = 2$ and $f'(2) = 3$. Let $g(x) = x^2 f(x)$ and $h(x) = \dfrac{f(x)}{x - 3}$.

a. Find an equation of the line tangent to $y = g(x)$ at $x = 2$.

b. Find an equation of the line tangent to $y = h(x)$ at $x = 2$.

3.4 Group Activity Problems (Drill Questions, you may do it after the recitation)

Find and simplify the derivatives.

38. $y = (2\sqrt{x} - 1)(4x + 1)^{-1}$

54. $f(z) = \left(\frac{z^2 + 1}{z}\right)e^z$

28. $f(x) = e^x \sqrt[3]{x}$

72–73. First and second derivatives *Find* $f'(x)$ *and* $f''(x)$.

72. $f(x) = \frac{x}{x + 2}$

3.5 Group Activity Problems

Dr. Tabanli's Spring 2020 Exam#1 Question

12. Calculate $f'(x)$. After calculating the derivative, do not simplify your answer.

(b) $f(x) = \dfrac{x^2 - 9}{\cos(x - 3)}$

Find and simplify the derivatives.

37. $y = x \cos x \sin x$

66–71. Trigonometric limits *Evaluate the following limits or state that they do not exist. (Hint: Identify each limit as the derivative of a function at a point.)*

68. $\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{6} + h\right) - \frac{1}{2}}{h}$

84. Continuity of a piecewise function Let

$$f(x) = \begin{cases} \frac{3 \sin x}{x} & \text{if } x \neq 0 \\ a & \text{if } x = 0. \end{cases}$$

For what values of a is f continuous?

3.9 Group Activity Problems

Evaluate:

32. $y = \ln(e^x + e^{-x})$

34. $y = e^x x^e$

70. $f(x) = \ln \dfrac{2x}{(x^2 + 1)^3}$

87. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

- **a.** The derivative of $\log_2 9$ is $1/(9 \ln 2)$.
- **b.** $\ln(x + 1) + \ln(x - 1) = \ln(x^2 - 1)$, for all x.
- **c.** The exponential function 2^{x+1} can be written in base e as $e^{2\ln(x+1)}$.