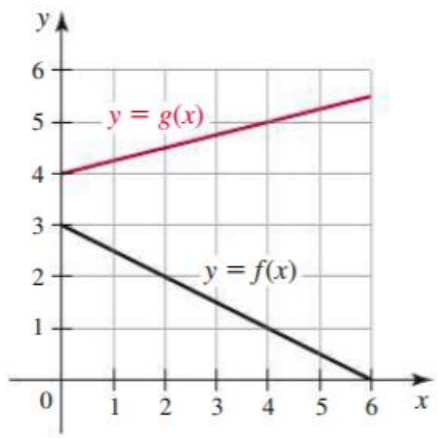
3.7 Group Activity Problems - Solutions

11. Given that $h(x) = f(g(x))$, find $h'(3)$ if $g(3) = 4$, $g'(3) = 5$, $f(4) = 9$, and $f'(4) = 10$.

12. Given that $h(x) = f(g(x))$, use the graphs of f and g to find $h'(4)$.



3.7.11 $h'(3) = f'(g(3))g'(3) = f'(4)g'(3) = 10 \cdot 5 = 50$. Note that the value of $f(4)$ isn't needed to calculate this.

3.7.12 $h'(4) = f'(g(4))g'(4) = f'(5)g'(4) = -\frac{1}{2} \cdot \frac{1}{4} = -\frac{1}{8}$.

Calculate the derivative of the following functions.

41. $y = \sqrt[4]{\dfrac{2x}{4x - 3}}$

46. $y = (\cos x + 2 \sin x)^8$

48. $y = (1 - e^x)^4$

3.7.41 With $u = \dfrac{2x}{4x-3}$ and $f(u) = u^{1/4}$ we have

$$\frac{dy}{dx} = \frac{1}{4}\left(\frac{2x}{4x-3}\right)^{-\frac{3}{4}} \cdot \left(\frac{2(4x-3) - 2x \cdot 4}{(4x-3)^2}\right) = -\frac{3}{2}\left(\frac{4x-3}{2x}\right)^{\frac{3}{4}} \cdot \frac{1}{(4x-3)^2} = -\frac{3}{2^{7/4}x^{3/4}(4x-3)^{5/4}}.$$

3.7.46 Take $g(x) = \cos x + 2\sin x$, and $n = 8$. Then $y' = n(g(x))^{n-1}g'(x) = 8(\cos x + 2\sin x)^7(2\cos x - \sin x)$.

3.7.48 Take $g(x) = 1 - e^x$, and $n = 4$. Then $y' = n(g(x))^{n-1}g'(x) = 4(1-e^x)^3(-e^x) = -4e^x(1-e^x)^3$.

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77. Explain why or why not Determine whether the following statements are true and give an explanation or counterexample.

a. The function $x \sin x$ can be differentiated without using the Chain Rule.

b. The function $e^{\sqrt{x+1}}$ should be differentiated using the Chain Rule.

c. The derivative of a product is *not* the product of the derivatives, but the derivative of a composition is a product of derivatives.

d. $\frac{d}{dx}P(Q(x)) = P'(x)Q'(x)$

3.7.77

a. True. The product rule alone will suffice.

b. True. This function is the composition of e^x with $\sqrt{x+1}$.

c. True. The derivative of the composition $f(g(x))$ is the product of $f'(g(x))$ with $g'(x)$, so it is the product of two derivatives.

d. False. In fact, $\frac{d}{dx}P(Q(x)) = P'(Q(x))Q'(x)$.

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85. Finding slope locations Let $f(x) = xe^{2x}$.

a. Find the values of x for which the slope of the curve $y = f(x)$ is 0.

b. Explain the meaning of your answer to part (a) in terms of the graph of f.

3.7.85

a. The slope is $f'(x) = e^{2x} + 2xe^{2x}$. This is zero when $e^{2x}(1 + 2x) = 0$, which occurs when $x = -\frac{1}{2}$.

b. The graph of f has a horizontal tangent line at $x = -1/2$.